بِسۡمِ اللّٰہِ الرَّحۡمٰنِ الرَّحِیۡمِ

(In the Name of Allah, the Most Merciful, the Most Compassionate.)

PHYSICS 11

PECTAA
Transformation, Innovation and Excellence

PUNJAB EDUCATION, CURRICULUM,
TRAINING AND ASSESSMENT AUTHORITY

Authors

- Prof. Muhammad Ali Shahid (Izaz-i-Fazeelat)
- Prof. Dr. (Rtd.) Ijaz Mujtaba Ghauri
- Mr. Shahzad Hussain
- Prof. (Rtd.) Muhammad Nisar
- Prof. Riaz Muhammad Khan
- Mr. Muhammad Saleem

Editor

- Prof. Dr. Muhammad Nouman Sarwar Qureshi

Reviewers

- Prof. Dr. Riaz Ahmad
- Prof. Muhammad Arshad
- Dr. Mohsin Rafique
- Prof. Muhammad Attiq-us-Salam
- Dr. Azhar Abbas Zaidi
- Mr. Aurangzeb Rehman
- Prof. Nasir Iqbal Sheikh
- Dr. Daniel Yousaf
- Dr. Waris Ali

- Director (Curriculum & Compliance): Aamir Riaz
- Deputy Director (Compliance-Sciences): Syed Saghir-ul-Hassnain Tirmizi
- Subject Coordinator: Abdul Rauf Zahid, Senior Subject Specialist, PECTAA, Lahore
- Incharge Art Cell: Aisha Sadiq
- Composing: Irfan Shahid
- Designing & Layout: Hafiz Inam-ul-Haq
- Illustrations: Hafiz Inam-ul-Haq, Aiyatullah

Experimental Edition

CONTENTS

Chapter 1

Measurements

Learning Objectives

After studying this chapter, the students will be able to:

- Make reasonable estimates of value of physical quantities [of those quantities that are discussed in the topics of this grade].
- Use the conventions for indicating units, as set out in the SI units.
- Express derived units as products or quotients of the SI base units
- Analyze the homogeneity of physical equations [Through dimensional analysis]
- Derive formulae in simple cases [Through using dimensional analysis]
- Analyze and critique the accuracy and precision of data collected by measuring instruments
- Justify why all measurements contain some uncertainty.
- Assess the uncertainty in a derived quantity [By simple addition of absolute, fractional or percentage uncertainties]
- Quote answers with correct scientific notations, number of significant figures and units in all experimental and numerical results.

Physics is the most fundamental branch of physical sciences. It provides the basic principles and laws which help to understand the mysteries of other branches of sciences such as astronomy, chemistry, geology, biology and health sciences. The tools, techniques and products of Physics have transformed our dreams into realities. The comforts and pleasures added in our lives are fruitful results of science, technology and engineering in everyday life.

Think over!

Computer chips are made from silicon, which is obtained from sand. It is up to us whether we make a sand castle or a computer out of it.

The information technology has entirely changed the outlook of mankind. The fast means of communication have brought people of the entire world in so close contact that the whole world has become a global village.

Physics is an experimental science and the scientific method emphasizes the need of accurate measurement of various measurable physical quantities. This chapter stresses in understanding the concept of measuring techniques and recording skills.

1.1 PHYSICAL QUANTITIES AND THEIR UNITS

The foundation of physics depends on physical quantities in terms of which the laws of Physics are expressed. Therefore, these quantities have to be measured accurately. These are mass, length, time, velocity, force, density, temperature, electric current, and numerous others.

Physical quantities are often divided into two categories: base quantities and derived quantities. Derived quantities are those which depend on base quantities. Examples of derived quantities are velocity, acceleration, force, etc. Base quantities are not defined in terms of other physical quantities. The base quantities are the independent physical quantities in terms of which the other physical quantities can be defined. Typical examples of base quantities are length, mass and time.

The measurement of a base quantity involves two steps: first, the choice of a standard, and second, the establishment of a procedure for comparing the quantity to be measured with the standard so that a number and a unit are determined as the measure of that quantity.

Measurements must be reliable and accurate so that they can be used, easily and effectively.

Areas of Physics

Mechanics
Heat & thermodynamics
Electromagnetism
Optics
Sound
Hydrodynamics
Special relativity
General relativity
Quantum mechanics
Atomic physics
Molecular physics
Nuclear physics
Solid state physics
Particle physics
Superconductivity
Superfluidity
Plasma physics
Magnetohydrodynamics
Space physics

Interdisciplinary areas of Physics

Astrophysics
Biophysics
Chemical physics
Engineering physics
Geophysics
Medical physics
Physical oceanography
Physics of music

1.2 INTERNATIONAL SYSTEM OF UNITS

In 1960, an international committee agreed on a set of definitions and standards to describe the physical quantities. The system that was established is called the System International (SI).

SI units are used by the world's scientific community and by almost all nations. The system International (SI) consists of two kinds of units: base units and derived units.

Base Units

There are seven base units for physical quantities namely: length, mass, time, temperature, electric current, light or luminous intensity and amount of substance (with special reference to the number of particles). Prefixes such as milli, micro, kilo, etc. may be used with them to express smaller or larger quantities.

Table 1.1

Physical Quantity	SI Unit	Symbol
Length	metre	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Thermodynamic temperature	kelvin	K
Intensity of light	candela	cd
Amount of substance	mole	mol

The names of base units for these physical quantities together with symbols are listed in Table 1.1.

Derived Units

Derived units are those units which depend on the base units. Some of the derived units are given in Table. 1.2. The units of plane angle and solid angle have also been included in the list of derived units since 1995.

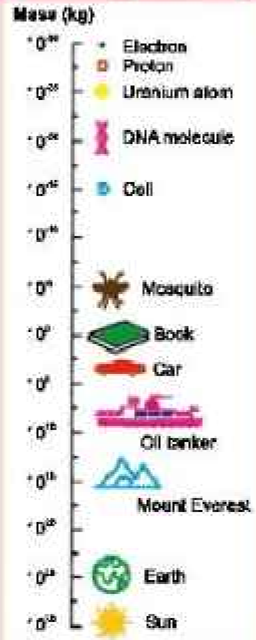
Table 1.2

Physical quantity	Unit	Symbol	In terms of base units
Plane angle	radian	rad	dimensionless
Solid angle	steradian	sr	dimensionless
Force	newton	N	$kg\ m\ s^{-2}$
Work	joule	J	$N\ m = kg\ m^2\ s^{-2}$
Power	watt	W	$J\ s^{-1} = kg\ m^2\ s^{-3}$
Electric charge	coulomb	C	A s
Pressure	pascal	Pa	$N\ m^{-2} = kg\ m^{-1}\ s^{-2}$

In addition to base and derived units, the SI permits the use of certain additional units, including:

- The traditional mathematical units for measuring angles (degree, arcminute, and arcsecond).
- The traditional units for standard time are (minute, hour, day, and year).
- The logarithmic units bel (and its multiples, such as the decibel).
- Two metric units commonly used in ordinary life: the litre for volume and the tonne (metric ton) for large masses.
- Two non-metric scientific units are atomic mass unit (μ) and the electron volt (eV).
- The nautical mile and knot; units traditionally used at sea and in meteorology.
- The acre and hectare, common metric units of land area.
- The bar is a unit of pressure and it is commonly used as the millibar in meteorology and the kilobar in engineering.
- The angstrom and the barn, units used in physics and astronomy.

Interesting Information



Order of magnitude of some masses.

Scientific Notation

Numbers are expressed in standard form called scientific notation, which employs powers of ten. The internationally accepted practice is that there should be only one non-zero digit left of decimal. Thus, the number 134.7 should be written as 1.347×10^2 and 0.0023 be expressed as 2.3×10^{-3}.

Prefixes

Most prefixes indicate order of magnitude in steps of 1000 and provide a convenient way to express large and small numbers, to eliminate non-significant digits. SI also includes four of the other prefixes to accommodate usage already established before the introduction of SI (Table 1.3). They are centi-(10^{-2}), deci-(10^{-1}), deca-(10^{1}) and hecto-(10^{2}).

Table 1.3
Some Prefixes for Powers of Ten

Factor	Prefix	Symbol
10^{-18}	atto	a
10^{-15}	femto	f
10^{-12}	pico	p
10^{-9}	nano	n
10^{-6}	micro	μ
10^{-3}	milli	m
10^{-2}	centi	c
10^{-1}	deci	d
10^{1}	deca	da
10^{2}	hecto	h
10^{3}	kilo	k
10^{6}	mega	M
10^{9}	giga	G
10^{12}	tera	T
10^{15}	peta	P
10^{18}	exa	E

Conventions for Using SI Units

Use of SI units requires special care, more particularly in writing prefixes. Some points to note are:

1. Each SI unit is represented by a symbol not an abbreviation. These symbols are the same in all languages. Hence, correct use of the symbol is very important.

   For example: For ampere, we should use "A" not "amp"; for seconds, "s" not "sec", SI not S.I.

2. Full name of unit does not begin with capital letter.

   For example: newton, metre, etc., except Celsius.

3. Symbols appear in lower case.

   For example: "m" for metre, "s" for second, etc., exception "L" for litre.

4. Symbols named after scientists have initial letters capital.

   For example: "N" for newton, "Pa" for pascal, "W" for watt.

5. Symbols and prefixes are printed in upright (roman) style regardless of the type style in surrounding text.

   For example: a distance of 50 m.

6. Symbols do not take plural form.

   For example: 1 mm, 100 mm, 1 kg, 60 kg.

7. No fullstop or dot is placed after the symbol except at the end of the sentence.

8. Prefix is written before and without space to base unit.

   For example: "mL" not m L or "ms" not m s.

9. Base units are written one space apart. Leave a space even between the number (value) and the symbol.

   For example: 1 kg, 10 m s^{-1}, etc.

For your information

	Interval (s)
Age of the universe	5×10^{17}
Age of the Earth	1.4×10^{17}
One year	3.2×10^{7}
One day	8.6×10^{4}
Time between normal heartbeats	8×10^{-1}
Period of audible sound waves	1×10^{-3}
Period of typical radiowaves	1×10^{-6}
Period of vibration of an atom in a solid	1×10^{-13}
Period of visible light waves	2×10^{-15}

Approximate Values of Some Time Intervals

Do you know?

Mass can be thought of as a form of energy. In effect, the mass is highly concentrated form of energy. Einstein's famous equation, $E=mc^2$ means:

Energy = mass x (speed of light)^2

According to this equation 1 kg mass is actually 9×10^{16} J of energy.

10. Compound prefixes are not allowed:

    For example: 1 μμF should be 1 pF.

11. When base unit of multiple is raised to a power, the power applies to whole multiple and not to base unit alone.

    For example: $1\ km^2 = 1\ (km)^2 = 1 \times (10^3\ m)^2 = 1 \times 10^6\ m^2$.

12. Use negative index notation ($m\ s^{-1}$) instead of solidus (m/s).

13. Use scientific notation, that is, one non-zero digit left of decimal.

    For example: $143.7 = 1.437 \times 10^2$.

14. Do not mix symbols and names in the same expression.

    For example: metre per second or $m\ s^{-1}$, not metre/sec or m/second.

15. Practical work should be recorded in most convenient units depending upon the instruments being used.

    For Example: Measurements using screw gauge should be recorded in mm but the final results must be recorded to the appropriate base units.

16. System International do not allow the use of former CGS System units such as dyne, erg, gauss, poise, torr, etc.

1.3 UNCERTAINTY IN MEASUREMENT

You can count the number of pages of a book exactly but measurement of its length needs some measuring instrument. Every instrument is calibrated to a certain smallest division mark on it and this fact puts a limit regarding its accuracy. When you take a reading with one instrument, its limit of measurement is the smallest division or graduation on its scale. Hence, every measured quantity has some uncertainty about its value. When a measurement is made, it is taken to the nearest graduation or marking on the scale. You can estimate the maximum uncertainty as being one smallest division of the instrument. This is called absolute uncertainty. It is one millimetre on a metre rule that is graduated in millimetres. For example, if one edge of the book coincides with 10.0 cm mark and the other with 33.5 cm, then the length with uncertainty is given by

$$(33.5 \pm 0.05)\ cm - (10.0 \pm 0.05)\ cm = (23{\cdot}5 \pm 0.1)\ cm$$

It means that the true length of the book is in between 23.4 cm and 23.6 cm. Hence, the maximum uncertainty is ±0.05 cm, which is equivalent to an uncertainty of 0.1 cm. Infact, it is equal to least count of the metre rule. Uncertainty may be recorded as:

$$\text{Fractional uncertainty} = \frac{\text{Absolute uncertainty}}{\text{Measured value}}$$

or

$$\text{Percentage uncertainty} = \frac{\text{Absolute uncertainty}}{\text{Measured value}} \times 100\%$$

Uncertainty in Digital Instruments

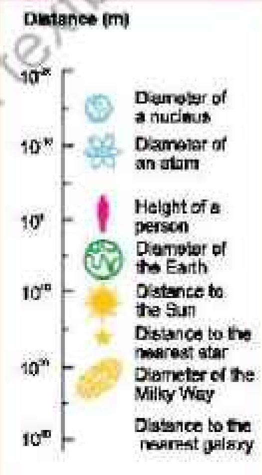
Some modern measuring instruments have a digital scale. We usually estimate one digit beyond what is certain: with a digital scale, this is reflected in some fluctuations of the last digit. If the last digit fluctuates by 1 or 2, write down that last digit. If fluctuation is more than 2 or so in the last digit, it may mean that the reading is being influenced by some factor such as air currents. Regardless of the reason, a large fluctuation may mean that the displayed digit is not really significant.

The indication of uncertainty in a recorded value has been simplified using significant figures. If a measurement is recorded using the knowledge of significant figures, then its last digit, which is an estimation, is an indication of the accuracy of the recorded value.

1.4 USE OF SIGNIFICANT FIGURES

The number of digits of a measurement about which we do feel reasonably sure are called significant figures. Infact, they reflect the use of actual instrument used for that measurement. While using a calculator, the result of any calculation contains many digits after the decimal point. The additional digits may mislead another person who uses those figures into believing them. Hence, they are to be rounded off to the correct number of significant figures. This can be done by keeping in view the uncertainty or the least count of the instrument while recording observations and also quoting results of any calculations to the correct numbers of significant figures. It is better to quote the result in scientific notation to avoid any ambiguity regarding the number of significant figures.

For your information



Order of magnitude of some distances

For example, weighing the same object with different balances:

Electronic balance : mass = 3.145 ± 0.001 g

Lever balance : mass = 3.1 ± 0.1 g

Usually, the uncertainty $\pm$ 0.001 g or $\pm$0.1 g is dropped, and it is understood that the number quoted has an uncertainty of at least 1 unit in the last digit. All digits which are quoted are called significant figures.

> In any measurement, the accurately known digits and the first estimated or doubtful digit are called significant figures.

Proper use of significant figures ensures that we correctly represent the uncertainty of our measurements. For example, scientists immediately realize that the reported mass 3.145 g is more accurate than a reported mass of 3.1 g, reflecting the use of a better or more precise instrument. As we improve the quality of our measuring instrument and techniques, we extend the result to more and more significant figures and correspondingly improve the experimental accuracy of the result.

Working with significant figures

(I) Counting significant digits

(a) All digits 1,2,3,4,5,6,7,8,9 are significant. However, zeros may or may not be significant. In case of zeros, the following rules may be adopted:

(b) A zero between two significant figures is itself significant.

(c) Zeros to the left most significant figure are not significant. For example, none of the zeros in 0.00467 or 02.59 are significant.

(d) Zeros to the right of a significant figure may or may not be significant. In decimal fraction, zeros to the right of a significant figure are significant. For example, all the zeros in 3.570 or 7.4000 are significant. However, in integers such as 8, 000 kg, the number of significant zeros is determined by the precision of the measuring instrument. If the measuring scale has a least count of 1 kg, then there are four significant figures written in scientific notation as 8.000×10^3 kg. If the least count of the scale is 10 kg, then the number of significant figures will be 3 written in scientific notation as 8.00×10^3 kg and so on.

(e) When a measurement is recorded in scientific notation or standard form, the figures other than the powers of ten are significant figures. For example, a measurement recorded as 8.70×10^4 kg has three significant figures.

(II) Multiplying or dividing numbers

Keep a number of significant figures in the product or quotient not more than that contained in the least accurate factor i.e., the factor containing the least number of significant figures. For example, the computation of the following using a calculator, gives

$$\frac{5.348 \times 10^{-2} \times 3.64 \times 10^{4}}{1.336} = 1.45768982 \times 10^{3}$$

As the factor 3.64×10^4, the least accurate in the above calculation has three significant figures, the answer should be written to three significant figures only. The other figures are insignificant and should be deleted. While deleting the figures, the last significant figure to be retained is rounded off for which the following rules are followed:

> **Remember Thumb Rule**
>
> **For calculation of end result:**
> - Addition / Subtraction: same precision.
> - Multiplication / Division: same accuracy (Same number of significant figures).

(a) If the first digit dropped is less than 5, the last digit retained should remain unchanged.

(b) If the first digit dropped is more than 5, the digit to be retained is increased by one.

(c) If the digit to be dropped is 5, the previous digit which is to be retained is increased by one if it is odd and retained as such if it is even. For example, the following numbers are rounded off to three significant figures as follows. The digits are deleted one by one.

43.75	is rounded off as	43.8
56.8546	is rounded off as	56.9
73.650	is rounded off as	73.6
64.350	is rounded off as	64.4

Following this rule, the correct answer of the computation given in section (ii) is 1.46×10^3.

Thumb Rule for Uncertainty

For average value of many readings:

- **Mean deviation from an average value.**
- **Periodic Uncertainty:** Divide least count of timing device by the number of oscillations.

(iii) In adding or subtracting numbers

The number of decimal places retained in the answer should be equal to the smallest number of decimal places in any of the quantities being added or subtracted. In this case, the number of significant figures is not important. It is the position of decimal that matters. For example, suppose we wish to add the following quantities expressed in metres.

	(a)	(b)
	72.1	2.7543
	3.42	4.10
	0.003	1.273
	75.523	8.1273
Correct answer:	75.5 m	8.13 m

Quick Quiz

1. Give the correct number of significant figures for 0.0054 m, 0.03030 m, 40.0 m, 0.6 m, 8.20×10^7 m.
2. Give the answer to the appropriate number of significant figures. 2602 kg + 36.02 kg + 54.1 kg = ?
3. Give the answer to the appropriate number of significant figures. 3.54 kg − 2.4 kg = ?
4. Give the answer to the appropriate number of significant figure. 2.45×10^2 m × 2.46 m / 3.6 m = ?

In case (a), the number 72.1 has the smallest decimal places, thus the answer is rounded off to the same position which is then 75.5 m. In case (b), the number 4.10 has the smallest number of decimal places and hence, the answer is rounded off to the same decimal position which is then 8.13 m.

Limitations of Significant Figures

Significant figures deal with only one source of uncertainties that inherent in reading the scale. Real experimental uncertainties have many contributions, including personal errors and sometimes hidden systematic errors. One cannot do better than that what the scale reading allows, but the total uncertainty may well be more than what the significant figure of the measurements would suggest.

1.5 PRECISION AND ACCURACY

Remember Thumb Rule

- Precision: Less absolute uncertainty.
- Accuracy: Less % age uncertainty.

The terms precision and accuracy are frequently used in physics measurements. They should be distinguished clearly. The precision of a measurement is determined by the instrument or device being used. The smaller the least count the more precise is the measurement. Accuracy is defined as the closeness of a measurement to the exact or accepted value of a physical quantity. It is expressed by the fractional or percentage uncertainty. The smaller the fractional or percentage uncertainty, the more accurate is the measurement.

For example, the length of an object is recorded as 25.5 cm by using a metre rule having smallest division in millimetre. Its precision or absolute uncertainty (least count) = ± 0.1 cm.

$$\text{Fractional uncertainty} = \frac{0.1\ \text{cm}}{25.5\ \text{cm}} = 0.004$$

$$\text{Percentage uncertainty} = \frac{0.1\ \text{cm}}{25.5\ \text{cm}} \times 100\% = 0.4\%$$

Another measurement taken by Vernier Callipers with least count 0.01 cm is recorded as 0.45 cm. It has precision or absolute uncertainty (least count) = ± 0.01 cm.

$$\text{Fractional uncertainty} = \frac{0.01\ \text{cm}}{0.45\ \text{cm}} = 0.02$$

$$\text{Percentage uncertainty} = \frac{0.01\ \text{cm}}{0.45\ \text{cm}} \times 100\% = 2\%$$

Thus, the reading 25.5 cm taken by metre rule is although less precise but is more accurate having less percentage uncertainty or error.

Whereas the reading 0.45 cm taken by Vernier Callipers is more precise but is less accurate. In fact, it is the relative measurement which is important. The smaller a physical quantity, the more precise instrument should be used. Here the measurement 0.45 cm demands that a more precise instrument, such as micrometer screw gauge, with least count 0.001 cm, should have been used. Hence, we can conclude that:

A precise measurement is the one which has less precision or absolute uncertainty and an accurate measurement is the one which has less fractional or percentage uncertainty.

We can never make an exact measurement. The best we can do is to come as close as possible with in the limitation of the measuring instrument.

1.6 ASSESSMENT OF TOTAL UNCERTAINTY IN THE FINAL RESULT

Knowing the uncertainties in all the factors involved in a calculation, the maximum possible uncertainty or error in the final result can be found as follows:

For your information

We use many devices to measure physical quantities, such as length, time, and temperature. They all have some limit of precision.

For your information

These are not decoration pieces of glass but are the earliest known exquisite and sensitive thermometers, built by the Accademia del Cimento (1657-1667), in Florence. They contained alcohol, some times coloured red for easier reading.

1. For addition and Subtraction

Absolute uncertainties are added. For example, the distance between two positions $x_1 = 15.4 \pm 0.1$ cm and $x_2 = 25.6$ cm ± 0.1 cm is recorded as:

$x = x_2 - x_1 = 10.2 \pm 0.2$ cm

and addition of two lengths is:

$\ell_1 = 8.5 \pm 0.1$ cm and $\ell_2 = 12.6 \pm 0.1$ cm recorded as:

$\ell = \ell_1 + \ell_2 = 21.1 \pm 0.2$ cm

For your information

Colour printing uses just four colours cyan, magenta, yellow and black to produce the entire range of colours. All the colours in this book have been made from just these four colours.

2. For multiplication and division

Percentage uncertainties are added. For example, the maximum possible uncertainty in the value of resistance R of a conductor determined by the potential difference V applied across the conductor resulting in current flowing through it is estimated as under:

Let $V = 3.4 \pm 0.1$ V

$I = 0.68 \pm 0.05$ A

using $R = \frac{V}{I}$

Percentage uncertainty in $V = \frac{0.1\text{ V}}{3.4\text{ V}} \times 100\% = 3\%$

Percentage uncertainty in $I = \frac{0.05\text{ A}}{0.68\text{ A}} \times 100\% = 7\%$

Thumb Rule for Total Uncertainty

- **For addition and subtraction:** Absolute uncertainties are added.
- **For multiplication and division:** Percentage uncertainties are added.
- **For power factor:** Power factor x Percentage uncertainty

Hence, total percentage uncertainty in the value of R is $3 + 7 = 10\%$

The value of R will be written as:

$$R = \frac{3.4\text{ V}}{0.68\text{ A}} = 5.0\text{ ohm}$$

Hence, $R = 5.0 \pm 0.5$ ohms, uncertainty being an estimate only, is recorded by one significant figure.

For your information

Travel time of light

Moon to Earth	1 min 20 s
Sun to Earth	8 min 20 s
Pluto to Earth	5 h 20 s

3. For Power Factor

The percentage uncertainty is multiplied by the power factor in the formula. For example, the calculation of cross-sectional area of a cylinder of radius $r = 1.25$ cm using formula for Area $A = \pi r^2$ is given by the %age uncertainty which is $A = 2$ x %age uncertainty in radius r. As uncertainty is multiplied by power factor, it increases the precision demand of measurement. When the radius of a small sphere is measured as 1.25 cm by Vernier Callipers with least count 0.01 cm, then

The radius r is recorded as $r = 1.25 \pm 0.01$ cm

%age uncertainty in radius r is $r = \frac{0.01}{1.25} \times 100\% = 0.8\%$

Total percentage uncertainty in area $A = 2 \times 0.8 = 1.6\%$

Thus $A = \pi r^2$

$= 3.14(1.25)^2 = 4.906\text{ cm}^2$ with 1.6% uncertainty

Thus, the result should be recorded as $A = 4.91 \pm 0.08\ \text{cm}^2$

Example 1.1 The length, breadth and thickness of a metal sheet are 2.03 m, 1.22 m and 0.95 cm respectively. Calculate the volume of the sheet correct up to the appropriate significant digits.

Solution Given Length $\ell = 2.03\ \text{m}$

Breadth $b = 1.22\ \text{m}$

Thickness $h = 0.95\ \text{cm} = 0.95 \times 10^{-2}\ \text{m}$

Volume $V = \ell \times b \times h = 2.03\ \text{m} \times 1.22\ \text{m} \times 0.95 \times 10^{-2}\ \text{m}$

$= 2.35277 \times 10^{-2}\ \text{m}^3$

As the factor 0.95 cm has minimum number of significant figures equal to two, therefore, volume is recorded up to 2 significant figures, hence, $V = 2.4 \times 10^{-2}\ \text{m}^3$

Example 1.2 The mass of a metal box measured by a lever balance is 3.25 kg. Two silver coins of masses 10.01 g and 10.02 g measured by a beam balance are added to it. What is now the total mass of the box correct up to the appropriate precision?

Solution

Total mass when silver coins are added to box $= 3.25\ \text{kg} + 0.01001\ \text{kg} + 0.01002\ \text{kg}$

$= 3.27003\ \text{kg}$

Since least precise mass is 3.25 kg, having two decimal places, hence, total mass should be reported to 2 decimal places which is the appropriate precision.

Thus Total mass = 3.27 kg

Example 1.3 The diameter and length of a metal cylinder measured with the help of Vernier Callipers of least count 0.01 cm are 1.25 cm and 3.35 cm, respectively. Calculate the volume V of the cylinder and uncertainty in it.

Solution Given

Diameter $d = 1.25$ cm with least count 0.01 cm

Length $\ell = 3.35$ cm with least count 0.01 cm

Absolute uncertainty in length $= 0.01$ cm

%age uncertainty in length $= (0.01\ \text{cm} / 3.35\ \text{cm}) \times 100\% = 0.3\%$

Absolute uncertainty in diameter $= 0.01$ cm

%age uncertainty in diameter $= (0.01\ \text{cm} / 1.25\ \text{cm}) \times 100\% = 0.8\%$

As Volume $= \pi r^2 \ell = \pi \dfrac{d^2}{4} \ell$

Total uncertainty in V = 2 (%age uncertainty in diameter) + (%age uncertainty in length)

$= 2 \times 0.8\% + 0.3\% = 1.9\%$

Then $V = 3.14 \times (1.25\ \text{cm})^2 \times 3.35\ \text{cm}/4 = 4.1089842\ \text{cm}^3$ with 1.9 % uncertainty

Thus $V = (4.11 \pm 0.08)\ \text{cm}^3$

where 4.11 cm^3 is calculated volume and 0.08 cm^3 is the uncertainty in it.

1.7 DIMENSIONS OF PHYSICAL QUANTITIES

Any physical quantity can be described by certain familiar properties such as length, mass, time, temperature, electric current, etc. These measurable properties are called dimensions. Dimensions deal with the qualitative nature of a physical quantity in terms of fundamental quantities. The quantities such as length, depth, height, diameter, light year are all measured in metre and denoted by the same dimension, basically known as length given by symbol L written within square brackets [L]. Similarly, the other fundamental quantities, mass, time, electric current and temperature are denoted by specific symbols [M], [T], [A] and [θ], respectively. These five dimensions have been chosen as being basic because they are easy to measure in experiments.

The dimensions of other quantities indicate how they are related to the basic quantities and are combination of fundamental dimensions. For example, speed v is measured in metres per second, so it has the dimensions of length [L] divided by time [T].

$$[v] = [L]/[T] = [L][T^{-1}] = [LT^{-1}]$$

As the acceleration $a = \Delta v / \Delta t$

Dimensions of acceleration are

$$[a] = [v]/[T] = [LT^{-1}]/[T] = [LT^{-2}]$$

Also, dimensions of force can be written as

$$[F] = [m][a] = [M][LT^{-2}] = [MLT^{-2}]$$

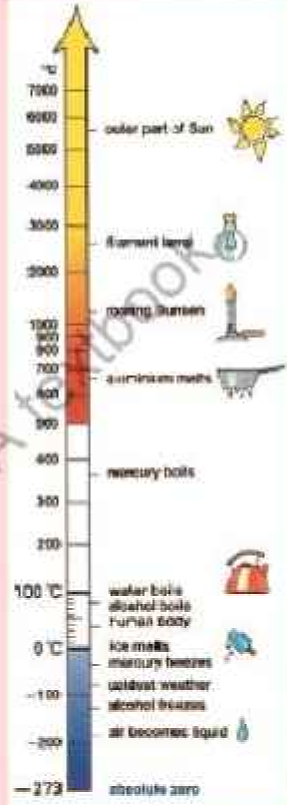
By the use of dimensionality, we can check the homogeneity (correctness) of a physical equation, and also, we can derive formula for a physical quantity.

Homogeneity of Physical Equations

The correctness of an equation can be checked by showing that the dimensions of quantities on both sides of the equation are the same. This is known as principle of homogeneity.

Suppose a car starts from rest ($v_i = 0$) and covers a distance S in time t moving with an acceleration a. The

Interesting Information



Some Specific Temperatures

For your information

Atomic Clock

The cesium atomic frequency standard at the National Institute of Standards and Technology in Colorado (USA). It is the primary standard for the unit of time.

equation of motion is given by,

$$S = v_i t + \frac{1}{2} a t^2$$

or

$$S = \frac{1}{2} a t^2$$

Numerical factors like 1/2 have no dimensions, so they can be ignored. By putting the dimensions of both sides of the equation:

$$[S] = [a][t^2]$$

Writing the symbols of dimensions $[L] = [LT^{-2}][T^2]$

$$[L] = [LT^{-2}T^2]$$

or

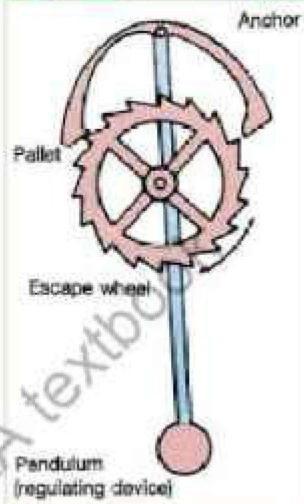
$$[L] = [L]$$

This shows that dimensions on both sides of equation are the same, therefore, the equation is dimensionally correct.

Derivation of a Formula

Dimensionality can be used to derive a possible formula for a physical quantity by correct estimation of various factors on which the quantity depends.

Do you know?



The device which made the pendulum clock practical.

Example 1.4 Derive a formula for the centripetal force required to keep an object moving along a circle with uniform speed. Assuming that centripetal force depends on mass of the object, radius of the circle and uniform speed.

Solution As force depends on mass m of the object, radius r of the circle and uniform speed v, we can write:

$$F \propto m^a v^b r^c$$

$$F = (\text{constant})\, m^a v^b r^c \quad .... \quad (i)$$

where the exponents (powers) a, b and c are to be determined. By the principle of homogeneity, the dimensions on both sides of the equation should be the same. Since, constant has no dimension so by ignoring it, we write the above equation in terms of dimensions as,

$$[F] = [m^a][v^b][r^c]$$

$$[MLT^{-2}] = [M^a][L/T]^b[L]^c$$

$$[MLT^{-2}] = [M^a][LT^{-1}]^b[L]^c$$

$$[MLT^{-2}] = [M^a L^{b+c} T^{-b}] \quad .... \quad (ii)$$

Comparing the powers of dimensions on both sides of the above equation, we have

$$a = 1$$

Beware!

Calculators are designed to yield as many digits as the memory of the calculator chip permits. Hence, be sure to round off the final answers of calculations down to correct number of significant figures.

$$b + c = 1$$
$$-b = -2$$

Solving the above equations, we have $a = 1$, $b = 2$, $c = -1$

Putting the values of a, b and c in equation (i), we have

$$F = (\text{constant})\, mv^2 r^{-1}$$

or

$$F = (\text{constant})\, mv^2/r$$

The numerical value of the constant cannot be determined by dimensional analysis. However, it can be found by experiments. In the above equation, numerical value of the constant happens to be "1", so the equation reduces to:

$$F = mv^2/r$$

Limitations in Dimensional Analysis

The dimensional method cannot identify where an equation is wrong. Even if an equation is proved correct, we can only say the equation might be correct, for the reason that the method does not provide a check on any numerical factor or constant. That can only be determined by experiments or plotting some suitable graph between the variables.

QUESTIONS

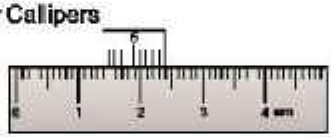
Multiple Choice Questions

Tick (✓) the correct answer.

1.1 The purpose of study and discoveries in Physics is:
- (a) the probing of interstellar spaces
- (b) the betterment of mankind
- (c) the development of destructive technology in warfare
- (d) development in aesthetics for the world

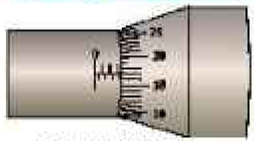
1.2 The length of a steel pipe is in between 0.7 m to 0.8 m. Identify from the following, the appropriate instrument to be used for an accuracy of 0.001 m.
- (a) A micrometer screw gauge
- (b) A metre rule
- (c) A ten metres measuring tape
- (d) A Vernier Callipers

1.3 The diameter of a steel ball is measured using a Vernier Callipers and its reading is shown in the figure. What is the diameter of the steel ball?



- (a) 1.30 cm
- (b) 1.39 cm
- (c) 1.40 cm
- (d) 1.31 cm

1.4 The figure shows the reading on a micrometer screw gauge used to measure diameter of a thin rod. One complete turn of the thimble is 0.50 mm and there are 50 lines on the circular scale. The diameter of the rod is:



(a) 3.67 mm (b) 3.17 mm (c) 4.17 mm (d) 4.20 mm

1.5 The number of significant figures of a measurement are defined as:

(a) they reflect the accuracy of the observation in a measurement

(b) they are the figures which are reasonably reliable

(c) they are the accurately known digits and the first doubtful digit of a measurement

(d) all of the above

1.6 The number of significant figures in the measured mass 2500.0 kg is:

(a) two (b) three (c) four (d) five

1.7 The sum 12 kg + 2.02 kg + 5.1 kg according to appropriate precision is:

(a) 19 kg (b) 19.0 kg (c) 19.1 kg (d) 19.12 kg

1.8 The answer to appropriate precision for the subtraction (1.126-0.97268) is:

(a) 0.15 (b) 0.153 (c) 0.1533 (d) 0.15332

1.9 The answer of the product (2.8723x1.6) to the appropriate number of significant figures is:

(a) 4.59568 (b) 4.595 (c) 4.59 (d) 4.6

1.10 The answer to the mathematical division ($45.2 \div 6.0$) in appropriate number of significant figures is:

(a) 7.5 (b) 7.53 (c) 7.533 (d) 7.5333

1.11 The answer to the following mathematical operation 24.4 m x 100 m / 5.0 m to the appropriate number of significant figures is:

(a) 4880 m (b) 4900 m (c) 4.88×10^3 m (d) 4.9×10^2 m

1.12 The ratio of the dimensions of force and energy is:

(a) T (b) T^{-1} (c) L (d) L^{-1}

1.13 Identify which pair from the following does not have identical dimensions.

(a) Work and torque

(b) Angular momentum and Planck's constant

(c) Moment of inertia and moment of force

(d) Impulse and momentum

1.14 The following figures are of the same Vernier Callipers. Figure (1) shows the reading when the jaws are closed while

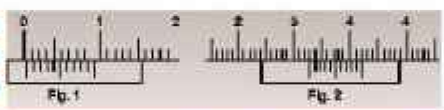


Fig. (2) shows the reading when a solid cylinder is placed between the jaws. The length of the cylinder is:

(a) 3.26 cm (b) 3.30 cm (c) 3.34 cm (d) 4.20 cm

1.15 The least count of an instrument determines:

(a) precision of a measurement
(b) accuracy of a measurement
(c) fractional uncertainty of a measurement
(d) percentage uncertainty of a measurement

1.16 A measuring tape has been graduated with a minimum scale division of 0.2 cm. The allowed reading using this tape may be:

(a) 80.5 cm (b) 80.6 cm (c) 80.65 cm (d) 80.7 cm

Short Answer Questions

1.1 What are base units and derived units? Give some examples of both these units.

1.2 How many significant figures should be retained in the following?
(i) Multiplying or dividing several numbers (ii) Adding or subtracting numbers

1.3 How is the Vernier scale related to the main scale of a Vernier Callipers? What is meant by L.C. of the Vernier Callipers?

1.4 Write the following numbers in scientific notation:
a) 143.7 (b) 206.4×10^{2}

1.5 Write the following numbers using correct prefixes:
(a) 580×10^{2} g (b) 0.46×10^{-3} s

1.6 Kinetic energy of a body of mass m moving with speed v is given by $1/2\ mv^2$. What are the dimensions of kinetic energy?

1.7 How many significant figures are there in the following measurements?
(i) 37 km (ii) 0.002953 m (iii) 7.50034 cm (iv) 200.0 m

1.8 Write the dimensions of: (i) Planck's constant (ii) angular velocity

Constructed Response Questions

1.1 Why do we find it useful to have two units for the amount of a substance, the kilogram and the mole?

1.2 Three students measured the length of a rod with a scale on which minimum division is 1 mm and recorded as: (i) 0.4235 m (ii) 0.42 m (iii) 0.424 m. Which record is correct and why?

1.3 Why is the kilogram (not the gram), the base unit of mass.

1.4 Consider the equation; $P = Q + R$
If Q and R both have the dimensions of [MLT], what are the dimensions of P? What

are the units of P in SI? If the dimensions of Q were different from those of R, could we determine dimensions of P?

1.5 What is the least count of a clock if it has:

(a) Hour's hand, minute's hand and second's hand

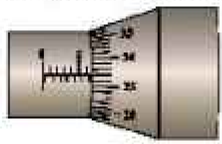
(b) Hour's hand and minute's hand

1.6 How can the diameter of a round pencil be measured using metre rule with the same accuracy as that of Vernier Callipers? Describe.

1.7 How would be the readings differ if the screw gauge is used instead of a Vernier Callipers to measure the thickness of a glass plate?

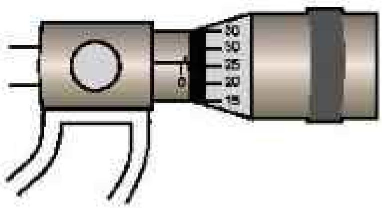
1.8 Write the correct reading of the length of a solid cylinder as shown in the figure if there is an error of +0.02 cm in the Vernier Callipers.

1.9 There are 50 divisions on the circular scale of a screw gauge. If the head (thimble) of the screw is given 10 revolutions, then the spindle advances by 5 mm. There is also zero error as the 2nd division of the circular scale coincides with the datum line and zero of circular scale is below the datum line. What is the thickness of a glass slab as measured by the described screw gauge shown in the figure?



1.10 What is meant by a dimensionless quantity? Give one example.

1.11 A student uses a screw gauge to determine the thickness of a sheet of paper. The student folds the paper three times and measures the total thickness of the folded sheet. Assume that there is no zero error in the screw gauge. The reading of screw gauge is shown in the figure. Find the thickness of the sheet.



1.12 Round off each of the following numbers to 3 significant figures and write your answer in scientific notation.

(a) 0.02055 (b) 4656.5

Comprehensive Questions

1.1 What is meant by uncertainty in a measurement? How the uncertainty in a digital instrument is indicated?

1.2 Differentiate between the terms precision and accuracy with reference to measurement of physical quantities.

1.3 (a) What is meant by significant figures? Write two reasons for using them in measurements. How to find the uncertainty in a timing experiment such as the time period of a simple pendulum?

(b) The mass of a solid cylinder is 12.85 g. Its length is 3.35 cm and diameter is 1.25 cm. Find the density of its material expressing the uncertainty in the density.

1.4 Explain with examples the writing of physical quantities into their dimensions. Write its two benefits.

1.5 Check the homogeneity of the relation:

$$v = \sqrt{\frac{T \times \ell}{m}}$$

where v is the speed of transverse wave on a stretched string of tension T, length ℓ and mass m.

Numerical Problems

1.1 Astronomers usually measure astronomical distances in light years. One light year is the distance that light travels in one year. If speed of light is 3×10^8 m s^{-1}, what is one light year in metres? (Ans: 9.5×10^{15} m)

1.2 Write the estimated answer of the following in standard form.

(a) How many seconds are there in 1 year?

(b) How many years are in 1 second?

(Ans: (a) 3.2×10^7 s (b) 3.1×10^{-8} years)

1.3 The length and width of a rectangular plate are measured to be 18.3 cm and 14.60 cm, respectively. Find the area of the plate and state the answer to correct number of significant figures. (Ans: 267 cm^2)

1.4 Find the sum of the masses given in kg up to appropriate precision:

(i) 3.197 (ii) 0.068 (iii) 13.9 (iv) 3.28 (Ans: 20.4 kg)

1.5 The diameter and length of a metal cylinder measured with the help of a Vernier Callipers of least count 0.01 cm are 1.22 cm and 5.35 cm respectively. Calculate its volume and uncertainty in it. (Ans: 6.2 ± 0.1 cm^3)

1.6 Show that the expression; $v_f^2 - v_i^2 = 2aS$ is dimensionally correct, where v_i is the initial velocity, a is the acceleration and v_f is the velocity after covering a distance S.

1.7 Show that the famous "Einstein equation" $E = mc^2$ is dimensionally consistent.

1.8 Derive a formula for the time period of a simple pendulum using dimensional analysis. The various possible factors on which the time period T may depend are:

(i) length of the pendulum ℓ

(ii) mass of the bob m

(iii) angle θ which the thread makes with the vertical

(iv) acceleration due to gravity g. (Ans: $T = \text{constant}\sqrt{\frac{\ell}{g}}$)

Chapter 2

Force and Motion

Learning Objectives

After studying this chapter, the students will be able to:

- Differentiate between scalar and vector quantities
- Represent a vector in 2-D as two perpendicular components
- Describe the product of two vectors (dot and cross-product) along with their properties
- Derive the equations of motion [For uniform acceleration cases only. Derive from the definitions of velocity and acceleration as well as graphically]
- Solve problems using the equations of motion [For the cases of uniformly accelerated motion in a straight line, including the motion of bodies falling in a uniform gravitational field without air resistance. This also includes situations where the equations of motion need to be resolved into vertical and horizontal components for 2-D motion]
- Evaluate and analyse projectile motion in the absence of air resistance
  [This includes solving problems making use of the below facts:
  (i) Horizontal component(V_x)of velocity is constant.
  (ii) Acceleration is in the vertical direction and is the same as that of a vertically free falling object.
  (iii) The horizontal motion and vertical motion are independent of each other. Situations may require students to determine for projectiles:
  - How high does it go?
  - How far would it go along the level land?
  - Where would it be after a given time?
  - How long will it remain in flight?

  Situations may also require students to calculate for a projectile launched from ground height the
  - launch angle that results in the maximum range.
  - relation between the launch angles that result in the same range.]
- Predict qualitatively how air resistance affects projectile motion. [This includes analysis of both the horizontal component and vertical component of velocity and hence predicting qualitatively the range of the projectile.]
- Apply the principle of conservation of momentum to solve simple problems [Including elastic and inelastic interactions between objects in both one and two dimensions.
  Knowledge of the concept of coefficient of restitution is not required.
  Examples of applications include:
  - karate chops to break a pile of bricks
  - car crashes
  - ball &bat
  - the motion under thrust of a rocket in a straight line considering short thrusts during which the mass remains constant]
- Predict and analyse motion for elastic collisions [This includes making use of the fact that for an elastic collision, total kinetic energy is conserved and the relative speed of approach is equal to the relative speed of separation]
- Justify how the momentum of a closed system is always conserved, some change in kinetic energy may take place.

BASIC CONCEPT OF SCALARS AND VECTORS

Scalars and vectors are basic concepts in physics. Many problems in physics require to distinguish between scalar and vector quantities to apply the correct mathematical and conceptual approaches. Understanding scalars and vectors help us to grasp how Physics applies to real-world situations, such as calculating the total distance travelled (scalar) or determining the magnitude and direction of force (vector). Learning these concepts develops critical thinking and problem-solving skills. This chapter is primarily concerned with vector algebra and its application in uniform accelerated motion, in a straight line, motion of freely falling bodies in uniform gravitational field, projectile motion, and interaction between objects in one and two dimensions.

2.1 SCALARS

Scalars are physical quantities that are described solely by a magnitude (size or amount) without any mention of direction. Thus, scalars are directionless and can be fully characterized by a single number and its associated unit.

Examples:

Mass: The amount of matter in an object. For example, 2 kg.

Distance: The total length of the path travelled by an object irrespective of the direction. For example, 50 m.

Speed: The rate at which an object covers distance. For example, 40 km h^{-1}.

Time: The duration between two events taking place. For example, 20 s.

Energy: The capacity to do work. For example, 25 J.

Temperature: A measure of the average kinetic energy of particles in a substance. For example, 20°C.

2.2 VECTORS

Those physical quantities which require magnitude as well as direction for their complete specification are known as vectors.

Examples:

Displacement: The change in position of an object. It has length, a distance (magnitude) and a direction (e.g. 10 m towards west).

Velocity: The speed of an object in a particular direction (e.g., 50 km h^{-1} towards west).

Acceleration: The rate of change of velocity that occurs in either speed or direction or both (e.g., 10 m s^{-2} upward).

Force: A push or pull acting on an object, determined by its magnitude and direction (e.g; 20 N to the right)

Graphical Representation of a Vector

A good way to represent a vector quantity is to use a vector diagram, in which vectors are often represented by arrows. The length of the arrows indicates the magnitude and the head of the arrow shows the direction of the vector. Vectors are typically denoted by bold face letters (e.g; **V**, **F**) or an arrow above symbol ($\vec{A}$).

Rectangular Components of a Vector

A component of a vector is its effective value in a given direction. A vector may be considered as the resultant of its component vectors along the specified directions. It is usually convenient to resolve a vector into its components along the mutually perpendicular directions. Such components are called rectangular components.

Let there be a vector **A** represented by a line OP making an angle θ with the x-axis. Draw projection OM of vector **A** on x-axis and projection ON of vector **A** on y-axis as shown in Fig.2.1. Projection OM being along x-direction is represented by $\mathbf{A}_x$ and projection ON along y-direction is represented by $\mathbf{A}_y$. By applying head to tail rule:

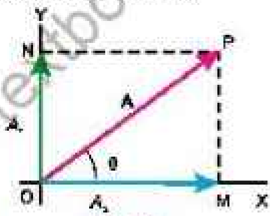


Fig. 2.1

$$\mathbf{A} = \mathbf{A}_x + \mathbf{A}_y \quad \text{..................... (2.1)}$$

Thus, $\mathbf{A}_x$ and $\mathbf{A}_y$ are the comoponants of vector **A**. Since these are at right angles to each other, they are called rectangular components of **A**. Considering the right angled triangle OMP, the magnitude of $\mathbf{A}_x$ or x-component of **A** is:

$$A_x = A\cos\theta \quad \text{..................... (2.2)}$$

And the magnitude of $\mathbf{A}_y$ or y-component of **A** is:

$$A_y = A\sin\theta \quad \text{..................... (2.3)}$$

Determination of a Vector from its Rectangular Components

If the rectangular components of a vector as shown in Fig.(2.1) are given, we can find out the magnitude of the vector by using Pythagorean Theorem.

In the right angle ΔOMP

$$(OP)^2 = (OM)^2 + (MP)^2$$

or $$A^2 = A_x^2 + A_y^2 \quad \text{..................... (2.4)}$$

or $$A = \sqrt{A_x^2 + A_y^2}$$

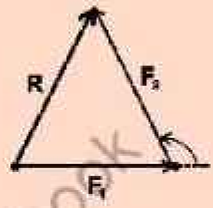
The direction θ is given by $$\tan\theta = \frac{MP}{OM} = \frac{A_y}{A_x}$$

or $$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right) \quad \text{.............. (2.5)}$$

Example 2.1 Find the angle between two forces of equal magnitude when the magnitude of their resultant is also equal to the magnitude of either of these forces.

Solution Let θ be the angle between two forces F_1 and F_2, where F_1 is along the x-axis. Then x-component of their resultant will be:

$$R_x = F_1 \cos 0° + F_2 \cos\theta$$
$$R_x = F_1 + F_2 \cos\theta$$



And y-component of their resultant is

$$R_y = F_1 \sin 0° + F_2 \sin\theta$$
$$R_y = F_2 \sin\theta$$

The resultant R is given by $R^2 = R_x^2 + R_y^2$

As $R = F_1 = F_2 = F$

Hence $F^2 = (F + F\cos\theta)^2 + (F\sin\theta)^2$

$$F^2 = F^2 + F^2\cos^2\theta + 2F^2\cos\theta + F^2\sin^2\theta$$

or $0 = 2F^2\cos\theta + F^2(\cos^2\theta + \sin^2\theta)$

or $0 = 2F^2\cos\theta + F^2,\ 0 = F^2(2\cos\theta + 1)$

As $F \neq 0$, so $2\cos\theta + 1 = 0$

or $\cos\theta = -0.5$

or $\theta = \cos^{-1}(-0.5) = 120°$

2.3 PRODUCT OF TWO VECTORS

There are two types of vector multiplications. The product of these two types are known as scalar product and vector product.

> If the product of two vectors results in a scalar quantity, it is called scalar product while if the product of two vectors results in a vector quantity, it is called vector product.

Scalar or Dot Product

The scalar product of two vectors **A** and **B** is written as **A.B** and is defined as:

$$\mathbf{A.B} = AB\cos\theta \text{ .... } (2.6)$$

where A and B are the magnitudes of vectors **A** and **B** and θ is the angle between them.

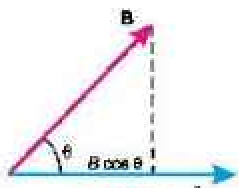


Fig. 2.2 (e)

For physical interpretation of dot product of two vectors **A** and **B**, these are first brought to a common origin (Fig.2.2-e) then,

A.B = A (projection of **B** on **A**)

or

A. B = **A** (magnitude of component of **B** in the direction of **A**), Fig.2.2(b)

$$= A\ (B\cos\theta)\ = AB\cos\theta$$

Similarly $\quad$ **B. A** = $B\ (A\cos\theta) = BA\cos\theta$

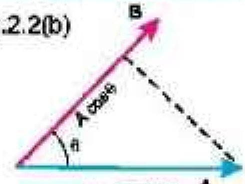


Fig. 2.2 (b)

This type of product when we consider the work done by a force **F** whose point of application moves a distance d in a direction making an angle θ with the line of action of **F**, as shown in Fig. 2.3.

Fig. 2.3

Work done = (Effective component of force in the direction of motion) x Distance moved

$$= (F\cos\theta)\,d\ = Fd\cos\theta$$

Using vector notation:

$$\mathbf{F.d}\ = Fd\cos\theta = \text{Work done}$$

Characteristics of Scalar Product

1. Since **A.B** = $AB\cos\theta$ and **B.A** = $BA\cos\theta$ = $AB\cos\theta$, hence, **A.B = B.A**. The order of multiplication is irrelevant. In other words, scalar product is commutative.
2. The scalar product of two mutually perpendicular vectors (θ = 90°) is zero.

$$\mathbf{A.B} = AB\cos 90° = 0$$

3. The scalar product of two parallel vectors is equal to the product of their magnitudes. Thus, for parallel vectors (θ = 0°).

$$\mathbf{A.B} = AB\cos 0° = AB$$

For antiparallel vectors (θ = 180°)

$$\mathbf{A.B} = AB\cos 180° = -AB$$

4. The self product of a vector **A** is equal to square of its magnitude.

$$\mathbf{A.A} = AA\cos 0° = A^2$$

5. Scalar product of two vectors **A** and **B** in terms of their rectangular components

$$\mathbf{A.B} = A_xB_x + A_yB_y + A_zB_z \quad \ldots\ldots\ldots\ldots\ldots (2.7)$$

Equation (2.6) can be used to find the angle between two vectors. Since,

$$\mathbf{A.B} = AB\cos\theta = A_xB_x + A_yB_y + A_zB_z$$

Therefore $\quad \cos\theta = \dfrac{A_xB_x + A_yB_y + A_zB_z}{AB} \quad \ldots\ldots\ldots\ldots (2.8)$

Vector or Cross Product

The vector product of two vectors **A** and **B**, is a vector which is defined as:

$$\mathbf{A} \times \mathbf{B} = AB \sin\theta \, \hat{n} \quad \text{.....................(2.9)}$$

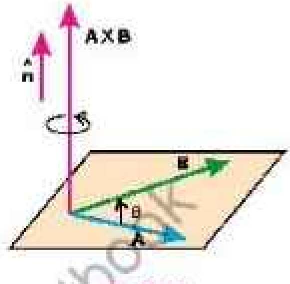


Fig. 2.4(a)

where $\hat{n}$ is a unit vector perpendicular to the plane containing **A** and **B** as shown in Fig. 2.4 (a). Its direction can be determined by right hand rule. For that purpose, place together the tail of vectors **A** and **B** to define the plane of vectors **A** and **B**. The direction of the product vector is perpendicular to this plane. Rotate the First vector **A** into **B** through the smaller of the two possible angles and curl the fingers of the right hand in the direction of rotation, keeping the thumb erect. The direction of the product vector will be along the erect thumb, as shown in the Fig 2.4 (b). Because of this direction rule, **B x A** is a vector opposite in sign to **A x B** (Fig. 2.4-c). Hence,

$$\mathbf{A} \times \mathbf{B} = -\mathbf{B} \times \mathbf{A} \quad \text{..............(2.10)}$$

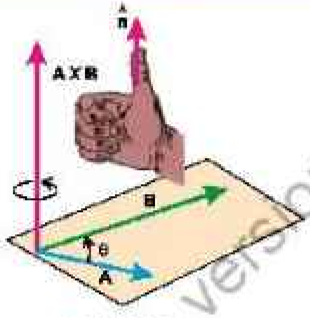


Fig. 2.4(b)

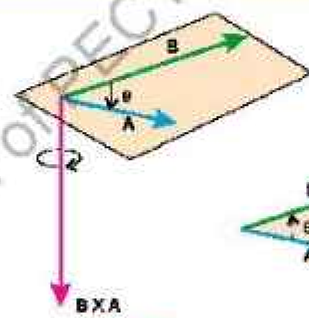


Fig. 2.4(c)

Fig. 2.4(d)

Characteristics of Cross Product

1. Since **A x B** is not the same as **B x A**, the cross product is non commutative. so,

   $$\mathbf{A} \times \mathbf{B} \neq \mathbf{B} \times \mathbf{A}$$

2. The cross product of two perpendicular vectors ($\theta = 90°$) has maximum magnitude.

   $$\mathbf{A} \times \mathbf{B} = AB \sin 90° \, \hat{n} = AB \, \hat{n}$$

3. The cross product of two parallel or anti-parallel vectors is a null vector, because for such vectors $\theta = 0°$ or $180°$. Hence,

   $$\mathbf{A} \times \mathbf{B} = AB \sin 0° \, \hat{n} = 0 \quad \text{or} \quad \mathbf{A} \times \mathbf{B} = AB \sin 180° \, \hat{n} = 0$$

   As a consequence **A x A = 0** ($\theta = 0°$)

4. The magnitude of **A x B** is equal to the area of the parallelogram formed with **A** and **B**

as two adjacent sides (Fig.2.4-d).

Examples of Vector Product

i. When a force **F** is applied on a rigid body at a point whose position vector is **r** from any point on the axis about which the body rotates, then the turning effect of the force called the torque τ is given by the vector product of **r** and **F**.

$$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$$

ii. The force **F** on a particle of charge *q* and velocity **v** in a magnetic field of strength **B** is given by vector product of **v** and **B**.

$$\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$$

2.4 EQUATIONS OF MOTIONS

Equations of motion can be used to describe the motion of an object in terms of its three kinematic variables: velocity *v*, position *S* and time *t*. There are three ways to pair these variables up: velocity-time, position-time and velocity-position. In this order, they are called first equation of motion, second equation of motion and third equation of motion, respectively.

These equations of motion can only be applied to those objects, which are moving in a straight line with constant acceleration.

Derivation of First Equation of Motion

Suppose a body is moving with uniform acceleration along a straight line with an initial velocity v_i . Suppose its velocity changes from initial value v_i to a final value v_f in time interval *t*. Then the acceleration produced in the body during this time interval is given as:

$$a = \frac{v_f - v_i}{t}$$

Rearranging, we can write

$$v_f - v_i = at$$

$$v_f = v_i + at \text{............(2.11)}$$

This is the first equation of motion. It correlates the final velocity attained by a body with initial velocity and the time interval *t*, when moving with constant acceleration *a*.

Derivation of First Equation of Motion By Graphical Method

First equation of motion can be derived using velocity-time graph for an object moving with initial velocity v_i final velocity v_f and constant acceleration *a*.

Let the velocity of a body at point A be v_i which changes to v_f at point B in time interval *t* as shown in Fig.2.5. A perpendicular BD is drawn from point B to x-axis and another perpendicular BE from B on y-axis, such that

$OA = v_i$ = Initial velocity of the body

$OE = DB = v_f$ = Final velocity of the body

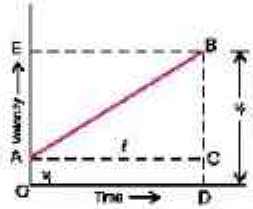


Fig. 2.5: Velocity-Time Graph

From the graph, it can observed that:

$$DB = DC + CB$$

$$DB = OA + CB \qquad (\text{As } OA = DC)$$

Therefore $\quad v_f = CB + v_i$ ………………(2.12)

The value of CB in the above equation can be determined by taking the slope of line AB, which is equal to acceleration a.

$$a = \frac{CB}{AC}$$

As $\quad AC = t$

So $\quad a = \dfrac{CB}{t}$

or $\quad BC = at$ …………………(2.13)

Combining Eqs. (2.12) and (2.13), we have

$$v_f = v_i + at$$

This is the first equation of motion.

Derivation of Second Equation of Motion

Suppose a body is moving with uniform acceleration a along a straight line with an initial velocity v_i, which become v_f after time interval t. Let it covers a distance S in a particular direction during time t, then using the definition of velocity as rate of change of displacement, we can write

Velocity = Displacement / Time

or $\quad$ Displacement = Velocity x Time

If velocity of the body is not constant, we can use average velocity instead of velocity.

Thus $\quad$ Displacement = Average velocity × Time

$$\text{Displacement} = \frac{(\text{Initial velocity}) + (\text{Final velocity})}{2} \times \text{Time}$$

$$S = \frac{(v_i + v_f)}{2} \times t$$

Using first equation of motion, $\quad S = \dfrac{(v_i + v_i + at)}{2} \times t$

$$S = \frac{(2v_i + at)}{2} \times t$$

$$2S = 2v_i t + at^2$$

$$S = v_i t + \frac{1}{2}at^2 \quad \text{.......................(2.14)}$$

This is the second equation of motion.

Derivation of Second Equation of Motion by Graphical Method

Second equation of motion can be derived using velocity-time graph for a body moving with initial velocity v_i which attains a final value v_f in time interval t. While moving with constant acceleration a, it covers a displacement S in time t.

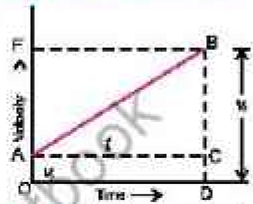


Fig. 2.6: Velocity-Time Graph

It can be seen from the graph that distance travelled by the body is, $S = v \times t$.

Also $\quad$ S = Area of the figure OABD

S = (Area of the rectangle OACD) + (Area of the triangle ABC)

$$S = (OA \times OD) + \frac{1}{2}(AC \times BC)$$

As $\quad$ OA = v_i and OD = AC = t. So, the above equation becomes:

$$S = v_i \times t + \frac{1}{2}(t \times BC)$$

Here BC = at (From graphical representation of first equation of motion). By putting this value in the above equation, we have

$$S = v_i t + \frac{1}{2}(t \times at)$$

$$S = v_i t + \frac{1}{2}at^2$$

This is the second equation of motion.

Derivation of third equation of motion

Consider a body moving along a straight line with an initial velocity v_i which attains a final value v_f in time t. Let the displacement of the body be S during this time interval. Then, we can write:

$$\text{Displacement} = \left(\frac{\text{Initial velocity} + \text{Final velocity}}{2}\right) \times \text{Time}$$

$$S = \frac{(v_i + v_f)}{2} \times t$$

$$2S = (v_i + v_f) \times t \quad \text{.................................. (2.15)}$$

Using the first equation of motion:

$$v_f = v_i + at$$

or $$t = \frac{v_f - v_i}{a}$$

Putting the value of t in Eq. (2.15)

$$2S = (v_f + v_i)\left(\frac{v_f - v_i}{a}\right)$$

$$2aS = v_f^2 - v_i^2$$

This is the third equation of motion.

Derivation of third Equation of Motion by Graphical method

In the speed-time graph shown in the figure, the total distance S travelled by a body is given by the area OABD under the graph, such that

$$S = \frac{1}{2} \text{ (Sum of parallel sides) x Height}$$

$$S = \frac{1}{2}(\text{OA} + \text{BD}) \times \text{OD}$$

Since $\quad \text{OA} = v_i, \; \text{BD} = v_f \;$ and $\; \text{OD} = t$

The above equation becomes:

$$S = \frac{1}{2}(v_i + v_f) \times t$$

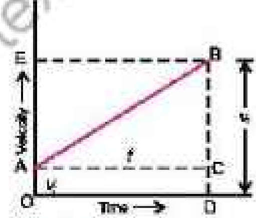


Fig. 2.7: Velocity-Time Graph

From first equation of motion,

$$t = \frac{v_f - v_i}{a}$$

Putting t in above equation

$$S = \frac{1}{2}(v_f + v_i)\,\frac{(v_f - v_i)}{a}$$

or

$$S = \frac{1}{2}(v_f + v_i)\,\frac{(v_f - v_i)}{a}$$

$$2aS = v_f^2 - v_i^2 \quad \text{.........................(2.16)}$$

This is the third equation of motion.

The equations of motion are useful in solving the problems relating to linear motion with uniform acceleration, when an object moves along a straight line. If its direction of motion does not change, then all the vector quantities can be manipulated like scalars. In such cases, initial velocity is taken as positive. A negative sign is assigned to quantities where direction is opposite to that of initial velocity. In the absence of air resistance, all objects in free fall at the surface of the Earth, move towards the Earth with a uniform acceleration. This acceleration is known as acceleration due to gravity, denoted by g and its average value at the Earth surface is taken as 9.8 m s^{-2} in the downward direction. The equation for uniformly accelerated motion can also be applied to free fall motion of the object by replacing a by g.

Example 2.2 A car travelling at 10 m s^{-1} accelerates uniformally at 2 m s^{-2}. Calculate its velocity after 5 s.

Solution $v_i = 10\, m\, s^{-1}$

$a = 2\, m\, s^{-2}$

$t = 5\, s$

$v_f = ?$

Using first equation of motion, we can write

$$v_f = v_i + at$$

$$v_f = 10\, m\, s^{-1} + 2\, m\, s^{-2} \times 5\, s$$

$$v_f = 10\, m\, s^{-1} + 10\, m\, s^{-1}$$

$$v_f = 20\, m\, s^{-1}$$

Example 2.3 A car travels with initial velocity of 15 m s^{-1}. It accelerates at a rate of 2 m s^{-2} for 4 seconds. Find the displacement of the car.

Solution

$v_i = 15\, m\, s^{-1}$

$a = 2\, m\, s^{-2}$

$t = 4\, s$

Displacement $S = ?$

By using 2nd equation of motion

$$S = v_i t + \frac{1}{2}at^2$$

Putting the values

$$S = (15\, m\, s^{-1} \times 4\, s) + \frac{1}{2}(2\, ms^{-2})(4\, s)^2$$

$$S = 76\, m$$

Example 2.4 In a short distance race, a contestant in a car starts from rest and reaches the velocity of 300 km h^{-1}, after covering a distance of 0.45 km at a constant acceleration. Find this constant acceleration.

Solution Initial velocity = $v_i = 0$

Final velocity = $v_f = 300\, km\, h^{-1}$

$$v_f = \frac{300 \times 1000}{60 \times 60}\, ms^{-1}$$

$= 83.33 \text{ m s}^{-1}$

Distance covered $= S = 0.45 \text{ km} = 0.45 \times 1000 \text{ m} = 450 \text{ m}$

Using third equation of motion, we have

$$v_f^2 - v_i^2 = 2aS$$

$$(83.33 \text{ m s}^{-1})^2 - (0)^2 = 2 \times a \times 450 \text{ m}$$

$$a = \frac{6943.88 \text{ m}^2 \text{ s}^{-2}}{900 \text{ m}}$$

$$a = 7.72 \text{ m s}^{-2}$$

2.5 MOTION UNDER GRAVITY

A body falling freely under the action of gravity is the most familiar example of uniformly accelerated rectilinear motion. According to Gallio, all bodies fall freely (in vacuum) under the acceleration due to gravity, denoted by 'g'. Its experimental value is 9.8 m s^{-2} in SI units. This means that different bodies, when allowed to fall from the same height, strike the ground with the same velocity. As regards the sign of g, it is taken positive for a falling body (when initial velocity is zero) and negative for a body projected vertically upward (when initial velocity is not zero).

The equations of motion for a freely falling body, on putting $a = g$, become

$$v_f = v_i + gt$$

$$S = h = v_i t + \frac{1}{2} g t^2$$

$$v_f^2 - v_i^2 = 2gh$$

Example 2.5 An iron ball of mass 1 kg is dropped from a tower. The ball reaches the ground in 3.34 s. Find: (a) the velocity of the ball on striking the ground, (b) the height of the tower.

Solution Since the ball is falling under the action of gravity, we shall putt $a = g$ in equations of motion.

Mass of the ball	$m = 1$ kg
Time taken to reach ground	$t = 3.34$ s
Initial velocity	$v_i = 0$
Final velocity	$v_f = ?$
Acceleration	$a = g = 9.8 \text{ m s}^{-2}$

(a) Using first equation of motion:

$$v_f = v_i + gt$$

$$v_f = 0 + (9.8\ \text{m s}^{-2})\ (3.34\ \text{s})$$

$$v_f = 32.7\ \text{m s}^{-1}$$

(b) Using third equation of motion:

$$v_f^2 - v_i^2 = 2gh$$

$$(32.7\text{m s}^{-1})^2 - (0)^2 = 2 \times 9.8\ \text{m s}^{-2} \times h$$

$$h = \frac{1069.29\ \text{m}^2\ \text{s}^{-2}}{19.6\ \text{m s}^{-2}}$$

$$h = 54.56\ \text{m}$$

2.6 PROJECTILE MOTION

Uptill now we have been studying the motion of a particle along a straight line i.e., motion in one dimension. Now we consider the motion of a ball, when it is thrown horizontally from certain height. It is observed that the ball travels forward as well as falls downward, until it strikes something such as ground. Suppose that the ball leaves the hand of the thrower at point A (Fig 2.8-a) and that its velocity at that instant is completely horizontal. Let this velocity be v_x. According to Newton's first law of motion, there will be no acceleration in horizontal direction, unless a horizontally directed force acts on the ball. Ignoring the air friction, only force acting on the ball during flight is the force of gravity. There is no horizontal force acting on it. So, its horizontal velocity will remain unchanged and will be v_x, until the ball hits the ground. The horizontal motion of ball is simple. The ball moves with constant horizontal velocity component. Hence, horizontal distance x is given by

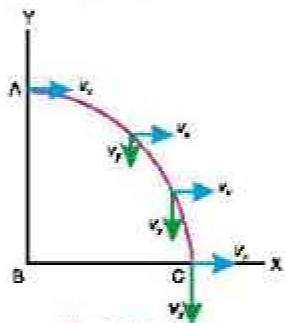


Fig. 2.8(a)

$$x = v_x \times t \ldots\ldots\ldots\ldots\ldots\ldots(2.17)$$

The vertical motion of the ball is also not complicated. It will accelerate downward under the force of gravity and hence $a = g$. This vertical motion is the same as for a freely falling body. Since initial vertical velocity is zero, hence, vertical distance y, using Eq. 2.14 is given by

$$y = \frac{1}{2}gt^2$$

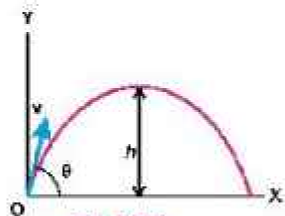


Fig. 2.8(b)

It is not necessary that an object should be thrown with some initial velocity in the horizontal direction. A football kicked off by a player; a ball thrown by a cricketer and a missile fired from a launching pad, all projected at some angles with the horizontal, are called projectiles.

> Projectile motion is two dimensional motion under constant acceleration due to gravity.

In such cases, the motion of a projectile can be studied easily by resolving it into horizontal and vertical components which are independent of each other. Suppose that a projectile is fired in a direction angle θ with the horizontal by velocity v_i as shown in Fig.2.8(b). Let components of velocity v_i along the horizontal and vertical directions be $v_i\cos\theta$ and $v_i\sin\theta$, respectively. The horizontal acceleration is $a_x = 0$ because we have neglected air resistance and no other force is acting along this direction, whereas the vertical acceleration is $a_y = g$. Hence, the horizontal component v_x remains constant and at any time t, we have

$$v_{fx} = v_{ix} = v_i \cos\theta \;\ldots\ldots\ldots\ldots\ldots\ldots (2.18)$$

Now we consider the vertical motion. The initial vertical component of the velocity is $v_i\sin\theta$ in the upward direction.

The vertical component v_{fy} at any instant t can be determined by considering the upward motion of projectile as free fall motion $(a_y = -g)$. Using 1st equation of motion:

$$v_{fy} = v_i \sin\theta - gt \quad \ldots\ldots\ldots \quad (2.19)$$

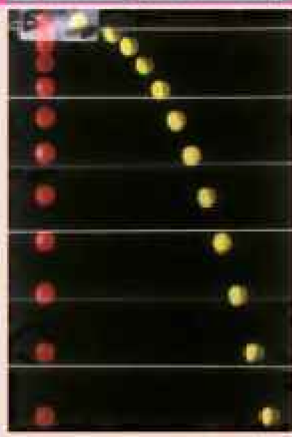
The magnitude of velocity at any instant is:

$$v = \sqrt{v_{fx}^2 + v_{fy}^2} \quad \ldots\ldots\ldots \quad (2.20)$$

The angle ϕ which this resultant velocity makes with the horizontal can be found from

$$\tan\phi = \frac{v_{fy}}{v_{fx}} \quad \ldots\ldots\ldots \quad (2.21)$$

Interacting Information



A photograph of two balls released simultaneously from a mechanism that allows one ball to drop freely while the other is projected horizontally. At any time the two balls are at the same level, i.e., their vertical displacements are equal.

In projectile motion one may wish to determine the height to which the projectile rises, the time of flight and horizontal range. These are described below.

Height of the Projectile

In order to determine the maximum height the projectile attains, we use the equation of motion:

$$2aS = v_f^2 - v_i^2$$

As body moves upward, $a = -g$, the initial vertical velocity $v_{iy} = v_i \sin\theta = v_i$ as $v_{fy} = 0 = v_f$, because the body comes to rest after reaching the highest point. Since

$$S = height = h$$

$$-2gh = 0 - v_i^2 \sin^2\theta$$

or $$h = \frac{v_i^2 \sin^2\theta}{2g} \quad \text{............ (2.22)}$$

The height of projectile will be reduced in presence of air resistance. In the presence of air resistance, the upward velocity of the projectile will decrease and hence its height will also decrease during time t.

Time of Flight

The time taken by body to cover the distance from the place of its projection to the place where it hits the ground is called the time of flight.

This can be obtained by taking $S = h = 0$, because body goes up and comes back to the same level, thus covering no vertical distance. If the body is projecting with velocity v_i making angle θ with the horizontal, then its vertical component will be $v_i \sin\theta$. Hence, the equation of motion is:

$$S = v_i t + \frac{1}{2} g t^2$$

$$0 = v_i \sin\theta\, t - \frac{1}{2} g t^2$$

$$t = \frac{2 v_i \sin\theta}{g} \quad \text{.................... (2.23)}$$

where t is the time of flight of the projectile when it is projected from the ground.

Range of the projectile

Maximum distance which a projectile covers in the horizontal direction is called the range of the projectile.

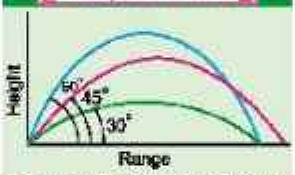
To determine the range R of the projectile, we multiply the horizontal component of the velocity of projection with total time taken by the body to hit the ground after leaving the point of projection. Thus,

$$R = v_{ix} \times t$$

or $$R = \frac{v_i \cos\theta \times 2 v_i \sin\theta}{g}$$

or $$R = \frac{v_i^2}{g} 2 \sin\theta \cos\theta$$

Do you know?



For an angle less than 45°, the height reached by the projectile and the range both will be less. When the angle of projectile is larger than 45°, the height attained will be more but the range is again less.

As $2\sin\theta\cos\theta = \sin 2\theta$, thus, the range of the projectile depends upon the velocity of projection and the angle of projection.

Therefore $$R = \frac{v_i^2}{g}\sin 2\theta \quad \text{................ (2.24)}$$

For maximum range R, the factor $\sin 2\theta = 1$, so

$$2\theta = \sin^{-1}(1) \quad \text{or} \quad 2\theta = 90^\circ \quad \text{or} \quad \theta = 45^\circ$$

Air resistance will slow down projectile forward motion, reducing its velocity v_i. The reduction in v_i will result in a decrease in the range of projectile.

Furthermore, air resistance is not constant throughout the flight of the object. As the object slows down, the air resistance experienced by it also decreases. This means that the object retards more slowly and accelerates more slowly as it falls down. This results in a trajectory that is not perfectly parabolic but is skewed, with steeper descent than ascent.

Example 2.6 A ball is thrown with a speed of 30 m s^{-1} in a direction 30° above the horizon. Determine the height to which it rises, the time of flight and the horizontal range.

Solution Initially

$$v_x = v_i\cos\theta = 30\text{ m s}^{-1} \times \cos 30^\circ = 25.98\text{ m s}^{-1}$$

$$v_y = v_i\sin\theta = 30\text{ m s}^{-1} \times \sin 30^\circ = 15\text{ m s}^{-1}$$

As the time of flight, is

$$t = \frac{2v_i\sin\theta}{g}$$

$$= \frac{2\times 30\text{ m s}^{-1}}{9.8\text{ m s}^{-2}} \times (0.5)$$

So $$t = \frac{2\times 15\text{ m s}^{-1}}{9.8\text{ m s}^{-2}} = 3.1\text{ s}$$

Height $$h = \frac{v_i^2\sin^2\theta}{2g}$$

So $$h = \frac{(30\text{ m s}^{-1})^2(0.5)^2}{19.6\text{ m s}^{-2}}$$

$$h = 11.5\text{ m}$$

Range $$R = \frac{v_i^2}{g}\sin 2\theta = \frac{v_i^2}{g}\sin 60^\circ$$

So $$R = \frac{(30\text{ m s}^{-1})^2 \times 0.866}{9.8\text{ m s}^{-2}}$$

$$= 79.53\text{ m}$$

For your information

In the presence of air friction the trajectory of a high speed projectile falls short of a parabolic path.

2.7 MOMENTUM

We are aware of the fact that moving object possesses a quality by virtue of which it exerts a force on anything that tries to stop it. The faster the object is travelling, the harder it is to stop it. Similarly, if two objects move with the same velocity, then it is more difficult to stop the massive of the two. Newton referred to this property as "momentum", a vector quantity defined as the product of an object's mass and velocity. This term is now called linear momentum **p** of the body and is defined by the relation:

$$\mathbf{p} = m\,\mathbf{v} \quad ....(2.25)$$

In this expression, $\mathbf{v}$ is the velocity of the mass m. Linear momentum is, therefore, a vector quantity and has the direction of velocity. The SI unit of momentum is kilogram metre per second (kg m s^{-1}). It can also be expressed as newton second (N s).

Momentum and Newton's Second Law of Motion

Consider a body of mass m moving with an initial velocity $\mathbf{v}_i$. Suppose an external force **F** acts upon it for time t after which velocity becomes $\mathbf{v}_f$. The acceleration **a** produced by this force is given by

$$\mathbf{a} = \frac{\mathbf{v}_f - \mathbf{v}_i}{t}$$

By Newton's second law, the acceleration is given as:

$$\mathbf{a} = \frac{\mathbf{F}}{m}$$

Equating the two expressions of acceleration, we have

$$\frac{\mathbf{F}}{m} = \frac{\mathbf{v}_f - \mathbf{v}_i}{t}$$

or

$$\mathbf{F} \times t = m\mathbf{v}_f - m\mathbf{v}_i \quad .... (2.26)$$

where $m\mathbf{v}_i$ is the initial momentum and $m\mathbf{v}_f$ is the final momentum of the body.

The equation (2.26) shows that change in momentum is equal to the product of force and the time for which force is applied. This form of the second law is more general than the form $\mathbf{F} = m\mathbf{a}$, because it can easily be extended to account for changes as the body accelerates when its mass also changes. For example, as a rocket accelerates, it loses mass because its fuel is burnt and ejected to provide greater thrust.

From Eq. (2.26)

$$\mathbf{F} = \frac{m\mathbf{v}_f - m\mathbf{v}_i}{t}$$

Thus, second law of motion can also be stated in terms of momentum as:

Point to ponder!

Which hurt you in the above situations (a) or (b) and think why?

Point to ponder!

Can a moving object experience impulse?

Do you know?

Your hair acts like a crumple zone on your skull. A force of 5 N might be enough to fracture your naked skull (cranium), but with a covering of skin and hair, a force of 50 N would be needed.

Time rate of change of momentum of a body is equal to the applied force.

Impulse

Sometimes we wish to apply the concept of momentum to cases where the applied force is not constant, it acts for very short time. For example, when a bat hits a cricket ball, the force certainly varies from instant to instant during the collision. In such cases, it is more convenient to deal with the product of force and time ($F \times t$) instead of either quantity alone. The product of average force F that acts during time t is called impulse given by

$$\text{Impulse} = F \times t = m\mathbf{v}_f - m\mathbf{v}_i \quad \text{.................} \quad (2.27)$$

Example 2.7 A 1500 kg car has its velocity reduced from 20 m s^{-1} to 15 m s^{-1} in 3.0 s. How large was the average retarding force?

Solution Using the Eq. (2.27)

$F \times t = mv_f - mv_i$

$F \times 3.0\text{ s} = 1500\text{ kg} \times 15\text{ m s}^{-1} - 1500\text{ kg} \times 20\text{ m s}^{-1}$

or $F = -2500\text{ kg m s}^{-2}$

$= -2500\text{ N} = -2.5\text{ kN}$

The negative sign indicates that the force is retarding one.

Law of Conservation of Momentum

Let us consider an isolated system. It is a system on which no external agency exerts any force. For example, the molecules of a gas enclosed in a glass vessel at constant temperature constitute an isolated system. The molecules can collide with one another because of their random motion, but being enclosed by glass vessel, no external agency can exert a force on them.

Consider an isolated system of two smooth hard interacting balls of masses m_1 and m_2, moving along the same straight line, in the same direction, with velocities $\mathbf{v}_1$ and $\mathbf{v}_2$ respectively. Both the balls collide and after collision, ball of mass m_1 moves with velocity $\mathbf{v}'_1$ vand m_2 moves with velocity $\mathbf{v}'_2$ in the same direction as shown in Fig. (2.9).

To find the change in momentum of mass m_1,

Using Eq. (2.27) as:

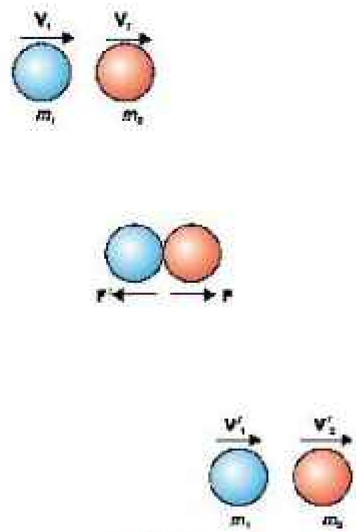


Fig. 2.9

$$\mathbf{F} \times t = m_1\mathbf{v}_1' - m_1\mathbf{v}_1$$

Similarly, for the ball of mass m_2, we have

$$\mathbf{F}' \times t = m_2\mathbf{v}_2' - m_2\mathbf{v}_2$$

Adding these two expressions, we have

$$(\mathbf{F} + \mathbf{F}')t = (m_1\mathbf{v}_1' - m_1\mathbf{v}_1) + (m_2\mathbf{v}_2' - m_2\mathbf{v}_2)$$

Since the action force **F** is equal and opposite to the reaction force **F′**, we have **F′** = – **F**, or **F**+**F′** = 0 so the left hand side of the equation is zero. Hence,

$$0 = (m_1\mathbf{v}_1' - m_1\mathbf{v}_1) + (m_2\mathbf{v}_2' - m_2\mathbf{v}_2)$$

In other words, change in momentum of 1st ball + change in momentum of the 2nd ball is zero.

or $$(m_1\mathbf{v}_1 + m_2\mathbf{v}_2) = (m_1\mathbf{v}_1' + m_2\mathbf{v}_2') \quad \text{..........(2.28)}$$

Do you know?

When a moving car stops quickly, the passengers move forward towards the windshield. Seatbelts change the forces of motion and prevent the passengers from moving. Thus, the chance of injury is greatly reduced.

Which means that total initial momentum of the system before collision is equal to the total final momentum of the system after collision. Consequently, the total change in momentum of the isolated two ball system is zero.

For such a group of objects, if one object within the group experiences a force, there must exist an equal but opposite reaction force on other object in the same group. As a result, the change in momentum of the group of objects as a whole is always zero. This can be expressed in the form of law of conservation of momentum, which states that:

The total linear momentum of an isolated system remains constant.

In applying the conservation law, we must notice that the momentum of a body is a vector quantity.

Example 2.8 Two spherical balls of 2.0 kg and 3.0 kg masses are moving towards each other with velocities of 6.0 m s^{-1} and 4 m s^{-1}, respectively. What must be the velocity of the smaller ball after collision, if the velocity of the bigger ball is 3.0 m s^{-1}?

Solution As both the balls are moving towards one another, so their velocities are of opposite sign. Let us suppose that the direction of motion of 2 kg ball is positive and that of the 3 kg is negative.

The momentum of the system before collision is:

$$m_1v_1 + m_2v_2 = 2\,\text{kg} \times 6\,\text{m s}^{-1} + 3\,\text{kg} \times (-4\,\text{m s}^{-1})$$
$$= 12\,\text{kg m s}^{-1} - 12\,\text{kg m s}^{-1} = 0$$

Do you know?

A motorcycle's safety helmet is padded so as to extend the time of any collision to prevent serious injury.

Momentum of the system after collision = $m_1v_1' + m_2v_2'$

$$= 2\text{ kg} \times v_1' + 3\text{ kg} \times (-3)\text{ m s}^{-1}$$

From the law of conservation of momentum

$$\begin{bmatrix}\text{Momentum of the system}\\ \text{before collision}\end{bmatrix} = \begin{bmatrix}\text{Momentum of the system}\\ \text{after collision}\end{bmatrix}$$

$$0 = 2\text{kg} \times v_1' - 9\text{ kg m s}^{-1}$$

$$v_1' = 4.5\text{ m s}^{-1}$$

2.8 ELASTIC AND INELASTIC COLLISIONS

When a tennis ball is dropped on the floor vertically, it may not rebound to its initial height. It is because, a portion of *K.E.* is lost, partly due to friction as the molecules in the ball move past one another when the ball distorts and partly due to its change into heat and sound energies. Similar is the case when two tennis balls collide with certain velocities, their final kinetic energy may be less than the total initial kinetic energy.

A collision in which the *K.E.* of the system is not conserved, is called inelastic collision.

Under certain special conditions, no kinetic energy is lost in the collision or impact on hitting the floor. Such type of collision is said to be elastic collision.

For example, when a hard ball is dropped onto a marble floor, it rebounds to very nearly the initial height. It loses negligible amount of energy in the collision with the floor. It is to be noted that momentum and total energy are conserved in all types of collisions. However, the *K.E.* is conserved only if it is an elastic collision.

Elastic Collisions in One Dimension

Consider two smooth, non-rotating balls of masses m_1 and m_2 moving initially with velocities $\mathbf{v}_1$ and $\mathbf{v}_2$ respectively, in the same direction. They collide and after collision, they move along the same straight line without rotation. Let their velocities after the collision be $\mathbf{v}'_1$ and $\mathbf{v}'_2$ respectively, as shown in Fig.(2.10).

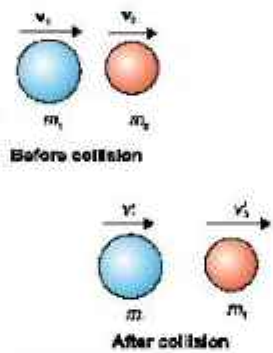


Fig. 2.10

We take the positive direction of the velocity and momentum to the right. By applying the law of conservation of momentum we have

$$(m_1\mathbf{v}_1 + m_2\mathbf{v}_2) = (m_1\mathbf{v}'_1 + m_2\mathbf{v}'_2)$$

$$m_1(\mathbf{v}_1 - \mathbf{v}'_1) = m_2(\mathbf{v}'_2 - \mathbf{v}_2) \quad \text{.............................} (2.29)$$

As the collision is elastic, so the *K.E.* is conserved. From the conservation of *K.E.*, we have $\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}m_1v_1'^2 + \frac{1}{2}m_2v_2'^2$

or $m_1(v_1^2 - v_1'^2) = m_2(v_2'^2 - v_2^2)$

or $m_1(v_1 + v_1')(v_1 - v_1') = m_2(v_2' + v_2)(v_2' - v_2)$ ....(2.30)

Dividing Eq. (2.29) by (2.30)

$$(v_1 + v_1') = (v_2' + v_2) \quad ....(2.31)$$

or $$(v_1 - v_2) = (v_2' - v_1') = -(v_1' - v_2')$$

We note that, before collision $(v_1 - v_2)$ is the velocity of first ball relative to the second ball. Similarly $(v'_1 - v'_2)$ is the velocity of the first ball relative to the second ball after collision. It means that relative velocities before and after the collision has the same magnitude but are reversed after the collision. In other words, the magnitude of relative velocity of approach is equal to the magnitude of relative velocity of separation.

In equations (2.29) and (2.30) m_1, m_2, v_1 and v_2 are known quantities. We solve these equations to find the values of v_1' and v_2' which are unknown. The results are

$$v_1' = \frac{m_1 - m_2}{m_1 + m_2}v_1' + \frac{2m_2}{m_1 + m_2}v_2 \quad ....(2.32)$$

$$v_2' = \frac{2m_1}{m_1 + m_2}v_1' + \frac{m_2 - m_1}{m_1 + m_2}v_2 \quad ....(2.33)$$

case (I)

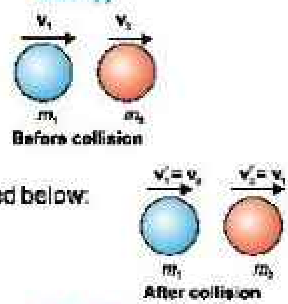


Fig. 2.11

There are some cases of special interest, which are discussed below:

(i) When $m_1 = m_2$

From Eq. (2.32) and (2.33), we find that

$$v_1' = v_2$$

and $$v_2' = v_1 \quad \text{(as shown in Fig. 2.11)}$$

(ii) When $m_1 = m_2$ and $v_2 = 0$

In this case, the mass m_2 be at rest, and $v_2 = 0$, then Eqs. (2.32) and (2.33) give

$$v_1' = 0 \quad ; \quad v_2' = v_1$$

case (II)

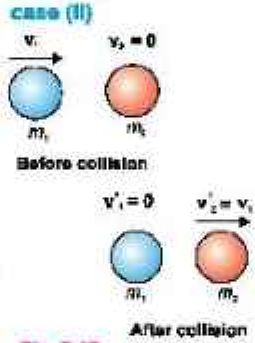


Fig. 2.12

When $m_1 = m_2$ then ball of mass m_1 after collision will come to a stop and m_2 will take off with the velocity that m_1 originally had, as shown in Fig.(2.12). Thus when a billiard ball m_1, moving on a table collides with exactly similar ball m_2 at rest, the ball m_1 stops while m_2 begins to move with the same velocity, with which m_1 was moving initially.

(iii) When a light body collides with a massive body at rest

In this case initial velocity $\mathbf{v}_2 = 0$ and $m_2 >> m_1$. Under these conditions m_1 can be neglected as compared to m_2. From Eq. (2.33) and (2.32), we have $\mathbf{v}'_1 = -\mathbf{v}_1$ and $\mathbf{v}'_2 = 0$.

The result is shown in Fig.(2.13). This means that m_1 will bounce back with the same velocity while m_2 will remain stationary. This fact is used of by the squash player.

case (iii)

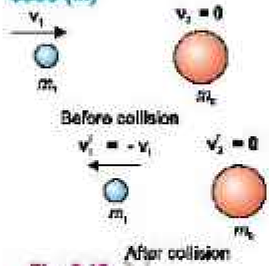


Fig. 2.13

(iv) When a massive body collides with light stationary body

In this case, $m_1 >> m_2$ and $\mathbf{v}_2 = 0$, so m_2 can be neglected in Eqs.(2.32) and (2.33). This gives $\mathbf{v}'_1 \approx \mathbf{v}_1$ and $\mathbf{v}'_2 \approx 2\mathbf{v}_1$. Thus, after the collision, there is practically no change in the velocity of massive body, but the lighter one bounces off in the forward direction with approximately twice the velocity of the incident body, as shown in Fig.(2.14).

case (iv)

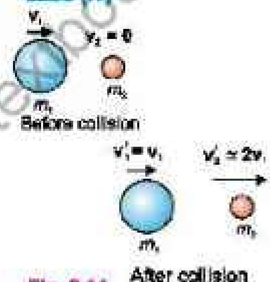


Fig. 2.14

2.9 INELASTIC COLLISION IN ONE DIMENSION

Consider two bodies having masses m_1 and m_2, moving with velocities $\mathbf{v}_1$ and $\mathbf{v}_2$, along the same line such that $\mathbf{v}_1 > \mathbf{v}_2$. In such a case m_1 is regarded as projectile and m_2 as target. After time t both the bodies make inelastic collision and stick together. Let their combined mass become $m_1 + m_2$ which moves with final velocity $\mathbf{v}_f$ after collision.

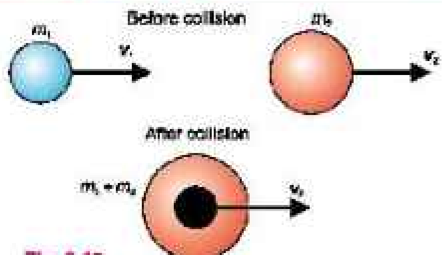


Fig. 2.15

Since the collision is perfectly inelastic, the total momentum of balls is conserved. Using law of conservation of momentum.

Total momentum of system before collision = Total momentum of the system after collision

$$m_1\mathbf{v}_1 + m_2\mathbf{v}_2 = (m_1 + m_2)\mathbf{v}_f$$

$$\mathbf{v}_f = \frac{m_1}{m_1 + m_2}\mathbf{v}_1 + \frac{m_2}{m_1 + m_2}\mathbf{v}_2 \quad ....(2.34)$$

Which gives the common velocity of the body after inelastic collision.

In a special case when the target m_2 is at rest, $v_2 = 0$, the above equation becomes:

$$\mathbf{v}_1 = \frac{m_1}{m_1 + m_2}\mathbf{v}_1$$

It shows that velocity of m_1 is reduced by the mass ratio i.e., $\frac{m_1}{m_1 + m_2}$.

Example 2.9 A 70 g ball collides with another ball of mass 140 g. The initial velocity of the first ball is 9 m s^{-1} to the right while the second ball is at rest. If the collision were perfectly elastic. What would be the velocity of the two balls after the collision?

Solution

$m_1 = 70$ g $\quad v_1 = 9$ m s^{-1} $\quad v_2 = 0$

$m_2 = 140$ g $\quad v'_1 = ?$ $\quad v'_2 = ?$

We know that;

$$v_1' = \frac{m_1 - m_2}{m_1 + m_2} v_1$$

$$= \left(\frac{70\text{ g} - 140\text{ g}}{70\text{ g} + 140\text{ g}}\right) \times 9\text{ m s}^{-1} = -3\text{ m s}^{-1}$$

$$v_2' = \frac{2m_1}{m_1 + m_2} v_1$$

$$= \left(\frac{2 \times 70\text{ g}}{70\text{ g} + 140\text{ g}}\right) \times 9\text{ m s}^{-1}$$

$$= 6\text{ m s}^{-1}$$

2.10 ELASTIC COLLISION IN TWO DIMENSIONS

Consider the motion of two balls of mass m_1 and m_2 in a straight line with velocities $\mathbf{v}_1$ and $\mathbf{v}_2$ respectively undergoing an elastic collision with each other as shown in Fig. 2.16

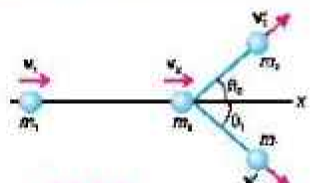


Fig. 2.16

Assume the bodies move off in different directions after collision with velocities $\mathbf{v}'_1$ and $\mathbf{v}'_2$ making angles θ_1 and θ_2 respectively with x- axis.

As, the collision is elastic, so we apply both the laws of conservation of momentum and law of conservation of kinetic energy. Momentum is a vector quantity, we resolve it into its rectangular components and apply the law of conservation of momentum along both axes.

Momentum conservation along x-axis is:

Momentum before collision = momentum after collision

$$m_1 v_1 + m_2 v_2 = m_1 v_1' \cos\theta_1 + m_2 v_2' \cos\theta_2 \quad \text{.................(2.35)}$$

Momentum conservation along y-axis:

Momentum before collision = Momentum after collision

$$0 = m_1 v_1' \sin\theta_1 - m_2 v_2' \sin\theta_2 \quad \text{.................(2.36)}$$

Conservation of Energy

Kinetic Energy before collision = Kinetic Energy after collision

$$\frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 = \frac{1}{2}m_1 v_1'^2 + \frac{1}{2}m_2 v_2'^2 \quad \text{.................(2.37)}$$

These equations (2.35 to 2.37) are useful in solving problems about elastic collisions in two dimensions.

2.11 INELASTIC COLLISION IN TWO DIMENSIONS

The macroscopic collisions are generally inelastic and do not conserve Kinetic energy.

The perfect inelastic collision is one in which the colliding objects stick together to make a single mass after collision. Its analysis can be carried out as follows:

Let us take two balls having masses m_1 and m_2 moving with velocities, v_1 and v_2, respectively, in a two-dimensional xy-plane. Assume that the first body is moving along the x-axis while the second body moves in a direction, making an angle θ with x-axis. Both the bodies collide at the origin as shown in the figure 2.17.

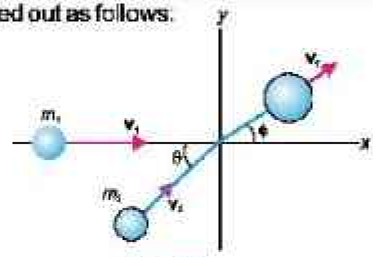


Fig. 2.17

After collision, bodies stick together, having combined mass $M = m_1 + m_2$, which moves with velocity v_f, making an angle ϕ with x-axis.

Momentum in the x-direction:

$$m_1 v_1 + m_2 v_2 \cos\theta = M v_f \cos\phi \quad \text{.................(2.38)}$$

Momentum in the y-direction:

$$0 + m_2 v_2 \sin\theta = M v_f \sin\phi \quad \text{.................(2.39)}$$

Equation 2.38 and 2.39 can be used to find the final velocity.

Kinetic Energy

Since collision is inelastic, the kinetic energy of colliding system is not conserved. The loss of kinetic energy is computed as follows:

Initial Kinetic Energy

The total initial kinetic energy $K.E_i$ of the system before the collision is:

$$(K.E)_i = \frac{1}{2}m_1v_i^2 + \frac{1}{2}m_2v_2^2 \quad \text{.............(2.40)}$$

Since $K.E.$ is a scalar quantity, so velocities involving in the formula of $K.E.$ does not require to break velocities into their components.

Final Kinetic Energy

The total final kinetic energy $K.E_f$ after the collision (when the objects stick together) is:

$$(K.E)_f = \frac{1}{2}Mv_f^2 \quad \text{.............(2.41)}$$

where v_f is magnitude of the final velocity which can be calculated from Eq. (2.41).

Energy Loss in the Collision

Since the collision is inelastic, there is a loss in kinetic energy, represented by $\Delta K.E.$

$\Delta K.E. = (K.E.)_f - (K.E.)_i$

This lost kinetic energy is transformed into other forms of energy, such as heat, sound, or in deformation.

Some examples of an inelastic collision:

(i) When a karate chop breaks a pile of bricks, it's an example of an inelastic collision. In this type of collision, the objects involved don't bounce back after impact. Instead, some of the energy from the strike is absorbed by the bricks, converting into heat, sound, and the force needed to break them. This means the energy goes into breaking the bricks rather than causing the hand to rebound. If the Karate chop is not perfectly vertical and involves some horizontal motion, the momentum transfer and the resulting forces will have both horizontal and vertical components.

(ii) In a car crash, the collision is an inelastic nature. When the vehicles collide and absorb the impact energy, causing them to crumple and deform. This energy absorption slows down the cars, stopping them from bouncing back. Most of the kinetic energy is lost, turning into heat, sound, and damage to the vehicles.

(iii) In real-world collisions, a ball and bat show an inelastic behaviour. When the bat hits the ball, some of the kinetic energy is lost because the ball deforms, and energy is also converted into heat and sound. Even though the bat is rigid, it does not transfer energy perfectly and absorbs some energy itself. The ball compresses upon impact, which leads to further energy loss. Consequently, not all of the initial kinetic energy is conserved, making the collision overall an inelastic.

2.12 ROCKET PROPULSION

Rockets move by expelling burning gas through engines at their rear. The ignited fuel turns to a high pressure gas which is expelled with extremely high velocity from the rocket engines (Fig. 2.18). The rocket gains momentum equal to the momentum of the gas expelled from the engine but in opposite direction. The rocket engines continue to expel gases after the rocket has begun moving and hence rocket continues to gain more and more momentum. So, instead of travelling at steady speed the rocket gets faster and faster so long the engines are operating.

A rocket carries its own fuel in the form of a liquid or solid hydrogen and oxygen. It can, therefore work at great heights where very little or no air is present. In order to provide enough upward thrust to overcome gravity, a typical rocket consumes about 10000 kg s^{-1} of fuel and ejects the burnt gases at speeds of over 4000 m s^{-1}. In fact, more than 80% of the launch mass of a rocket consists of fuel only. One way to overcome the problem of mass of fuel is to make the rocket from several rockets linked together.

When one rocket has done its job, it is discarded leaving others to carry the space craft further up at ever greater speed.

If m is the mass of the gases ejected per second with velocity v relative to the rocket, the change in momentum per second of the ejecting gases is mv. This equals the thrust produced by the engine on the body of the rocket. So, the acceleration 'a' of the rocket is

$$a = \frac{mv}{M} \quad \text{..........} (2.42)$$

where M is the mass of the rocket. When the fuel in the rocket is burned and ejected, the mass M of rocket decreases and hence the acceleration increases.

Fig. 2.18

Fuel and oxygen mix in the combustion chamber. Hot gases exhaust the chamber at a very high velocity. The gain in momentum of the gases equals the gain in momentum of the rocket. The gas and rocket push against each other and move in opposite directions.

QUESTIONS

Multiple Choice Questions

Tick (✓) the correct answer.

2.1 The angle at which dot product becomes equal to cross product:
(a) 65° (b) 45° (c) 75° (d) 30°

2.2 The projectile gains its maximum height at an angle of:
(a) 0° (b) 45° (c) 60° (d) 90°

2.3 The scalar product of two vectors is maximum if they are:
(a) perpendicular (b) parallel (c) at 30° (d) at 45°

2.4 The range of projectile is same for two angles which are mutually:
(a) perpendicular (b) supplementary
(c) complementary (d) 270°

2.5 The acceleration at the top of a trajectory of projectile is:
(a) maximum (b) minimum (c) zero (d) *g*

2.6 SI unit of impulse is:
(a) $kg\ m\ s^{-2}$ (b) N m (c) N s (d) $N\ m^2$

2.7 The rate of change of momentum is:
(a) force (b) impulse (c) acceleration (d) power

2.8 As rocket moves upward during its journey, then its acceleration goes on:
(a) increasing (b) decreasing
(c) remains same (d) it moves with uniform velocity

2.9 Elastic collision involves:
(a) loss of energy
(b) gain of energy
(c) no gain, no loss of energy
(d) no relation between energy and elastic collision

Short Answer Questions

2.1 State right hand rule for two vectors with reference to vector product.

2.2 Define impulse and show how it is related to momentum.

2.3 Differentiate between an elastic and an inelastic collision.

2.4 Show that rate of change in momentum is equal to force applied. Also state Newton's second law of motion in terms of momentum.

2.5 State law of conservation of linear momentum. Also state condition under which it holds.

2.6 Show that range of projectile is maximum at an angle of 45°.

2.7 Find the time of flight of a projectile to reach the maximum height.

2.8 The maximum horizontal range of a projectile is 800 m. Find the value of height attained by the projectile at $\theta = 60°$.

Constructed Response Questions

2.1 Why does a hunter aiming a bird in a tree miss the target exactly at the bird?

2.2 A person falling on a heap of sand does not hurt more as compared to a person falling on a concrete floor. Why?

2.3 State the conditions under which birds fly in air.

2.4 Describe the circumstances for which velocity and acceleration of a vehicle are:

(i) v is zero but a is not zero

(ii) a is zero but v is not zero

(iii) perpendicular to one another

2.5 Describe briefly effects of air resistance on the range and maximum height of a projectile.

Comprehensive Questions

2.1 Define and explain scalar product. Write down its important characteristics.

2.2 Define and explain vector product of two vectors. Discuss important characteristics of vector product.

2.3 Derive three equations of motion by graphical method.

2.4 What is projectile motion? Explain.

2.5 Derive the following expressions for projectile motion:

(i) time of flight

(ii) height attained

(iii) range for projectile.

2.6 Explain elastic collision in one dimension. Show that magnitude of relative velocities before and after collision are equal.

2.7 Explain elastic collision in two dimensions.

2.8 Explain an inelastic collision in one and two dimensions.

Numerical Problems

2.1 The magnitude of cross and scalar products of two vectors are $4\sqrt{3}$ and 4, respectively. Find the angle between the vectors. (Ans: 60°)

2.2 A helicopter is ascending vertically at the rate of 19.6 m s^{-1}. When it is at a height of 156.8 m above the ground, a stone is dropped. How long does the stone take to reach the ground? (Ans: 8.0 s)

2.3 If $|\mathbf{A}+\mathbf{B}| = |\mathbf{A}-\mathbf{B}|$, then prove that **A** and **B** are perpendicular to each other.

(Ans: $\theta = 90°$)

2.4 A body of mass M at rest explodes into 3 pieces, two of which of mass $M/4$ each are thrown off in perpendicular directions with velocities of $3\ m\ s^{-1}$ and $4\ m\ s^{-1}$, respectively. Find the velocity of 3rd piece with which it will be flown away.

(Ans: $2.5\ m\ s^{-1}$, opposite to resultant velocity vector of two pieces)

2.5 A cricket ball is hit upward with velocity of $20\ m\ s^{-1}$ at an angle of 45° with the ground. Find its:

(a) time of flight (b) maximum height (c) how far away it hits the ground

(Ans: 2.9 s, 41 m, 10.2 m)

2.6 A 20 g ball hits the wall of a squash court with a constant force of 50 N. If the time of impact of force is 0.50 s, find the impulse. (Ans: 25 N s)

2.7 A ball is kicked by a footballer. The average force on the ball is 240 N, and the impact lasts for a time interval of 0.25 s.

(a) Calculate change in momentum

(b) State the direction of change in momentum

[Ans: (a) 60 N s, (b) In the direction of force]

2.8 An aeroplane is moving horizontally at a speed of $200\ m\ s^{-1}$ at a height of 8 km to drop a bomb on a target. Find horizontal distance from the target at which the bomb should be released. (Ans: 8.08 km)

2.9 Why does range R of a projectile remain the same when angle of projection is changed from θ to $\theta' = 90° - \theta$. Also show that for complementary angles of projection, the ratio R/R' is equal to 1.

2.10 A trolley of mass 1.0 kg moving with velocity $1.0\ m\ s^{-1}$ collides with a similar trolley at rest:

(i) after collision, the 1st trolley comes to rest whereas the second starts moving with velocity of $1.0\ m\ s^{-1}$ in the same direction. Show that it is an example of an elastic collision.

(ii) after the collision, they stick together and move away with a velocity of $0.5\ m\ s^{-1}$. Show that it is an example of an inelastic collision.

2.11 A railway wagon of mass 4×10^4 kg moving with velocity of $3\ m\ s^{-1}$ collides with another wagon of mass 2×10^4 kg which is at rest. They stick together and move off together. Find their combined velocity. (Ans: $2\ m\ s^{-1}$)

2.12 A car with mass 575 kg moving at $15.0\ m\ s^{-1}$ smashes into the rear end of a car with mass 1575 kg moving at $5\ m\ s^{-1}$ in the same direction.

(a) What is the final velocity if the wrecked car lock together?

(b) How much kinetic energy is lost in the collision?

[Ans: (a) $7.67\ m\ s^{-1}$, (b) 2.11×10^4 J]

Chapter 3 Circular and Rotational Motion

Learning Objectives

After studying this chapter, the students will be able to:

- Express angles in radians
- Define and calculate angular displacement, angular velocity and angular acceleration [This involves use of $S = r\theta$, $v = r\omega$, $\omega = 2\pi r/T$, $a = r\omega^2$, and $a = v^2/r$ to solve problems]
- Use equations of angular motion to solve problems involving rotational motions.
- Discuss qualitatively motion in a curved path due to a perpendicular force.
- Define and calculate centripetal force [Use $F_c = mr\omega^2$, $F_c = mv^2/r$]
- Analyze situations involving circular motion in terms of centripetal force [e.g. situations in which centripetal acceleration is caused by a tension force, a frictional force, a gravitational force, or a normal force.]
- Define and calculate average orbital speed for a satellite. [from the equation $v = 2\pi r/T$ where r is the average radius of the orbit and T is the orbit period; apply this equation to solve numerical problems]
- Explain why the objects in orbiting satellites appear to be weightless.
- Describe how artificial gravity is created to counter weightlessness.
- Define and calculate moments of inertia of a body and angular momentum.
- Derive and apply the relation between torque, moment of inertia and angular acceleration. Illustrate the applications of conservation of angular momentum in real life. [such as by flywheels to store rotational energy, by gyroscopes in navigation systems, by ice skaters to adjust their angular velocity]
- Describe how a centrifuge is used to separate materials using centripetal force

Among all possible motions of the material bodies, the circular motion is one that appears to be working in the most of the natural world. Satellites moving in circular orbits around the Earth, orbital and spin motion of the Earth itself, a car turning around a curved road, and a stone whirled around by a string are the familiar examples. When objects move in circular paths, their direction is continuously changing. Since velocity is a vector quantity, this change of direction means that their velocities are not constant.

In this chapter, we will study, circular motion, rotational motion, moment of inertia, angular momentum and the related topics.

3.1 ANGULAR MEASUREMENTS

Consider an angle subtended at the centre 'O' of a circle by an arc 'AB' as shown in Fig. 3.1. If the length of the arc 'AB' is equal to the radius 'r' of the circle, then the angle is called one radian. It is the SI unit of angular measurement and its symbol is "rad".

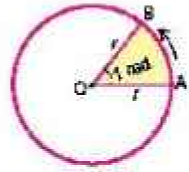


Fig. 3.1

Angular Displacement

Consider the motion of a single particle P of mass m in a circular path of radius r. Suppose this motion is taking place by attaching the particle P at the end of a massless rigid rod of length r whose other end is pivoted at the centre O of the circular path, as shown in Fig. 3.2 (a). As the particle is moving on the circular path, the rod OP rotates in the plane of the circle. The axis of rotation passes through the pivot O and is normal to the plane of rotation. Consider a system of axes as shown in Fig. 3.2 (b). The z-axis is taken along the axis of rotation with the pivot O as origin of coordinates. Axes x and y are taken in the plane of rotation. While OP is rotating, suppose at any instant t, its position is OP_1, making angle θ with x-axis. At a later time $t + \Delta t$, let its position be OP_2 making angle $\theta + \Delta\theta$ with x-axis (Fig. 3.2-c).

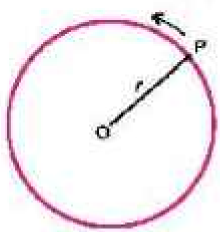


Fig. 3.2(a)

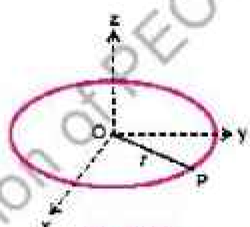


Fig. 3.2(b)

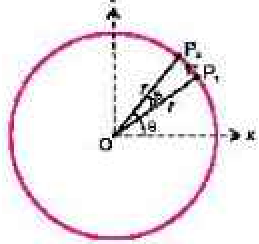


Fig. 3.2(c)

Angle $\Delta\theta$ defines the angular displacement of OP during the time interval Δt. For very small values of $\Delta\theta$, the angular displacement is a vector quantity.

The angular displacement $\Delta\theta$ is assigned a positive sign when the sense of rotation of OP is counter clock wise.

The direction associated with $\Delta\theta$ is along the axis of rotation and is given by right hand rule as shown in Fig 3.2 (d) which states that:

Grasp the axis of rotation in right hand with fingers curling in the direction of rotation; the thumb points in the direction of angular displacement.

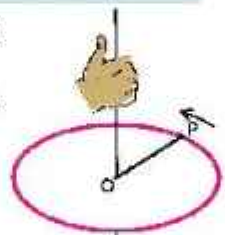


Fig. 3.2(d)

Three units are generally used to express angular displacement, namely degrees, revolution and radian. Consider an arc of length S of a circle of radius r (Fig. 3.3) which subtends an angle θ at the centre of the circle. Its value in radians (rad) is given as:

$$\theta = \frac{S}{r}$$

or $\qquad S = r\theta \qquad$ (where θ is in radian) ..... (3.1)

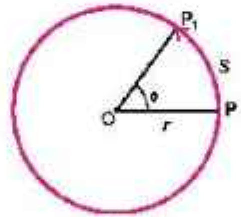


Fig. 3.3

If OP is rotating, the point P covers a distance $S = 2\pi r$ in one revolution of P. In radian, it would be:

$$\frac{S}{r} = \frac{2\pi r}{r} = 2\pi$$

So $\qquad$ 1 revolution = 2π rad = 360°

or $\qquad 1 \text{ rad} = \frac{360^\circ}{2\pi} = 57.3^\circ$

Angular Velocity

Very often, we are interested in knowing how fast or how slow a body is rotating. It is determined by its angular velocity defined as the rate at which the angular displacement is changing with time. Referring to Fig. 3.2(c), if $\Delta\theta$ is the angular displacement during the time interval Δt, the average angular velocity ω_{av} during this interval is given by

$$\omega_{av} = \frac{\Delta\theta}{\Delta t} \qquad \text{.........} \qquad (3.2)$$

The instantaneous angular velocity ω is the limit of the ratio $\Delta\theta/\Delta t$ as Δt, approaches to zero.

Thus $\qquad \omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} \qquad$ .... (3.3)

In the limit when Δt approaches zero, the angular displacement would be infinitesimally small. So, it would be a vector quantity and the angular velocity as defined by Eq. 3.3 would also be a vector. Its direction is along the axis of rotation and is given by right hand rule as described earlier.

Angular velocity is measured in radians per second which is the SI unit. Sometimes it is also given in terms of revolution per minute (rpm).

Angular acceleration

When we switch on an electric fan, we notice that its angular velocity goes on increasing till it becomes uniform. We say that it has an angular acceleration. We define angular acceleration as the rate of change of angular velocity. If ω_i and ω_f are the values of instantaneous velocity of a rotating body at instants t_i and t_f, the average angular acceleration during the interval $t_f - t_i$ is given by

$$\alpha_{av} = \frac{\omega_f - \omega_i}{t_f - t_i} = \frac{\Delta\omega}{\Delta t} \qquad \text{..........} \qquad (3.4)$$

The instantaneous angular acceleration is the limit of the ratio $\frac{\Delta\omega}{\Delta t}$ as Δt approaches zero. Therefore, instantaneous angular acceleration α is given by

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} \quad .... \quad (3.5)$$

The angular acceleration is also a vector quantity whose magnitude is given by Eq. 3.5 and its direction is along the axis of rotation. Angular acceleration is expressed in rad s^{-2}.

Till now we have been considering the motion of a particle P on a circular path. The point P was fixed at the end of a rotating massless rigid rod. Now consider the rotation of a rigid body as shown in Fig. 3.4. Imagine a point P on the rigid body. Line OP is the perpendicular dropped from P on the axis of rotation usually referred as the reference line. As the body rotates, line OP also rotates with the same angular velocity and angular acceleration. Thus, the rotation of a rigid body can be described by the rotation of the reference line OP and all the terms that we defined with the help of rotating line OP are also valid for the rotational motion of a rigid body. Henceforth, while dealing with rotation of a rigid body, we will replace it by its reference line OP.

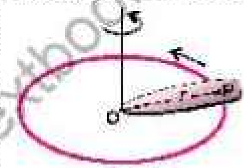


Fig. 3.4

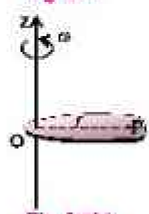


Fig. 3.5(a)

Relation between Angular and Linear Velocities

Consider a rigid body rotating about z-axis with an angular velocity ω as shown in Fig. 3.5 (a).

Imagine a point P in the rigid body at a perpendicular distance r from the axis of rotation. OP represents the reference line of the rigid body. As the body rotates, the point P moves along a circle of radius r with a linear velocity **v** whereas the line OP rotates with angular velocity ω as shown in Fig. 3.5 (b). We are interested in finding out the relation between ω and **v**. As the axis of rotation is fixed, so the direction of ω always remains the same and ω can be manipulated as a scalar. As regards the linear velocity of the point P, let us consider only its magnitude only which can also be treated as a scalar.

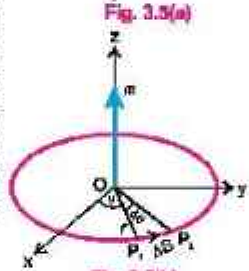


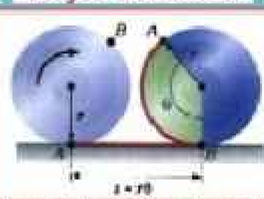
Fig. 3.5(b)

$$P_1P_2 = \Delta S$$

Suppose during the course of its motion, the point P moves through a distance $P_1P_2 = \Delta S$ in a time interval Δt during which reference line OP covers an angular displacement $\Delta\theta$ radian. So, ΔS and $\Delta\theta$ are related by Eq. 3.1 as:

$$\Delta S = r\Delta\theta$$

For your information



As the wheel turns through an angle θ, it lays out a tangential distance $S = r\theta$.

Dividing both sides by Δt

$$\frac{\Delta S}{\Delta t}=r\frac{\Delta \theta}{\Delta t} \quad \dots\dots\dots\dots \quad (3.6)$$

In the limit when linear $\Delta t \to 0$ the ratio $\Delta S/\Delta t$ represents v, the magnitude of the linear velocity with which point P is moving on the circumference of the circle. Similarly $\Delta\theta/\Delta t$ represents the angular velocity ω of the reference line OP. So, Eq. 3.6 becomes:

$$v = r\omega \quad \dots\dots\dots\dots \quad (3.7)$$

Point to ponder!

You may feel scared at the top of roller coaster ride in the amusement parks but you never fall down even when you are upside down. Why?

From Fig. 3.5(b), it can be seen that the point P is moving along the arc P_1P_2. In the limit when $\Delta t \to 0$, the length of arc P_1P_2 becomes very small and its direction represents the direction of tangent to the circle at point P_1. Thus, the velocity with which point P is moving on the circumference of the circle has a magnitude v and its direction is always along the tangent to the circle at that point. That is why, the linear velocity of the point P is also known as tangential velocity.

Similarly, Eq. 3.7 shows that if the reference line OP is rotating with an angular acceleration α, the point P will also have a linear or tangential acceleration a_t. Using Eq. 3.7 it can be shown that the two accelerations are related by

$$a_t = r\alpha \quad \dots\dots\dots\dots \quad (3.8)$$

Eqs. 3.7 and 3.8 show that on a rotating body, points that are at different distances from the axis do not have the same speed or acceleration, but all points on a rigid body rotating about a fixed axis do have the same angular displacement, angular speed and angular acceleration at any instant. Thus, by the use of angular variables, we can describe the motion of the entire body in a simple way.

Equations of Angular Motion

The equations (3.2, 3.3, 3.4 and 3.5) of angular motion are exactly analogous to those in linear motion if θ, ω and α be replaced by S, v and a, respectively. As the other equations of linear motion were obtained by algebraic manipulation of these equations, it follows that analogous equations will also apply to angular motion. Given below are angular equations together with their linear counterparts.

Linear Equations	Angular Equations	
$v_f = v_i + at$	$\omega_f = \omega_i + \alpha t$	(3.9)
$2aS = v_f^2 - v_i^2$	$2\alpha\theta = \omega_f^2 - \omega_i^2$	(3.10)
$S = v_i t + \frac{1}{2}at^2$	$\theta = \omega_i t + \frac{1}{2}\alpha t^2$..........	(3.11)

The angular equations 3.9 to 3.11 hold true only in the case when the axis of rotation is fixed, so that all the angular vectors have the same direction. Hence, they can be manipulated as scalars.

Example 3.1 An electric fan rotating at 3 rev s^{-1} is switched off. It comes to rest in 18.0 s. Assuming deceleration to be uniform, find its value. How many revolutions did it turn before coming to rest?

Solution In this problem, we have

$\omega_i = 3.0 \text{ rev s}^{-1}$, $\omega_f = 0$, $t = 18.0\text{ s}$ and $\alpha = ?$, $\theta = ?$

From Eq. 3.9, we have

$$\alpha = \frac{\omega_f - \omega_i}{t} = \frac{(0-3.0)\text{ rev s}^{-1}}{18.0\text{ s}} = -0.167 \text{ rev s}^{-2}$$

and from Eq 3.11, we have

$$\theta = \omega_i t + \frac{1}{2}\alpha t^2$$

$= 3.0 \text{ rev s}^{-1} \times 18.0 \text{ s} + \frac{1}{2}(-0.167 \text{ rev s}^{-2}) \times (18.0 \text{ s})^2$

$= 27$ rev

Do you know?

Direction of motion changes continuously in circular motion.

3.2 CENTRIPETAL FORCE

Newton's second law of motion states that when a force acts on a body, it produces acceleration in the same direction. A force acting on a moving body along the direction of its velocity will change magnitude of the velocity (speed) keeping the direction unchanged. On the other hand, a constant force acting perpendicular to the velocity of a body moving in a circular path will change the direction but magnitude of velocity (speed) will remain the same. Such force makes the body move in a circle by producing a radial (or centripetal) acceleration and is called centripetal force (Centre seeking) force. Figure 3.6(a) shows a ball tied at the end of a string is whirled in a horizontal surface. It would not continue in a circular path if the string is snapped. Careful observation shows at once that if the string snaps, when the ball is at the point A, in Fig. 3.6 (b), the ball will follow the straight line path AB which is tangent AB at point A.

Fig. 3.6(a)

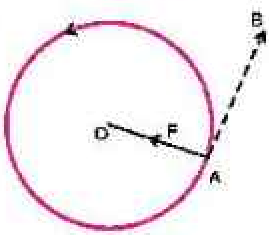


Fig. 3.6(b)

Thus, a force is needed to change the direction of velocity or motion of a body continuously at each point in circular motion moving with uniform speed. The force that does not alter speed but only direction at each point is a perpendicular force which acts along the radius of the circular path. This force always pulls the object towards the centre of the circular path. Its direction is perpendicular to the tangential velocity at each point.

The force needed to bend the straight path of the particle into a circular path is called the centripetal force.

For a body of mass m moving with velocity v in a circular path of radius r, centripetal force F_c is given by

$$F_c = ma_c = \frac{mv^2}{r} \quad .... \quad (3.12)$$

where $a = v^2/r$ is the centripetal acceleration and its direction is towards the centre of the circle. As $v = r\omega$, so the above equation becomes:

$$F_c = mr\omega^2 \quad .... \quad (3.13)$$

Example 3.2 If a CD spins at 210 rpm, what is the radial acceleration of a point on the outer rim of the CD? The CD is 12 cm in diameter.

Solution We convert 210 rpm into a frequency in revolutions per second (Hz).

Thus $f = 210 \frac{\text{rev}}{\text{min}} \times \frac{1}{60} \frac{\text{min}}{\text{s}} = 3.5 \frac{\text{rev}}{\text{s}} = 3.5 \text{ Hz}$

For each revolution, the CD rotates through an angle of 2π radians. The angular velocity is:

$$\omega = 2\pi f = 2\pi \text{ rad} \times 3.5 \text{ s}^{-1} = 7.0\pi \text{ rad s}^{-1}$$

The radial acceleration is:

$$a = \omega^2 r = (7.0\pi \text{ rad s}^{-1})^2 \times 0.06 \text{ m} = 29 \text{ m s}^{-2}$$

Do you know?

Curved flight at high speed requires a large centripetal force that makes the stunt dangerous even if the air planes are not so close.

Example 3.3 A ball tied to the end of a string, is swung in a vertical circle of radius r under the action of gravity as shown in Fig. 3.7. What will be the tension in the string when the ball is at the point A of the path and its speed is v at this point?

Solution For the ball to travel in a circle, the force acting on the ball must provide the required centripetal force. In this case, at point A, two forces act on the ball, the pull of the string and the weight w of the ball. These forces act along the radius at A, and so their vector sum must furnish the required centripetal force. We, therefore, have

$$T + w = \frac{mv^2}{r} \qquad \text{as} \quad w = mg, \text{ therefore,}$$

$$\therefore \quad T = \frac{mv^2}{r} - mg = m\left(\frac{v^2}{r} - g\right)$$

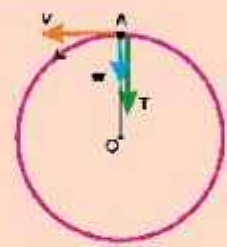


Fig. 3.7

If $\frac{v^2}{r} = g$, then T will be zero and the centripetal force is just equal to the weight.

Examples of centripetal force

In every circular or orbital motion, centripetal force is needed which is provided by some agency.

1. When a ball is whirled in a horizontal circle with the help of a string, then tension in the string provides necessary centripetal force.
2. For an object placed on a turntable, the friction is the centripetal force.
3. The gravitational force is the cause of the Earth orbiting around the Sun, Moon and artificial satellites revolving around the Earth.
4. A normal or perpendicular magnetic force compels a charge particle moving along a straight path into a circular path.
5. When a vehicle takes turn on a road, it also needs centripetal force which is provided by the friction between the tyres and the road. If the road is slippery, then at high speed, the friction may not be sufficient enough to provided necessary centripetal force.

Tidbits

Banked tracks are needed for turns that are taken so quickly that friction alone cannot provide energy for centripetal force.

Hence, vehicle will not be able to take turn and may skid or may even be toppled. To overcome this difficulty, the highway road is banked on turns. That is, the outer edge of the track is kept slightly higher than that of the inner edge.

Applications of Centripetal Force

We know that an object moves in a circle because of centripetal force. If the magnitude of applied force falls short of required centripetal force then the object will move away from the centre of the circle. The centrifuge (Fig. 3.8-a) functions on this basic principle.

Fig. 3.8 (a)

Centrifuge: It is one of the most useful laboratory device. It helps to separate out denser and lighter particles from a mixture. The mixture is rotated at high speed for a specific time. In a laboratory setup, sample tubes are used where the denser particles will settle at the bottom and lighter particles will rise to the top of the sample tubes (Fig. 3.8-b).

Fig. 3.8 (b)

The **dryer** of the washing machine also functions on the principle of centrifuge. The dryer consists of a long cylinder with hundreds of small holes on its wall. Wet clothes are piled up in this cylinder, which is then rotated rapidly about its axis. Water moves outward to the walls of the cylinder and thus, drained out through the holes. In this way, clothes become dry quickly.

Fig. 3.9

Cream separator is another practical device which is used to separate cream form the milk. In this machine, milk is whirled rapidly. Since milk is a mixture of light and heavy particles, when it is rotated, the light particles gather near the axis of rotation whereas the heavy particles will go outwards and hence, cream can easily be separated from milk.

Fig. 3.10

3.3 ARTIFICIAL SATELLITES

Satellites are objects that orbit in nearly circular path around the Earth. They are put into orbit by rockets and are held in orbits by the gravitational pull of the Earth. The low flying Earth satellites have acceleration 9.8 m s^{-2} towards the centre of the Earth. If there is no gravitational pull, they would fly off in a straight line along tangent to the orbit. When the satellite is moving in a circle, it has an acceleration $\frac{v^2}{r}$. In a circular orbit around the Earth, the centripetal acceleration is supplied by gravity and we have

$$g = \frac{v^2}{R} \qquad \text{..........} \qquad (3.14)$$

where v is the orbital velocity and R is the radius of the Earth (6400 km). From Eq. 3.14, we have

$$v = \sqrt{gR}$$

$$= \sqrt{9.8\ \text{m s}^{-2} \times 6.4 \times 10^6\ \text{m}}$$

$$= 7.9 \times 10^3\ \text{m s}^{-1} \quad = 7.9\ \text{km s}^{-1}$$

This is the minimum velocity necessary to put a satellite into the orbit, called the critical velocity. The period T is given by

$$T = \frac{2\pi R}{v} \quad = 2 \times 3.14 \times \frac{6400\ \text{km}}{7.9\ \text{km s}^{-1}}$$

$$= 5060\ \text{s} = 84\ \text{min approx.}$$

If, however, a satellite in a circular orbit is at a distance h much greater than R above the Earth's surface, we must take into account the experimental fact that the gravitational acceleration decreases inversely as the square of the distance from the centre of the Earth (Fig. 3.11).

The higher the satellite, the slower will be the required speed and longer it will take to complete one revolution around the Earth.

For your information

Satellites Orbits

Do you know?

Newton had predicated about the artificial satellites 300 years ago. The above figure has been taken from his well-known book "Principia Mathematica". According to this book, if an object is thrown horizontally with a particular speed from a place which is sufficiently high, it will start revolving around the Earth.

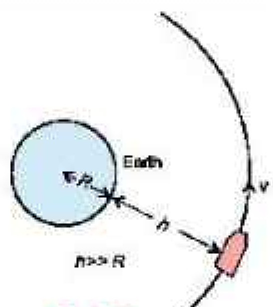


Fig. 3.11

Orbital Velocity

Figure 3.12 shows a satellite going round the Earth in a circular path. Let the mass of the satellite be m_s and v is its orbital speed. The mass of the Earth is M and r represents the radius of the orbit. A centripetal force m_sv^2/r is required to hold the satellite in the orbit. This force is provided by the gravitational force of attraction between the Earth and the satellite. Equating the gravitational force to the required centripetal force, we have

$$\frac{Gm_sM}{r^2}=\frac{m_sv^2}{r}$$

or $$v=\sqrt{\frac{GM}{r}} \quad \ldots\ldots\ldots \quad (3.15)$$

This shows that the mass of the satellite is unimportant in describing the satellite's orbit. Thus, any satellite orbiting at distance r from the Earth's centre must have the orbital speed given by Eq. 3.15. Any speed less than this will bring the satellite tumbling back to the Earth.

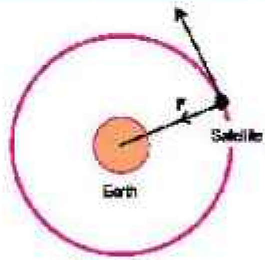


Fig. 3.12

Tidbits

The moment you switch on your mobile phone, your location can be tracked immediately by global positioning system.

Example 3.4 An Earth satellite is in circular orbit at distance of 384,000 km from the Earth's surface. What is its period of one revolution in days? Take mass of the Earth $M = 6.0 \times 10^{24}$ kg and its radius $R = 6400$ km.

Solution

As $r = R + h = (6400 + 384000)\ km = 390400\ km$

Using $v=\sqrt{\frac{GM}{r}}=\sqrt{\frac{6.67\times10^{-11}\ N\ m^2\ kg^{-2}\times 6\times10^{24}\ kg}{390400\ km}}$

$= 1.025\ km\ s^{-1}$

Also

$T=\frac{2\pi R}{v}=2\times3.14\times390400\ km\times\frac{1}{1.025\ km\ s^{-1}}\times\frac{1\ day}{60\times60\times24\ s}$

$= 27.7$ days

Tidbits

In 1984, at a height of 100 km above Hawaii island with a speed of 29000 km h^{-1} Bruce McCandless stepped into space from a space shuttle and became the first human satellite of the Earth.

Weightlessness in Satellites

When a satellite is launched by a rocket in its desired orbit around the Earth, then it has been observed practically that everything inside the satellite experiences weightlessness because the satellite is accelerating towards the centre of the Earth as a freely falling body.

Do you know?

Your weight slightly changes when the velocity of the elevator changes at the start and end of a ride, not during the rest of the ride when that velocity is constant.

Consider a satellite of mass M revolving in its orbit of radius r around the Earth. A body of mass m inside the satellite suspended by a spring balance from the ceiling of the satellite is under the action of two forces. That is, its weight mg acting downward, while the supporting force, called normal force F_N or tension in the spring acting upward, as shown in Fig. 3.13. Their resultant force is equal to the centripetal force required by the mass m which is acting towards the centre of the Earth, and is expressed as:

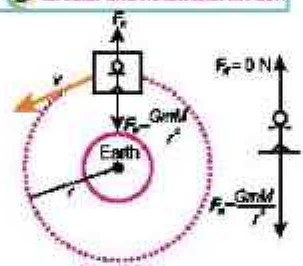


Fig. 3.13

Can you tell?

When a bucket full of water is rapidly whirled in a vertical circular path, water does not fall out even if the bucket is inverted at the maximum height. Why is it so?

$$F_c = mg - F_N \quad \text{........} \quad (3.16)$$

where $\quad F_c = \dfrac{mv^2}{r}$

Hence $$\frac{mv^2}{r} = mg - F_N \quad \text{........} \quad (3.16\text{-a})$$

It may be noted that the centripetal force responsible for the revolution of the satellite of mass M around the Earth is provided by the gravitational force of attraction between the Earth and the satellite.

$$F_g = F_c$$

$$Mg = \frac{Mv^2}{r}$$

$$g = \frac{v^2}{r}$$

Hence, Eq. 3.16(a) becomes:

$$mg = mg - F_N$$

or $$F_N = 0$$


Astronaut floating inside the cabin of a spaceship.

This shows that the supporting force which is acting on a body inside the satellite is zero. Therefore, the bodies as well as the astronauts in a satellite find themselves in a state of apparent weightlessness.

Artificial Gravity

In a gravity free space, there will be no force that will push anybody to any side of the spacecraft. If this spacecraft is to stay in the orbit over an extended period of time, the weightlessness may affect the performance of the astronauts present in that spacecraft. To overcome this difficulty, an artificial gravity can be created in the spacecraft. This could enable the crew of the space ships to function in an almost normal manner. For this situation to prevail, the spaceship is set into rotation around its own axis. The astronaut then is pressed towards the outer rim and exerts a force on the 'floor' of the spaceship in much the same way as on the Earth.

Consider a spacecraft of the shape as shown in Fig. 3.14. The outer radius of the spaceship is R and it rotates around its own central axis with angular speed ω, then its angular acceleration a_c is

$$a_c = R\,\omega^2$$

But $\omega = \frac{2\pi}{t}$ where t is the period of revolution of spaceship

Hence $$a_c = R\frac{(2\pi)^2}{t^2} = R\frac{4\pi^2}{t^2}$$

As frequency $f = 1/t$, therefore,

$$a_c = R\,4\pi^2 f^2$$

or $$f^2 = \frac{a_c}{4\pi^2 R} \quad \text{or} \quad f = \frac{1}{2\pi}\sqrt{\frac{a_c}{R}}$$

As described above, the force of gravity provides the required centripetal acceleration, therefore,

$$a_c = g$$

So $$f = \frac{1}{2\pi}\sqrt{\frac{g}{R}} \qquad \text{..........} \qquad (3.17)$$

When the spaceship rotates with this frequency, the artificial gravity like the Earth is provided to the inhabitants of the spaceship.

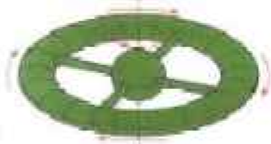


Fig. 3.14

Do you know?

The surface of the rotating space-ship pushes on an object with which it is in contact and thereby provides the centripetal force needed to keep the object moving on a circular path.

3.4 MOMENT OF INERTIA

Consider a mass m attached to the end of a massless rod as shown in Fig. 3.15. Assume that the bearing at the pivot point O is frictionless. Let the system be in a horizontal plane. A force F is acting on the mass perpendicular to the rod and hence, this will accelerate the mass according to:

$$F = ma$$

In doing so, the force will cause the mass to rotate about O. Since tangential acceleration a_T is related to angular acceleration α by the equation.

$$a_t = r\alpha$$

So $$F = mr\alpha$$

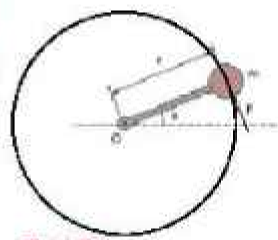


Fig. 3.15

The force F causes a torque about the axis O and gives the mass m an angular acceleration α about the pivot point.

As turning effect is produced by torque τ, it would, therefore, be better to write the equation for rotation in terms of torque. This can be done by multiplying both sides of the above equation by r. Thus,

$$rF = \tau = \text{torque} = mr^2\alpha$$

which is rotational analogue of the Newton's second law of motion, $F = ma$.

Here F is replaced by τ, a by α and m by mr^2. The quantity mr^2 is known as the moment of inertia and is represented by I. The moment of inertia plays the same role in angular motion as the mass in linear motion. It may be noted that moment of inertia depends not only on mass m but also on r^2.

Most rigid bodies have different mass concentration at different distances from the axis of rotation, which means the mass distribution is not uniform. As shown in Fig. 3.16(a), the rigid body is made up of n small pieces of masses m_1, m_2, ... at distances r_1, r_2, ... from the axis of rotation O. Let the body be rotating with the angular acceleration α, so the magnitude of the torque acting on m_1 is

$$\tau_1 = m_1 r_1^2 \alpha_1$$

Similarly, the torque on m_2 is

$$\tau_2 = m_2 r_2^2 \alpha_2$$

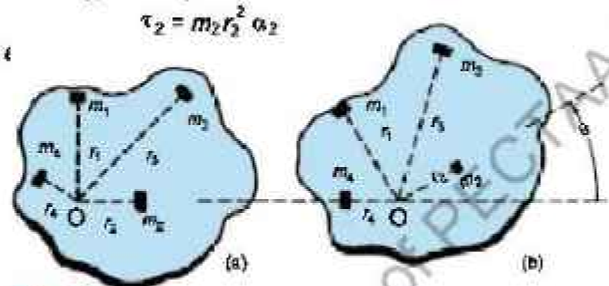


Fig. 3.16

Each small piece of mass within a large, rigid body undergoes the same angular acceleration about the pivot point.

Since the body is rigid, so all the masses are rotating with the same angular acceleration α.

Total torque τ_{total} is then given by

$$\tau_{total} = (m_1 r_1^2 + m_2 r_2^2 + ... + m_n r_n^2)\,\alpha$$

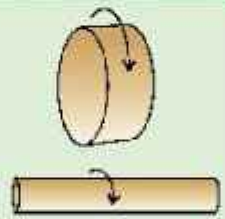
$$= \left(\sum_{i=1}^{n} m_i r_i^2\right)\alpha$$

or $\tau = I\alpha$ .... (3.18)

where I is the moment of inertia of the body and is expressed as

$$I = \sum_{i=1}^{n} m_i r_i^2 \quad .... \quad (3.19)$$

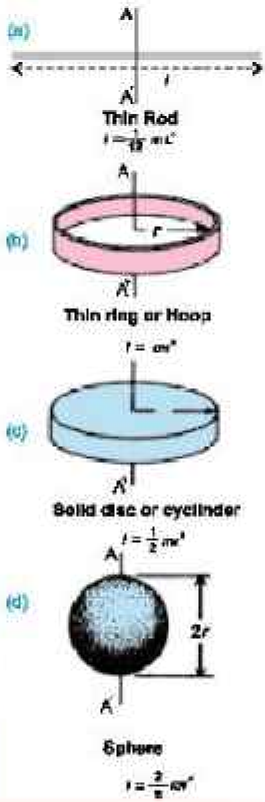
Do you know?



Two cylinders of equal mass. The one with the larger diameter has the greater rotational inertia.

For your information

Moments of inertia of various bodies about axis AA'



3.5 ANGULAR MOMENTUM

We have already seen that linear momentum plays an important role in translational motion of bodies. Similarly, another quantity known as angular momentum has important role in the study of rotational motion.

A particle is said to possess an angular momentum about a reference axis if it so moves that its angular position changes relative to that reference axis.

The angular momentum **L** of a particle of mass m moving with velocity **v** and momentum **p** (Fig. 3.17) relative to the origin O is defined as:

$$\mathbf{L} = \mathbf{r} \times \mathbf{p} \quad \text{..........} \quad (3.20)$$

Fig. 3.17

where **r** is the position vector of the particle at that instant relative to the origin O. Angular momentum is a vector quantity. Its magnitude is:

$$L = rp\sin\theta = m\,rv\sin\theta$$

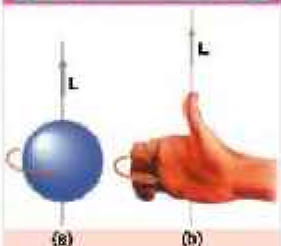
where θ is the angle between **r** and **p**. The direction of **L** is perpendicular to the plane formed by **r** and **p** and its sense is given by the right hand rule of vector product. SI unit of angular momentum is kg m^2 s^{-1} or J s.

If the particle is moving in a circle of radius r with uniform angular velocity ω, then angle between r and tangential velocity is 90°. Hence,

$$L = mrv\sin 90^\circ = mrv$$

But $\quad v = r\omega \quad$ Hence $\quad L = mr^2\omega$

For your information



The sphere in (a) is rotating in the sense given by the gold arrow. Its angular velocity and angular momentum are taken to be upward along the rotational axis, as shown by the right-hand rule in (b).

Now consider a symmetric rigid body rotating about a fixed axis through the centre of mass as shown in Fig 3.18. Each particle of the rigid body rotates about the same axis in a circle with an angular velocity ω. The magnitude of the angular momentum of the particle of mass m_i is $m_i v_i r_i$ about the origin O. The direction of **L** is the same as that of ω. Since $v_i = r_i\omega$, the angular momentum of the ith particle is $m_i r_i^2\omega$. Summing this over all particles gives the total angular momentum of the rigid body.

$$\mathbf{L} = \left(\sum_{i=1}^{n} m_i r_i^2\right)\omega = I\omega$$

where I is the moment of inertia of the rigid body about the axis of rotation.

Fig. 3.18

Example 3.5 The mass of Earth is 6.00×10^{24} kg. The distance r from Earth to the Sun is 1.50×10^{11} m. As seen from the direction of the North Star, the Earth revolves counter-clockwise around the Sun. Determine the orbital angular momentum of the Earth about the Sun, assuming that it traverses a circular orbit about the Sun once a year (3.16×10^7s).

Solution To find the Earth's orbital angular momentum, we must first know its orbital speed from the given data. When the Earth moves around a circle of radius r, it travels a distance of $2\pi r$ in one year. Its orbital speed v_o is thus, $v_o = \frac{2\pi r}{t}$

Orbital angular momentum of the Earth = $L_o = mv_o r$

$$L = \frac{2\pi r^2 m}{t}$$

$$= \frac{2\pi(1.50 \times 10^{11}\text{m})^2 \times (6.00 \times 10^{24}\text{kg})}{3.16 \times 10^7\text{s}}$$

$$L = 2.67 \times 10^{40}\ \text{kg m}^2\ \text{s}^{-1}$$

The sign is positive because the revolution is counter clockwise.

3.6 LAW OF CONSERVATION OF ANGULAR MOMENTUM

The law of conservation of angular momentum states that if no external torque acts on a system, the total angular momentum of the system remains constant.

$$L_{total} = L_1 + L_2 + \ldots = \text{constant}$$

The law of conservation of angular momentum is one of the fundamental principles of Physics. It has been verified from the cosmological to the sub microscopic level. The effect of the law of conservation of angular momentum is readily apparent if a single isolated spinning body alters its moment of inertia.

If a body of moment of inertia I_1 spinning with angular speed ω_1 alters its moment of inertia to I_2, then its angular speed ω_2 also changes so that its angular momentum remains constant.

Hence $I_1\omega_1 = I_2\omega_2$

The angular momentum is a vector quantity with direction along the axis of rotation. Hence, the direction of angular momentum along the axis of rotation also remains fixed. This is illustrated by the fact given below:

The axis of rotation of an object will not change its orientation unless an external torque causes it to do so.

Do you know?

If you try to sit on a bike at rest, it falls. But if the bike is moving, the angular momentum of the spinning wheel resists any tendency to change and helps to keep the bike upright and stable.

Do you know?

The ball's speed increases as the string wraps around the finger.

This fact is of great importance for the Earth as it moves around the Sun. No other sizeable torque is experienced by the Earth, because the major force acting on it is the pull of the Sun. The Earth's axis of rotation, therefore, remains fixed in one direction with reference to the universe around us.

Examples of conservation of angular momentum

A man diving from a diving board

A diver jumping from a springboard has to take a few somersaults in air before touching the water surface, as shown in Fig.3.19. After leaving the springboard, he curls his body by rolling arms and legs in. Due to this, his moment of inertia decreases, and he spins in midair with a large angular velocity. When he is about to touch the water surface, he stretches out his arms and legs. He enters the water at a gentle speed and gets a smooth dive. This is an example of the law of conservation of angular momentum.

Fig. 3.19
A man diving from a diving board.

The spinning ice skater

An ice skater as shown in Fig.3.20 can increase his angular velocity by folding arms and bringing the stretched leg close to the other leg. By doing so, he decreases his moment of inertia. As a result, angular speed increases. When he stretches his hands and a leg outward, the moment of inertia increases and hence angular velocity decreases.

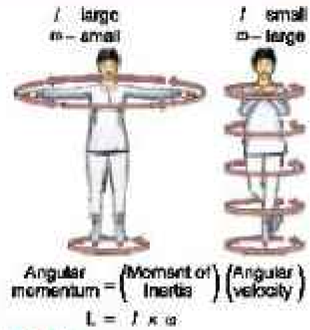


Fig. 3.20
An ice skater using angular momentum

A person holding some weight in his hands standing on a turntable.

A person is standing on a turntable with heavy mass (dumb-bell) in his hands stretched out on both sides as shown in Fig. 3.21. As he draws his hands inward, his

(a) (b)

Fig. 3.21
Man with masses in his hands on a turntable. Conservation of angular momentum requires that as the man pulls his arms in, the angular velocity increases.

For your information

It has been noticed that when ice on the polar caps of Earth melts and water flows away in the form of river, the moment of inertia of water and hence that of Earth about its axis of rotation increases due to conservation of angular momentum. Hence, the angular velocity of Earth will decrease, therefore, the duration of day increases slightly.

angular speed at once increases. This is because the moment of inertia decreases on drawing the hands inwards, which increases the angular speed.

Flywheel

Flywheel is a mechanical device which consists of a heavy wheel with an axle (Fig. 3.22). It is used to store rotational energy, smooth out output fluctuations and provides stability in a wide range of applications such as bicycles and other vehicles, industrial machinery, gyroscopes, ships and spacecrafts.

Do you know?

The flywheel called the balance wheel regulates the time keeping mechanism in mechanical clocks and watches by maintaining controlled oscillations rate.

Fig. 3.22: A flywheel

Point to ponder!

Why does the coasting rotating system slow down as water drips into the beaker?

When a fly wheel spins, its angular momentum resists changes to its orientations, maintaining stability. This is useful in systems that need precise control over their orientation without external interference.

The Gyroscope

A gyroscope is a device which is used to maintain its orientation relative to the Earth's axis or resists changes in its orientation. It consists of a mounted flywheel pivoted in supporting rings as shown in Fig. 3.23. It works on the basis of law of conservation of angular momentum due to its large moment of inertia. When the gyroscope spins at a large angular speed, it gains large angular momentum. It is then difficult to change the orientation of the gyroscope's rotational axis due to its large moment of inertia. A change in orientation requires a change in its angular momentum. To change the direction of a large angular momentum, a corresponding large torque is required. Even if gyroscope is tilted (Fig. 3.24), it still keeps levitated without falling. Hence, it is a reason why a gyroscope can be used to maintain orientation. The

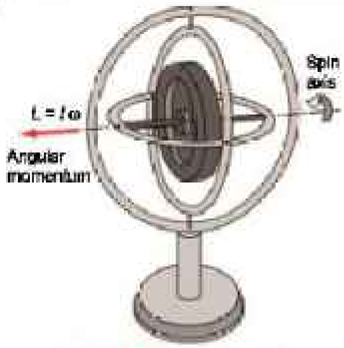


Fig. 3.23: The gryscope

Fig. 3.24

main applications of gyroscope are in the guiding system of aeroplanes, submarines and space vehicles in order to maintain a specific direction in space to keep steady course.

Point to ponder!

Planets move around the Sun in elliptical orbits with Sun situated at one of its foci, thus, distance of a planet from the Sun is not constant when it is nearer the Sun. Its orbital velocity increases automatically. Why?

QUESTIONS

Multiple Choice Questions

Tick (✓) the correct answer.

3.1 The ratio of angular speed of minute's hand and hour's hand of a watch is:
(a) 1 : 6 (b) 6 : 1
(c) 1 : 12 (d) 12 : 1

3.2 A body travelling in a circle at constant speed:
(a) has constant velocity (b) has an inward radial acceleration
(c) is not accelerated (d) has an outward radial acceleration

3.3 A stone at the end of long string is whirled in vertical circle at a constant speed. The tension in the string will be minimum when the stone is:
(a) at the top of the circle (b) half way down
(c) at the bottom of circle (d) any where in the circle

3.4 Every point of rotating rigid body has:
(a) same angular velocity (b) same linear velocity
(c) same linear acceleration (d) same linear distance

3.5 The minimum velocity necessary to put a satellite into the orbit is called:
(a) terminal velocity (b) critical velocity
(c) artificial velocity (d) angular velocity

3.6 An astronaut is orbiting around the Earth in a large capsule. Then,
(a) he will be in a state of weightlessness with respect to capsule
(b) he is freely falling towards the Earth
(c) a ball projected at an angle has a straight line path as observed by him
(d) all the above

3.7 An object in uniform circular motion makes 10 revolutions in 2 seconds. Which of the following statement is true?
(a) Its period is 2.0 s (b) Its period is 20 s
(c) Its frequency is 5 Hz (d) Its frequency is 0.2 Hz

3.8 A man inside the artificial satellite feels weightlessness because the force of attraction due to the Earth is:
(a) zero at pole
(b) balanced by the force of attraction due to the moon
(c) equal to the centripetal force
(d) non-effective due to some particular design of the satellite

3.9 A bottle of soda water is grasped from the neck and swung briskly in a vertical circle. Near which portion of the bottle do the bubbles collect?
(a) Near the bottom (b) In the middle of bottle
(c) Bubbles remain distributed throughout the volume of the bottle
(d) Near the neck of the bottle

3.10 The moment of inertia of body depends upon:
(a) mass of the body and its distribution about axis of rotation
(b) volume of the body
(c) kinetic energy of the body
(d) angular momentum of the body

Short Answer Questions

3.1 State second law of motion in case of rotation.
3.2 What is the effect of changing the position of a diver while diving in the pool?
3.3 How do we get butter from the milk by using centrfuge?
3.4 Mass is a measure of inertia in linear motion. What is its analogue in rotational motion? Describe briefly.
3.5 Why is it harder for a car to take turn at higher speed than at lower speed?
3.6 What are the benefits of using rear wheels of heavy vehicles consisted of double tyres?
3.7 When a moving car turns around a corner to the left, in what direction do the occupants tend to fall? Explain briefly.
3.8 Why is the acceleration of a body moving uniformly in a circle, directed towards the centre?
3.9 How does an astronaut feel weightlessness while orbiting from the Earth in a spaceship?

Constructed Response Questions

3.1 If angular velocity of different particles of a rigid body is constant, will the linear velocity of these particles be also constant?
3.2 A loaf of bread is lying on rotating plate. A crow takes away the loaf of bread and the rotation of the plate increases. Why?

3.3 Why do we tumble when we take the sharp turn with large speed?

3.4 What will be time period of a simple pendulum in an artificial satellite at a certain height?

3.5 Is the motion of a satellite in its orbit, uniform or accelerated?

3.6 What are the advantages that radian has been preferred as SI unit to degree?

3.7 In uniform circular motion, what are the average velocity and average acceleration for one revolution? Explain.

3.8 In a rainstorm with a strong wind, what determines the best position to hold an umbrella?

3.9 A ball is just supported by a string without breaking. If it is whirled in a vertical circle, it breaks. Explain why.

3.10 How is the centripetal force supplied in the following cases;

(a) a satellite orbiting around the Earth?

(b) a car taking a turn on a level road?

Comprehensive Questions

3.1 What is meant by angular momentum? Explain the law of conservation of angular momentum with daily life examples.

3.2 Show that orbital angular momentum is; $L = I\omega$.

3.3 Define moment of inertia. Prove that torque acting on rotating rigid body is equal to the product of its moment of inertia and angular acceleration.

3.4 What are artificial satellites? Calculate the minimum time period necessary to put a satellite into the orbit near the surface of the Earth.

3.5 Define orbital velocity and derive an expression for the same.

3.6 Write a note on artificial gravity. Derive an expression for frequency with which the spaceship rotates to provide artificial gravity.

3.7 Prove that; (i) $v = r\omega$ and (ii) $a = r\alpha$

Numerical Problems

3.1 A laser beam is directed from the Earth to the moon. The beam spreads over a diameter of 2.50 m at the moon surface. What is divergence angle of the beam? The distance of moon from the Earth is 3.8×10^8 m. (Ans: 6.6×10^{-9} rad)

3.2 A car is moving with a speed of 108 km h^{-1}. If its wheel has a diameter of 60 cm, find its angular speed in rad s^{-1} and rev s^{-1}. (Ans: 100 rad s^{-1}, 16 rev s^{-1})

3.3 An electric motor is running at 1800 rev min^{-1}. On switching off, it comes to rest in 20 s. If angular retardation is uniform, find the number of revolutions it makes before stopping. (Ans: 300 rev)

3.4 A string 0.5 m long holding a stone can withstand maximum tension of 35.6 N. Find the maximum speed at which a stone of 0.5 kg mass can be whirled with it in a vertical circle. (Ans: 5.5 m s^{-1})

3.5 The flywheel of an engine is rotating at 2100 rev min^{-1} when the power source is shut off. What torque is required to stop it in 3 minutes? The moment of inertia of the flywheel is 36 kg m^2. (Ans: 44 N m)

3.6 What is the moment of inertia of a 200 kg sphere whose diameter is 60 cm? (Ans: 7.2 kg m^2)

3.7 A satellite is orbiting the Earth at an altitude of 200 km. Assuming the Earth's radius is 6400 km, calculate the orbital speed of the satellite. (Ans: 7.77 km s^{-1})

3.8 A space station has a radius of 20 m and rotates at an angular velocity of 0.5 rad s^{-1}. What is the artificial gravity experienced by the astronauts on the space station? (Ans: 5 m s^{-2})

3.9 A bicycle wheel has an angular momentum of 10 kg m^2 s^{-1} and angular velocity of 2 rad s^{-1}. Find the value of its moment of inertia. (Ans: 5 kg m^2)

3.10 A diver comes off a board with arms straight up and legs straight down, giving him a moment of inertia of 18 kg m^2 about his rotation axis. Then tucks into a small ball, decreasing his moment of inertia to 3.6 kg m^2. While tucked, he makes two complete rotations in 1.0 second. If he had not tucked at all, how many revolutions would he have made in 1.5 s from board to water? (Ans: 0.6 rev)

Chapter 4 Work, Energy and Power

Learning Objectives

After studying this chapter, the students will be able to:

- Derive the formula for kinetic energy [using the equations of motion]
- Derive an expression for absolute potential energy of a body at a certain position in the gravitational field [including escape velocity]
- Deduce the work done from force-displacement graph.
- Differentiate between conservative and non-conservative forces.
- State and use the work - energy theorem in a resistive medium to solve problems.

Work is often thought in terms of physical or mental effort. In Physics, however, the term work involves two things (i) force, and (ii) displacement. We shall begin with a simple situation in which work is done by a constant force.

4.1 WORK DONE BY A CONSTANT FORCE

Let us consider an object which is being pulled by a constant force **F**. The force displaces the object through a displacement **d** in the direction of force. In such a case, work W is defined as the product of the magnitude of the force F and magnitude of the displacement **d**. This can be written as:

$$W = Fd \qquad (4.1)$$

Equation (4.1) shows that if the displacement is zero, no work is done even if a large force is applied. For example, pushing on a wall may tire your muscles, but work done is zero as shown in Fig. 4.1.

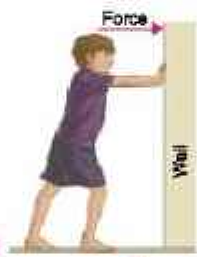


Fig. 4.1

The force applied on a body may not always be in the direction of force as shown in Fig. (4.2). If the force **F** makes an angle θ with the displacement **d** (Fig. 4.3), the work done is equal to the product of the component of force along the direction of the displacement and the magnitude of displacement. Then

Fig. 4.2

$$W = (F\cos\theta)\, d = Fd\cos\theta \quad \ldots\ldots\ldots \quad (4.2)$$

or

$$W = \mathbf{F} \cdot \mathbf{d} \quad \ldots\ldots\ldots \quad (4.3)$$

Equation (4.3) shows that work is a scalar quantity.

The unit of work is joule (J). From Eq. (4.1), we have

$$1\,\mathrm{J} = 1\,\mathrm{N\,m}$$

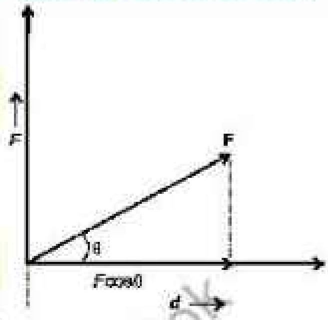


Fig. 4.3

When a constant force acts through a distance d, the event can be plotted on a simple graph (Fig. 4.4). The distance is normally plotted along x-axis and the force along y-axis. As the force does not vary, in this case, the graph will be a horizontal straight line. If the constant force F (newton) and the displacement **d** (metre) are in the same direction, then the work done is Fd (joule). Clearly shaded area in Fig. 4.4 is also Fd. Hence, the area under a force-displacement curve can be taken as to represent the work done by the constant force. In case the force F is not in the direction of displacement, the graph is plotted between $F\cos\theta$ and d.

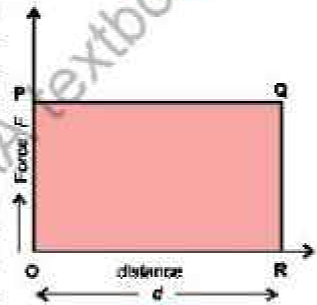


Fig. 4.4

From the definition of work, we find that:

(i) Work is a scalar quantity.

(ii) If $\theta < 90°$, work is done and it is said to be positive work.

(iii) If $\theta = 90°$, no work is done.

(iv) If $\theta > 90°$, the work done is said to be negative.

(v) SI unit of work is N m, also known as joule (J).

4.2 WORK DONE BY A VARIABLE FORCE

In many cases, the force does not remain constant during the process of doing work. For example, as a rocket moves away from the Earth, work is done against the force of gravity, which varies as the inverse square of the distance from the Earth's centre. Similarly, the force exerted by a spring increases with the amount of stretch. How can we calculate the work done in such situations?

Figure 4.5 shows the path of a particle in the x y plane as it moves from point P to point Q. The path has been divided into n short intervals of displacements $\Delta\mathbf{d}_1$, $\Delta\mathbf{d}_2$, ..., $\Delta\mathbf{d}_n$ and $\mathbf{F}_1$, $\mathbf{F}_2$, ..., $\mathbf{F}_n$ are the forces acting during these intervals, respectively.

During each small interval, the force is supposed to be approximately constant. So, the work done for the first interval can then be written as

$\Delta W_1 = \mathbf{F}_1 \cdot \Delta \mathbf{d}_1 = F_1 \cos\theta_1 \Delta d_1$

and in the second interval

$\Delta W_2 = \mathbf{F}_2 \cdot \Delta \mathbf{d}_2 = F_2 \cos\theta_2 \Delta d_2$

and so on. The total work done in moving the object can be calculated by adding all these terms.

Fig. 4.5

A particle acted upon by a variable force, moves along the path shown from point P to point Q.

$$W_{total} = \Delta W_1 + \Delta W_2 + ... + \Delta W_n$$

$$= F_1 \cos\theta_1 \Delta d_1 + F_2 \cos\theta_2 \Delta d_2 + ... + F_n \cos\theta_n \Delta d_n$$

or $$W_{total} = \sum_{i=1}^{n} F_i \cos\theta_i \Delta d_i \quad .... (4.4)$$

We can examine this graphically by plotting $F\cos\theta$ verses d as shown in Fig. 4.6. The displacement d has been sub-divided into the same n equal intervals. The value of $F \cos\theta$ at the beginning of each interval is indicated in the figure.

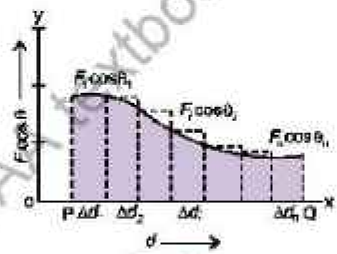


Fig. 4.6

Now the i[th] shaded rectangle has an area $F_i \cos\theta_i \Delta d$ which is the work done during the i[th] interval. Thus, the work done given by Eq. 4.4 equals the sum of the areas of all the rectangles. If we sub-divide the distance into a large number of intervals so that each Δd becomes very small, the work done given by Eq. 4.4 becomes more accurate. If we let each Δd to approach zero, then we obtain an exact result for the work done, such as:

$$W_{total} = \lim_{\Delta d \to 0} \sum_{i=1}^{n} F_i \cos\theta_i \Delta d_i \quad .... (4.5)$$

If this limit Δd approaches zero, the total area of all the rectangles (Fig. 4.6) approaches the area between the $F\cos\theta$ verses d curve and x-axis from P to Q as shown shaded in Fig. 4.7.

Thus, the work done by a variable force in moving a particle between two points is equal to the area under the $F\cos\theta$ verses d curve between the two points P and Q as shown in Fig. 4.7.

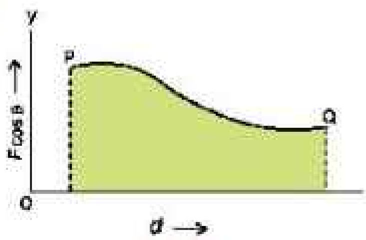


Fig. 4.7

Example 4.1 A force F acting on an object varies with distance d as shown in Fig. 4.8. Calculate the work done by the force as the object moves from $d = 0$ to $d = 6$ m.

Solution The work done by the force is equal to the total area under the curve from $d = 0$ to $d = 6$ m. This area is equal to the area of the rectangular section from $d = 0$ to

$d = 4$ m, plus the area of triangular section from $d = 4$ m to $d = 6$ m.

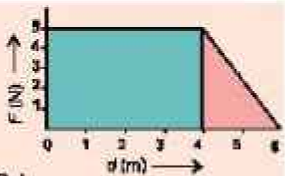


Fig. 4.8

Hence

Work done represented by the area of rectangle $= 4\text{ m} \times 5\text{ N}$

$= 20\text{ N m} = 20\text{ J}$

Work done represented by the area of triangle $= \frac{1}{2} \times 2\text{ m} \times 5\text{ N}$

$= 5\text{ N m} = 5\text{ J}$

Therefore, the total work done $= 20\text{ J} + 5\text{ J} = 25\text{ J}$

4.3 CONSERVATIVE AND NON-CONSERVATIVE FORCES

Conservative Forces

The space around the Earth in which its gravitational force acts on a body is called the gravitational field. When an object is moved in the gravitational field, the work is done by the gravitational force. If displacement is in the direction of gravitational force, the work is positive. If the displacement is against the gravitational force, the work is said to be negative.

There is an interesting property of the gravitational force that when an object is moved from one place to another, the work done by the gravitational force does not depend on the choice of the path. Let us explore it.

Consider an object of mass m being displaced with constant speed from point A to B along various paths in the presence of a gravitational force (Fig. 4.9). In this case, the gravitational force is equal to the weight mg of the object.

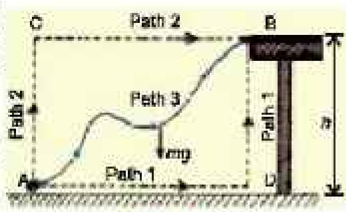


Fig. 4.9

The work done by the gravitational force along the path 1 (ADB) can be split into two parts (path AD and path DB). The work done along AD is zero, because the weight mg is perpendicular to this path, whereas the work done along DB is $(-mgh)$ because the direction of mg is opposite to that of the displacement i.e., $\theta = 180°$. Hence, the work done in displacing a body from A to B through path 1 is:

$$W_{ADB} = 0 + (-mgh) = -mgh$$

If we consider the path 2 (ACB), the work done along AC is also $(-mgh)$. Since the work done along CB is zero, therefore,

$$W_{ACB} = -mgh + 0 = -mgh$$

Now consider path 3, i.e., a curved one. Imagine the curved path to be broken down into a series of horizontal and vertical steps as shown in Fig. 4.10. There is no work done along the horizontal steps, because mg is perpendicular to the displacement for these steps. Work is done by the force of gravity only along the vertical displacements. During the segment CD, mg is not negative; it is positive. But here all Δy elements are negative, so the products of mg and Δy for all the elements will again be negative. Therefore, we can write:

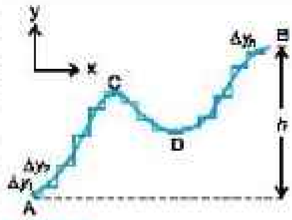


Fig. 4.10

A smooth path may be replaced by a series of infinitesimal x and y displacements. Work is done only during the y displacements.

$$W_{AB} = -mg(\Delta y_1 + \Delta y_2 + \Delta y_3 + \ldots\ldots + \Delta y_n)$$

As $\quad \Delta y_1 + \Delta y_2 + \Delta y_3 + \ldots\ldots + \Delta y_n = h$

Hence $\quad W_{AB} = -mgh$

The net amount of work done along the curved path AB is still $(-mgh)$. We conclude from the above discussion that:

Work done by gravitational force is independent of the path followed.

Can you prove that the work done, along a closed path, such as ACBA or ADBA (Fig. 4.9), by the gravitational force is zero?

If the work done by a force in moving an object between two points is independent of the path followed or the work done in a closed path be zero, the force is called a conservative force.

As shown above, the gravitational force is a conservative force, other examples of conservative force are electrostatic force and elastic spring force.

Non-Conservative Forces

All types of forces are not conservative forces.

A force is non-conservative if the work done by it in moving an object between two points or in a closed path depends on the path of motion.

The kinetic frictional force is a non-conservative force. When an object slides over a surface, the kinetic frictional force always acts opposite to the motion and does negative work equal in magnitude to the frictional force multiplied by the length of the path. Thus, greater amount of work is done over a longer path between any two points. Hence, the work depends on the choice of path. Moreover, the total work done by a non-conservative force in a closed path is not zero.

Other examples of non-conservative force are air resistance, tension in a string, normal force and propulsion force of a rocket.

4.4 POWER

In the definition of work, it is not clear, whether the same amount of work is done in one second or in one hour. The rate, at which work is done, is often of interest in practical applications.

Power is the measure of the rate at which work is being done.

If work ΔW is done in a time interval Δt, then the average power P_{av} during the interval Δt is defined as:

$$P_{av} = \frac{\Delta W}{\Delta t} \quad \text{..............................} \quad (4.6)$$

If work is expressed as a function of time, the instantaneous power P at any instant is defined as:

$$P = \underset{\Delta t \to 0}{\text{Limit}} \frac{\Delta W}{\Delta t} \quad \text{..............................} \quad (4.7)$$

where ΔW is the work done in short interval of time Δt.

Since $\quad \Delta W = \mathbf{F}.\Delta \mathbf{d}$

Hence $\quad P = \frac{\mathbf{F}.\Delta \mathbf{d}}{\Delta t} = \mathbf{F}.\frac{\Delta \mathbf{d}}{\Delta t}$

Since $\quad \frac{\Delta \mathbf{d}}{\Delta t} = \mathbf{v}$

Hence $\quad P = \mathbf{F}.\mathbf{v}$ .... (4.8)

Thus, power may also be defined as the scalar product of **F** and **V**.

The SI unit of power is watt, defined as one joule of work done in one second.

Sometimes, for example, in the electrical measurements, the unit of work is expressed as watt second. However, a commercial unit of electrical energy is kilowatt-hour.

One kilowatt- hour is the work done in one hour by an agency whose power is one kilowatt.

Therefore $\quad 1\,\text{kW h} = 1000\,\text{W} \times 3600\,\text{s}$

or $\quad 1\,\text{kW h} = 3.6 \times 10^6\,\text{J} = 3.6\,\text{M J}$

For your information

Approximate Powers

Device	Power (W)
Jumbo Jet Aircraft	1.3×10^8
Car at 90 km h	1.1×10^5
Electric heater	2×10^3
Coloured TV	120
Flash light (two cells)	1.5
Pocket calculator	7.5×10^{-4}

Example 4.2 A 70 kg man runs up a long flight of stairs in 4.0 s. The vertical height of the stairs is 4.5 m. Calculate his power output in watts.

Solution Work done = mgh

$$\text{Power} = \frac{mgh}{t}$$

$$P = \frac{70\,\text{kg} \times 9.8\,\text{m s}^{-2} \times 4.5\,\text{m}}{4\,\text{s}}$$

$$P = 7.7 \times 10^2\,\text{kg m}^2\text{s}^{-3} = 7.7 \times 10^2\,\text{W}$$

Do you know?

It takes about 9×10^9 J of energy to make a car and the car then uses about 1×10^{12} J of energy from petrol in its life time.

4.5 ENERGY

Energy of a body is its capacity to do work. There are two basic forms of energy:

(i) Kinetic energy (ii) Potential energy

Kinetic energy is the energy possessed by a body due to its motion and potential energy is the energy possessed by a body due to its changed position.

The kinetic energy and the potential energy both are the kinds of mechanical energy.

Kinetic Energy

Let us derive a formula for the kinetic energy of a moving body. Consider a car running with a constant speed on a road. If its engine is switched OFF, it will still cover some distance before stopping. As long as it is moving, it is doing work against the force of friction of the road. In other words, during this interval, it will exert a force equal in magnitude to the force of friction f. Let the distance travelled before coming to rest be d, then the work done by the car would be fd. This work is done by the car due to its motion. The ability of a body to do work due to its motion is its kinetic energy. Therefore, kinetic energy of the car is equal to fd. The acceleration can be found by using Newton's second law of motion, i.e.,

$$F = ma$$

As the car slows down and finally stops, its acceleration a is negative because it is produced by force of friction f acting apposite to the direction of motion. Thus,

$$f = -ma$$

or $$a = -\frac{f}{m}$$

We can now determine the value of (fd) by using the third equation of motion, i.e;

$$2aS = v_f^2 - v_i^2$$

Here, Initial velocity $v_i = v$

Final velocity $v_f = 0$

Distance $S = d$

Acceleration $a = -\frac{f}{m}$

Putting values in the above equation of motion, we have

$$2\times\left(\frac{-f}{m}\right)d = 0 - v^2$$

$$fd = \frac{1}{2}mv^2$$

As fd is equal to the kinetic energy of body, therefore,

$$\text{Kinetic energy} = \frac{1}{2}mv^2 \qquad (4.9)$$

For your information

Approximate Energy Values

Source	Energy (J)
Burning 1 ton coal	30×10^{9}
Burning 1 litre petrol	5×10^{7}
K.E. of a car at 90 km h^{-1}	1×10^{6}
Running Person at 10 km h^{-1}	3×10^{2}
Fission of one atom of uranium	1.8×10^{-11}
K.E. of a molecule of air	6×10^{-21}

Since, kinetic energy is equal to work which the body is capable of doing, so the unit of kinetic energy must be that of work, i.e. joule (J).

Example 4.3 A car weighing 18620 N is running with a speed of 16 m s^{-1}. Brakes are applied and it is brought to rest in a distance of 60 m. Determine the average force of friction acting on it.

Solution

Given that $v = 16\text{ m s}^{-1}$, $d = 80\text{ m}$, $w = 18620\text{ N}$ and $f = ?$

The kinetic energy of the car is equal to the work done by it before stopping, i.e.,

$$\frac{1}{2}mv^2 = fd$$

Here $$m = \frac{w}{g} = \frac{18620\text{ N}}{9.8\text{ m s}^{-2}} = 1900\text{ kg}$$

Putting the value in the above equation, we have

$$\frac{1}{2} \times 1900\text{ kg} \times (16\text{ m s}^{-1})^2 = f \times 80$$

or $$f = 3040\text{ N}$$

Potential Energy

The potential energy is possessed by a body because of its position in a force field, e.g. gravitational field or because of its constrained state.

The energy stored in a compressed spring is the potential energy possessed by the spring due to its compressed or stretched state. This form of energy is called the elastic potential energy.

Absolute Potential Energy

> The absolute gravitational potential energy of an object at a certain position is the work done by the gravitational force in displacing the object from that position to infinity where the force of gravity becomes zero.

The relation for the calculation of the work done by the gravitational force or potential energy is *mgh*, which is true only near the surface of the Earth where the gravitational force is nearly constant. But if the body is displaced through a large distance in space, let it be from point 1 to N (Fig. 4.11) in the gravitational field, then the gravitational force will not remain constant, since it varies inversely to the square of the distance.

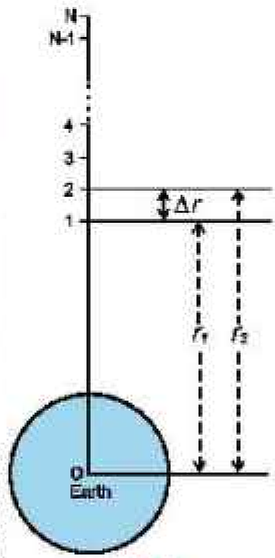


Fig. 4.11

In order to overcome this difficulty, we divide the distance between points 1 and N into small steps each of length Δr so that the value of the force remains constant for each small step. Hence, the total work done can be calculated by adding the work done during all these steps. If r_1 and r_2 are the distances of points 1 and 2 respectively, from the centre O of the Earth (Fig. 4.11.), the work done during the first step i.e., displacing a body from point 1 to point 2 can be calculated as below. The distance between the centre of this step and centre of the Earth will be:

$$r = \frac{r_1 + r_2}{2}$$

As $r_2 - r_1 = \Delta r$ then $r_2 = r_1 + \Delta r$

Hence $r = \frac{r_1 + r_1 + \Delta r}{2} = r_1 + \frac{\Delta r}{2}$ .... (4.10)

The gravitational force F at the centre of this step is:

$$F = G\frac{Mm}{r^2} \quad .... (4.11)$$

where m = mass of an object , M = mass of the Earth
and G = Gravitational constant

Squaring Eq. 4.10

$$r^2 = \left(r_1 + \frac{\Delta r}{2}\right)^2$$

$$r^2 = r_1^2 + 2r_1\frac{\Delta r}{2} + \left(\frac{\Delta r}{2}\right)^2$$

As $(\Delta r)^2 << r_1^2$, so $(\Delta r)^2$ can be neglected as compared to r_1^2.

Hence $r^2 = r_1^2 + r_1 \Delta r$

Putting the value of $\Delta r = r_2 - r_1$

$$r^2 = r_1^2 + r_1(r_2 - r_1) = r_1 r_2$$

Hence, Eq. 4.11 becomes

$$F = G\frac{Mm}{r_1 r_2} \quad .... (4.12)$$

As this force is assumed to be constant during the interval Δr, so the work done is:

$$W_{1\rightarrow 2} = \mathbf{F}.\Delta\mathbf{r} = F\Delta r \cos 180^\circ = -GMm\frac{\Delta r}{r_1 r_2}$$

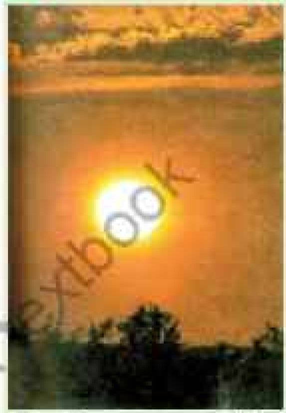
The negative sign indicates that the work has to be done on the body from point 1 to 2 because displacement is opposite to gravitational force. Putting the value of Δr, we have

$$W_{1\rightarrow 2} = -GMm\frac{r_2 - r_1}{r_1 r_2}$$

or

$$W_{1\rightarrow 2} = -GMm\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

Do you know?



There is more energy reaching Earth in 10 days of sunlight than in all the fossil fuels on the Earth.

Similarly, the work done during the second step in which the body is displaced from point 2 to 3 is:

$$W_{2\to 3} = -GMm\left(\frac{1}{r_2}-\frac{1}{r_3}\right)$$

Tidbits

More coal has been used since 1945 than was used in the whole of history before that.

and the work done in the last step is:

$$W_{N-1\to N} = -GMm\left(\frac{1}{r_{N-1}}-\frac{1}{r_N}\right)$$

Hence, the total work done in displacing a body from point 1 to N is calculated by adding up the work done during all these steps.

$$W_{total} = W_{1\to 2} + W_{2\to 3} + \ldots\ldots\ldots + W_{N-1\to N}$$

$$= -G\,Mm\left[\left(\frac{1}{r_1}-\frac{1}{r_2}\right)+\left(\frac{1}{r_2}-\frac{1}{r_3}\right)+\ldots\ldots+\left(\frac{1}{r_{N-1}}-\frac{1}{r_N}\right)\right]$$

On simplification, we have

$$W_{total} = -GMm\left(\frac{1}{r_1}-\frac{1}{r_N}\right)$$

If the point N is situated at an infinite distance from the Earth, then

$r_N = \infty$ so $\frac{1}{r_N}=\frac{1}{\infty}=0$

Hence $W_{total} = -\frac{GMm}{r_1}$

This total work by definition is the absolute potential energy (P.E) as stated earlier represented by U.

$$U = -\frac{GMm}{r}$$

This is also known as the absolute value of gravitational potential energy of a body at a distance r from the centre of the Earth.

Note that when r increases, U becomes less negative i.e., U increases. It means when we raise a body above the surface of the Earth, its P.E. increases. Therefore, if we want to raise the body up to infinite distance, we will have to do work on it equal to $\frac{GMm}{R}$, so that its P.E. becomes zero.

Now the absolute potential energy on the surface of the Earth is found by putting $r = R$ (Radius of the Earth), so

$$\text{Absolute potential energy} = U_g = -\frac{GMm}{R} \quad \ldots\ldots \quad (4.13)$$

The negative sign shows that the Earth's gravitational field for mass m is attractive. The above expression gives the work or the energy required to take the body out of the

Earth's gravitational field, where its potential energy with respect to Earth is zero.

4.6 ESCAPE VELOCITY

It is our daily life experience that an object projected upward comes back to the ground after rising to a certain height. This is due to the force of gravity acting downward. With increased initial velocity, the object rises to the greater height before coming back. If we go on increasing the initial velocity of the object, a stage comes when it will not return to the ground. It will escape from the influence of gravity.

> The initial velocity of an object with which it goes out of the Earth's gravitational field, is known as escape velocity.

For your Information

Some Escape speeds ($km\ s^{-1}$)

Heavenly body	Escape speed
Moon	2.4
Mercury	4.3
Mars	5.0
Venus	10.4
Earth	11.2
Uranus	22.4
Neptune	25.4
Saturn	37.0
Jupiter	61

The escape velocity corresponds to the initial kinetic energy gained by the body, which carries it to an infinite distance from the surface of the Earth.

$$\text{Initial } K.E. = \frac{1}{2} m v_{esc}^2$$

We know that the work done in lifting a body from Earth's surface to an infinite distance is equal to the increase in potential energy.

$$\text{Incerase in } P.E. = 0-(-G\frac{Mm}{R}) = G\frac{Mm}{R}$$

where M and R are the mass and radius of the Earth respectively. The body will escape out of the gravitational field if the initial $K.E.$ of the body is equal to increase in $P.E.$ Then

$$\frac{1}{2} m v_{esc}^2 = G\frac{Mm}{R}$$

or $$v_{esc} = \sqrt{\frac{2GM}{R}} \quad \text{..........} \quad (4.14)$$

As $$g = \frac{GM}{R^2} \quad \text{or} \quad gR = \frac{GM}{R}$$

Hence $$v_{esc} = \sqrt{2gR} \quad \text{..........} \quad (4.15)$$

The value of v_{esc} comes out to be approximately 11 $km\ s^{-1}$.

4.7 WORK-ENERGY THEOREM

Whenever work is done on a body, it increases its energy. For example, if a force F acts on a body of mass m, initially moving with velocity v_i, through a distance d and increases its velocity to v_f, then the acceleration produced will be:

$$2ad = v_f^2 - v_i^2$$

or $$a = \frac{1}{2d}(v_f^2 - v_i^2) \quad \ldots\ldots\ldots \quad (4.16)$$

From the second law of motion:

$$F = ma$$

or $$a = \frac{F}{m} \quad \ldots\ldots\ldots \quad (4.17)$$

Tidbits

All the food you eat in one day has about the same energy as 1/3 litre of petrol.

Comparing Eqs. 4.16 and 4.17, we have

$$\frac{F}{m} = \frac{1}{2d}(v_f^2 - v_i^2)$$

or $$Fd = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 \quad \ldots\ldots\ldots \quad (4.18)$$

This expression is the work-energy theorem. It states that:

The change in kinetic energy of an object is equal to the work done on it by a net force.

W = Change in kinetic energy $= (K.E)_f - (K.E)_i = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$

This is also known as work-energy principle.

The work-energy theorem is applicable for any direction of the force relative to the displacement. For instance, an object with kinetic energy can perform work if it is allowed to push or pull on another object. In this case, the work will be taken as negative and the kinetic energy of the object will decrease. The theorem remains valid even if the force may vary from point to point.

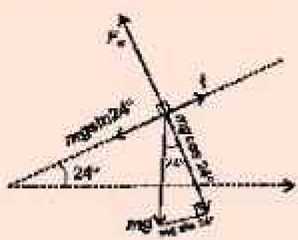
Example 4.4 A motorcycle rider weighing 60 kg is coasting down a 24° slope. The weight of motorcycle is 30 kg. At the top of the slope, the speed of motorcycle is 3.2 m s^{-1}. If the kinetic frictional force is 100 N, what will be the speed of the motorcycle 72 m downhill?

Solution The normal force F_N is balanced by the component of weight ($mg\cos 24°$) perpendicular to the slope. Let the kinetic frictional force is f, then the net force F is:

$F = mg\sin 24° - f$ where m = total mass = 60 kg + 30 kg = 90 kg

or $F = (90\text{ kg} \times 9.8\text{ m s}^{-2} \times 0.4) - 100\text{ N}$

$F = 252.8\text{ N}$



Wok done $W = Fd = 252.8\text{ N} \times 72\text{ m} = 18201.6\text{ J}$

As work is positive, so applying work – energy theorem,

$$W = (K.E.)_f - (K.E.)_i$$

Form here,

$$(K.E.)_f = W + (K.E.)_i$$

Putting the values, we have

$$\frac{1}{2}mv_f^2 = W + \frac{1}{2}mv_i^2$$

$$\frac{1}{2}\times 90\text{ kg}\times v_f^2 = 18201\text{ J} + \frac{1}{2}\times 90\text{ kg}\times(3.2\text{ m s}^{-1})^2$$

This gives, $v_f^2 = 414.7\text{ m}^2\text{ s}^{-2}$

or $v_f = \sqrt{414.7\text{ m}^2\text{ s}^{-2}} = 20.4\text{ m s}^{-1}$

4.8 INTERCONVERSION OF POTENTIAL ENERGY AND KINETIC ENERGY

Consider a body of mass m at rest, at a height h above the surface of the Earth as shown in Fig. 4.12. At position A, the body has $P.E. = mgh$ and $K.E. = 0$. We release the body and as it falls, we can examine how kinetic and potential energies associated with it interchange.

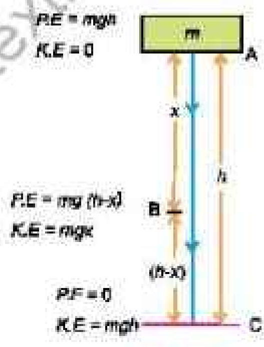


Fig. 4.12

Let us calculate P.E. and K.E. at the position B when the body has fallen a distance x, ignoring air friction.

Now, height from the ground is $(h-x)$, so that

$$P.E. = mg\,(h-x)$$

and $$K.E. = \frac{1}{2}mv_B^2$$

Velocity v_B, at position B, can be calculated from the relation,

$$v_f^2 = v_i^2 + 2gS$$

as $v_f = v_B$, $v_i = 0$, $S = x$

$$v_B^2 = 0 + 2gx$$

$$v_B^2 = 2gx$$

Therefore $$K.E. = \frac{1}{2}m\,(2gx)$$

$$= mgx$$

Total energy at position B = P.E. + K.E.

Total energy $= mg\,(h - x) + mgx = mgh$ .... (4.19)

At position C, just before the body strikes the Earth, P.E. = 0 and $K.E. = \frac{1}{2}mv_C^2$, where v_C can be found out by the following expression.

$$v_C^2 = v_i^2 + 2gh = 2gh \quad \text{as} \quad v_i = 0$$

i.e., $$K.E = \frac{1}{2}mv_C^2 = \frac{1}{2}m \times 2gh = mgh$$

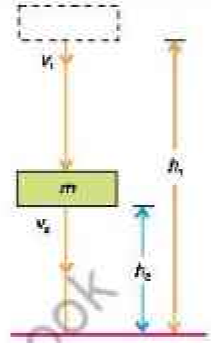


Fig. 4.13

Thus, at point C, kinetic energy is equal to the original value of the potential energy of the body. Actually, when a body falls, its velocity increases i.e., the body is being accelerated under the action of gravity. The increase in velocity results in the increase in its kinetic energy. On the other hand, as the body falls, its height decreases and hence, its potential energy also decreases. Thus, we see (Fig. 4.13) that:

Loss in *P.E.* = Gain in *K.E*

$$mg(h_1 - h_2) = \frac{1}{2}m(v_2^2 - v_1^2) \quad .... \quad (4.20)$$

where v_1 and v_2 are he velocities of the body at the heights h_1 and h_2 respectively. This result is true only when frictional force is not considered.

If we assume that a frictional force f is present during the downward motion, then a part of P.E. is used in doing work against friction equal to fh. The remaining *P.E.* = $mgh - fh$ is converted into K.E.

Hence $$mgh - fh = \frac{1}{2}mv^2$$

or $$mgh = \frac{1}{2}mv^2 + fh \quad .... \quad (4.21)$$

Thus Loss in *P.E.* = Gain in *K.E.* + Work done against friction

Conversely,

Loss of *K.E.* = Gain in *P.E.* + Work done against friction

Example 4.5

A car weighing 1100 kg is moving with a velocity of 12 m s^{-1}. When it is at point P, its engine stops. If the frictional force is 120 N, what will be its velocity at point Q? How far beyond Q will it go before coming to rest?

Solution

The kinetic energy possessed by the car at point P will partly be converted into *P.E.* and partly used up in doing work against friction as it reaches point Q. Therefore,

Loss of *K.E.* = Gain in *P.E.* + Work against friction

$$\frac{1}{2}m(v_i^2 - v_f^2) = wh + fd$$

$$\frac{1}{2}\times 1100\text{ kg}(144\text{ m}^2\text{ s}^{-2} - v_f^2) = (1100\text{ kg}\times 9.8\text{ m s}^{-2}\times 1.5\text{ m}) + 120\text{ N}\times 24\text{ m}$$

$$550\text{ kg}(144\text{ m}^2\text{ s}^{-2} - v_f^2) = 16170\text{ kg m}^2\text{ s}^{-2} + 2880\text{ kg m}^2\text{ s}^{-2}$$

$$(144\text{ m}^2\text{ s}^{-2} - v_f^2) = \frac{16170\text{ kg m}^2\text{ s}^{-2} + 2880\text{ kg m}^2\text{ s}^{-2}}{550\text{ kg}} = 34.6\text{ m}^2\text{ s}^{-2}$$

$$v_f^2 = 144\text{ m}^2\text{ s}^{-2} - 34.6\text{ m}^2\text{ s}^{-2} = 109.4\text{ m}^2\text{ s}^{-2}$$

Velocity at point Q, $v_f = \sqrt{109.4\text{ m}^2\text{ s}^{-2}} = 10.5\text{ m s}^{-1}$

Now if the car stops at point R, then using the formula:

$$\frac{1}{2}mv^2 = fS$$

$$\frac{1}{2}\times 1100\text{ kg}\times 109.4\text{ m}^2\text{ s}^{-2} = 120\text{ kg m s}^{-2}\times S$$

$$S = 501\text{ m approximately}$$

Example 4.6 An object of mass 3 kg falls from a height of 15 m. If it strikes the ground with a velocity of 16 m s^{-1}, calculate the average frictional force of the air.

Solution Loss of $P.E.$ = Gain in $K.E.$ + Work done against friction

$$\because v_i = 0,\quad mgh = \frac{1}{2}mv^2 + fh$$

$$3\text{ kg}\times 9.8\text{ m s}^{-2}\times 15\text{ m} = \frac{1}{2}\times 3\text{ kg}(16\text{ m s}^{-1})^2 + f\times 15\text{ m}$$

$$441\text{ kg m}^2\text{ s}^{-2} = 384\text{ kg m}^2\text{ s}^{-2} + 15\text{ m}\times f$$

or $$f = \frac{441\text{ kg m}^2\text{ s}^{-2} - 384\text{ kg m}^2\text{ s}^{-2}}{15\text{ m}} = 3.8\text{ kg m s}^{-2} = 3.8\text{ N}$$

QUESTIONS

Multiple Choice Questions

Tick (✓) the correct answer.

4.1 A 1 kg mass has potential energy of 1 joule relative to the ground when it is at a height of:

(a) 0.102 m (b) 1 m (c) 9.8 m (d) 32 m

4.2 An iron sphere whose mass is 30 kg has the same diameter as an aluminium sphere whose mass is 10.5 kg. The spheres are simultaneously dropped from a cliff. When they are 10 m from the ground, they have identical:

(a) accelerations (b) momentums (c) potential energies (d) kinetic energies

4.3 A body at rest may have:

(a) speed (b) velocity (c) momentum (d) energy

4.4 The height above the ground of a child on a swing varies from 0.5 m of his lowest point to 1.5 m at his highest point. The maximum speed of the child is approximately:

(a) 1.5 m s^{-1} (b) 4.4 m s^{-1}

(c) 9.8 m s^{-1} (d) Depends upon child's mass

4.5 When a ball is thrown vertically upward and then falls back to the ground, which force can be considered conservative in this scenario?

(a) Air resistance (b) Gravity

(c) Friction between ball and air (d) Contact force with hand

4.6 According to work-energy principle in linear motion, the work done on body is equal to:

(a) change in *K.E.* (b) change in *P.E.*

(c) zero (d) sum of *K.E.* and *P.E.*

4.7 Power of a lamp is 6 W. How much energy does a lamp give out in 2 min?

(a) 12 J (b) 20 J (c) 3 J (d) 720 J

4.8 A dry battery can deliver 3000 J of energy to a 2 W small electric motor before the battery is exhausted. For how many minutes does the battery run?

(a) 1500 min (b) 100 min (c) 50 min (d) 25 min

4.9 The kinetic energy acquired by a mass m after travelling a fixed distance from rest under the action of a constant force is directly proportional to:

(a) $\sqrt{m}$ (b) $1/\sqrt{m}$ (c) m (d) independent of m

4.10 A body moves a distance of 10 m along a straight line under the action of 5 N force. If the work done is 25 J, the angle which the force makes with the direction of motion of the body is:

(a) 0° (b) 30° (c) 60° (d) 90°

Short Answer Questions

4.1 Why is electrical power required at all when the elevator is descending? Why should be there a limit on the number of passengers in this case?

4.2. A body is being raised to a height H from surface of the Earth. What is the sign of work done by both (body and the Earth)? Justify.

4.3 A body falls towards the Earth in air. Will its total mechanical energy be conserved during fall? Justify.

4.4 Calculate power of a crane in kilowatt which lifts a mass of 1000 kg to a height of 100 m in 20 second.

4.5 A trolley of mass 1500 kg carrying sand bags of 500 kg is moving uniformly with a speed of 40 km h^{-1} on a frictionless track. After some time, sand starts leaking out of whole sand bags on the road at a rate of 0.05 kg s^{-1}. What is the speed of the trolley after entire sand bags are empty?

4.6 Give absolute and gravitational units of work in M.K.S and C.G.S systems.

4.7 A body dropped from a height of H reaches the ground with a speed of $1.2\sqrt{gH}$. Calculate work done by air friction.

4.8 A bicycle has a *K.E.* of 150 J. What *K.E.* would the bicycle have if it had:

(i) same mass but has speed double?

(ii) three times mass and was moving with one half of the speed?

4.9 What will be the effect on *K.E.* of the body having mass *m*, moving with velocity *v* when its momentum becomes double? Justify.

4.10 Does the international space station have gravitational *P.E.* or kinetic energy or both? Explain.

Constructed Response Questions

4.1 When will you say that a force is conservative? Give two conditions.

4.2 A light and a heavy body have same linear momentum, which one has greater *K.E.*?

4.3 A motorcycle is running with constant speed on a horizontal track. Is any work being done on the motorcycle, if no net force is acting on it?

4.4 A force acts on a ball moving with 14 m s^{-1} speed and brings its speed to 6 m s^{-1}. Has the force done positive or negative work? Explain your answer.

4.5 A slow moving truck can have more kinetic energy than a fast moving car. How is this possible?

4.6 Why work done against friction is non-conservative in nature? Explain briefly.

4.7 Does wind contain kinetic energy? Explain.

Comprehensive Questions

4.1 Define *K.E.* and derive an expression for the same.

4.2 How is work done by a:

(i) constant force (ii) variable force?

4.3 Define conservative field. Show that gravitational field is conservative in nature.

4.4 What is meant by absolute P.E.? Derive an expression for absolute P.E.

4.5 State and explain work-energy theorem in a resistive medium.

4.6 Define escape velocity. Show that an expression for escape velocity can be expressed as $\sqrt{2Rg}$, where *R* and *g* denote radius of the Earth and acceleration due to gravity, respectively. Also find its numerical value near the surface of the Earth.

Numerical Problems

4.1 A machine gun fires 6 bullets per minute with a velocity of 700 m s^{-1}. If each bullet has a mass of 40 g, then find power developed by the gun? (Ans: 980 W)

4.2 A family uses 10 kW of power. Direct solar energy is incident on horizontal surface at an average rate of 300 W per square metre. If 75% of this energy can be converted into useful electrical energy, how large area is needed to supply 10 kW?

(Ans: 44.44 m^2)

4.3 The mass of the Earth is 6.0×10^{24} kg and mass of the Sun is 1.99×10^{30} kg. The Sun is 160 million km away from the Earth. Find the value of gravitational *P.E.* of the Earth. (Ans: -4.97×10^{33} J)

4.4 An object weighing 98 N is dropped from a height of 10 m. Its speed just before hitting the ground is 12 m s^{-1}. What is the frictional force acting on it? (Ans: 26 N)

4.5 A 75 watt fan is used for 8 hours daily for 30 days. Find:

(i) energy consumed in electrical units

(ii) electricity bill if one unit costs Rs. 22.5? [Ans: (i) 18 units (ii) Rs. 405]

4.6 If an object of mass 2 kg thrown up from ground reaches a height of 5 m and falls back to the Earth (neglecting air resistance), calculate:

(i) work done by gravity when the object reaches at 5 m height.

(ii) work done by gravity when the object comes back to the Earth.

(iii) total work done by gravity in upward and downward motion. Also mention physical significance of the result.

[Ans: (i) -98 J (ii) +98 J (iii) 0 J work done in a closed path in a conservative field is zero]

4.7 An electrical motor of one horse power is used to run a water pump. Water pump takes 15 minutes to fill a tank of 400 litres at a height of 10 m (1 hp = 746 watts). Find:

(a) actual input work done by electric motor to fill the tank

(b) actual output work done [Ans: (a) 671.4 kJ, (b) 39.2 kJ]

4.8 A passenger just arrives at the airport and dragging his suitcase to luggage checks in at the desk. He pulls strap with a force of 200 N at an angle of 45° to the floor to displace it 50 m to the desk. Determine the value of work done by him on the suitcase. (Ans: 7 kJ)

4.9 A 1200 kg car is running at a speed of 40 km h^{-1}. How much power will be expended by it to accelerate at 2 m s^{-2}? (Ans: 26.67 kW)

4.10 A 200 g apple is lifted to 10 m and then dropped. What is its velocity when it hits the ground? Assume that 75% of work done in lifting the apple is transferred to *K.E.* by the time it hits the ground. (Ans: 12.1 m s^{-1})

Chapter 5 Solids and Fluid Dynamics

Learning Objectives

After studying this chapter, the students will be able to:

- Distinguish between the structures of crystalline, amorphous, and polymeric solids.
- Describe that deformation of solids in one dimension [That it is caused by a force and that in one dimension, the deformation can be tensile or compressive.]
- Define and use the terms stress, strain and the Young's modulus
- Describe an experiment to determine the Young's modulus of a metal wire.
- Describe and use the terms elastic deformation, plastic deformation and elastic limit
- Justify why and apply the fact that the area under the force-extension graph represents the work done
- Determine the elastic potential energy of a material [That is deformed within its limit of proportionality from the area under the force-extension graph. Also state and use $E_p = \frac{1}{2}kx^2$ for a material deformed within its limit of proportionality]
- State and use Archimedes' principle and flotation
- Justify how ships are engineered to float in the sea
- Define and apply the terms: steady (streamline or laminar) flow, incompressible flow and non - viscous flow as applied to the motion of an ideal fluid.
- State and use equation of continuity to solve problems
- Explain that squeezing the end of a rubber pipe results in increase in flow velocity
- Justify that the equation of continuity is a form of the principle of conservation of mass.
- Justify that the pressure difference can arise from different rates of flow of a fluid [Bernoulli effect]
- Explain and apply Bernoulli's equation for horizontal and vertical fluid flow.
- Explain why real fluids are viscous fluids.
- Describe how viscous forces in a fluid cause a retarding force on an object moving through it.
- Describe super fluidity [As the state in which a liquid will experience zero viscosity. Students should know the implications of this state e.g. this allows for super fluids to creep over the walls of containers to 'empty' themselves. It also implies that if you stir a superfluid, the vortices will keep spinning indefinitely.]
- Analyze the real-world applications of the Bernoulli effect [For example, atomizers in perfume bottles, the swinging trajectory of a spinning cricket ball and the lift of a spinning golf ball (the Magnus effect), the use of Venture ducts in filter pumps and car engineers to adjust the flow of fluid, etc.]

Materials have specific uses depending upon their characteristics and properties, such as hardness, ductility, malleability, etc. What makes a metal hard and other soft? It depends upon the structure, the particular order and bonding of atoms and molecules in a material. Similarly, the study of fluids in motion is relatively complicated but analysis can be simplified by making a few assumptions. The analysis is further simplified by the use of two important conservation principles, the conservation of mass and conservation of energy. The law of conservation of mass gives us the equation of continuity while the law of conservation of energy is the basis of Bernoulli's equation.

5.1 CLASSIFICATION OF SOLIDS

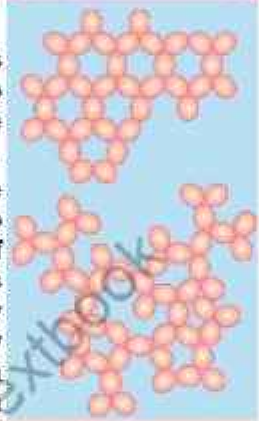
Crystalline Solids

In crystalline solids, there is a regular arrangement of atoms and molecules. The neighbours of every molecule are arranged in a regular pattern that is consistent throughout the crystal. There is, thus, an ordered structure in crystalline solids.

Most solids, like metals and ceramics have a crystalline structure. This means their atoms, molecules or ions are arranged in a regular pattern. The arrangement of molecules, atoms or ions within all types of crystalline solids can be studied using various techniques such as X-ray Diffraction (XRD) and Transmission Electron Microscopy (TEM). It should be noted that atoms, molecules or ions in a crystalline solid are not static. For example, each atom in a crystal vibrates about a fixed point with an amplitude that increases with rise in temperature. It is the average atomic positions which are perfectly ordered over large distances.

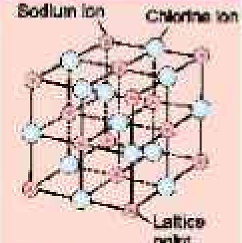
The cohesive forces between atoms, molecules or ions in crystalline solids maintain the strict long-range order inspite of atomic vibrations. For every crystal, however, there is a temperature at which the vibrations become greater that the structure suddenly breaks up, and the solid melts. The transition from solid (order) to liquid (disorder) is, therefore, abrupt or discontinuous. Every crystalline solid has a definite melting point e.g., Quartz, Calcite, Sugar, Mica, diamond, etc.

For your information



Glassy and crystalline-solid-short and long-range order.

For your information

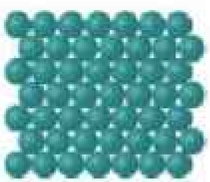


NaCl crystal lattice

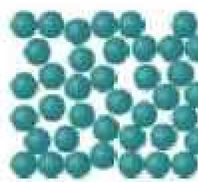
Amorphous or Glassy Solids

The word amorphous means without form or structure. Thus, in amorphous solids there is no regular arrangement of molecules like that in crystalline solids. We can, therefore, say that amorphous solids are more like liquids with the disordered structure frozen in.

For example, ordinary glass, which is a solid at ordinary temperature, has no regular arrangement of molecules. On heating, it gradually softens into a paste like state before it becomes a very viscous liquid at almost 800°C. Thus, amorphous solids are also called glassy solids. This type of solids has no definite melting point e.g., plastic, glass, fused silicon, etc.



Crystalline solids



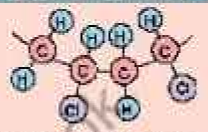
Amorphous solids

Polymeric Solids

Polymers are solid materials with a structure that is intermediate between order and disorder. They can be classified as partially or poorly crystalline solids.

Polymers form a large group of naturally occurring and synthetic materials. Plastics and synthetic rubbers are termed as polymers because they are formed by polymerization reactions in which relatively simple molecules are chemically combined into massive long chain molecules, or three-dimensional structures. These materials have rather low specific gravity compared with even the lightest of metals, and yet exhibit good strength to weight ratio.

For your information



Parts of a PVC molecule

Polymers consist wholly or in part of chemical combinations of carbon with oxygen, hydrogen, nitrogen and other metallic or non metallic elements. Polythene, polystyrene and nylon, etc. are examples of polymers. Natural rubber is composed in the pure state entirely of a hydrocarbon with the formula $(C_5H_8)_n$.

5.2 MECHANICAL PROPERTIES OF SOLIDS

Deformation in Solids

If we hold a soft rubber ball in our hand and then squeeze it, the shape or volume of the ball will change. However, if we stop squeezing the ball, and open our hand, the ball will return to its original spherical shape. This has been illustrated schematically in Fig. 5.1.

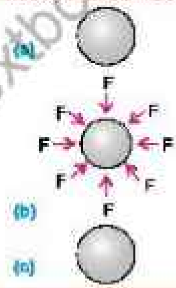
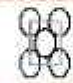


Fig. 5.1:
(a) Original rubber ball
(b) Squeezed rubber ball subjected force F by the hand
(c) Rubber ball after removing force

Similarly, if we hold two ends of a rubber string in our hands, and move our hands apart to some extent, the length of the string will increase under the action of the applied force exerted by our hands. Greater the applied force, larger will be the increase in length. Now on removing the applied force, the string will return to its original length. From these examples, it is concluded that deformation (i.e., change in shape, length or volume) is produced when a body is subjected to some external force.


(a) Unstretched unit cell


(b) Unit cell under outward stretching force


(c) Unit cell under inward applied force


(d) Unit cell after removing applied force

Fig. 5.2

In crystalline solids, atoms are usually arranged in a certain order. These atoms are held about their equilibrium position, which depends on the strength of the inter-atomic cohesive force between them. Under the influence of external force, distortion occurs in the solid bodies because of the displacement of the atoms from their equilibrium position and the body is said to be in a state of stress. After the removal of external force, the atoms return to their equilibrium position, and the body regains its original shape, provided that external applied force was not too great. The ability of the body to return to its original shape is called elasticity. Figure 5.2. illustrates deformation produced in a unit cell of a crystal subjected to an external applied force.

5.3 STRESS, STRAIN AND YOUNG'S MODULUS

The results of mechanical tests are usually expressed in terms of stress and strain, which are defined in terms of applied force and deformation.

Stress

It is defined as the force applied per unit area to produce any change in the shape, volume or length of a body. Mathematically, it is expressed as:

$$\text{Stress } (\sigma) = \frac{\text{Force } (F)}{\text{Area } (A)} \quad \ldots\ldots\ldots \quad (5.1)$$

The SI unit of stress (σ) is newton per square metre (N m^{-2}), which is given the name pascal (Pa). Stress may cause a change in length, volume and shape. When a stress changes length, it is called the tensile stress, when it changes the volume, it is called the volume stress and when it changes the shape, it is called the shear stress.

Strain

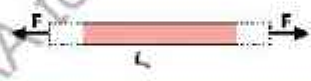


Fig. 5.3(a): Tensile strain

Strain is a measure of the deformation of a solid when stress is applied to it. In the case of deformation in one dimension, strain is defined as the fractional change in length. If ΔL is the change in length and L_o is the original length (Fig. 5.3-a), then strain is given by

$$\text{Strain } (\varepsilon) = \frac{\text{Change in length } (\Delta L)}{\text{Original length } (L_o)} \quad \ldots\ldots\ldots \quad (5.2)$$

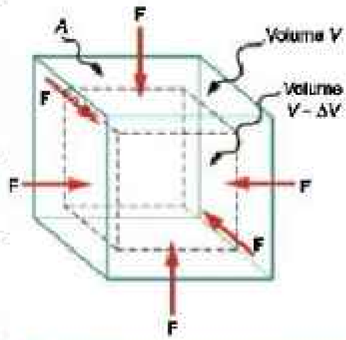


Fig. 5.3(b): Volumetric strain

Since strain is the ratio of lengths, it is dimensionless and therefore, has no units. If strain ε is due to tensile stress σ, it is called tensile strain, and if it is produced as a result of compressive stress σ, it is termed as compressive strain.

In case when the applied stress changes the volume, the change in volume per unit volume is known as volumetric strain as shown in Fig. 5.3 (b), thus

$$\text{Volumetric strain } (\varepsilon_v) = \frac{\Delta V}{V_o} \quad \ldots\ldots\ldots \quad (5.3)$$

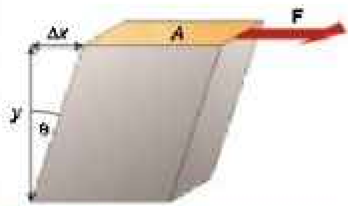


Fig. 5.3(c): Shear strain

Let y be the distance between two opposite faces of a rigid body (Fig. 5.3-c), which are subjected to shear stress one of its face slides through a distance Δx, then shear strain is produced which is is given by

$$\text{Shear strain } (\gamma) = \frac{\Delta x}{y} = \tan\theta \quad \ldots\ldots\ldots \quad (5.4)$$

However, for small value of angle θ, measured in radian $\tan\theta \approx \theta$, so that

$$\gamma = \theta \quad \text{..................(5.5)}$$

For your information

Although it is named after the 19th century British Scientist Thames Young, the concept was developed in 1727 by Leonhard Euler.

Young's Modulus

The stress applied per unit strain is called Young's modulus

i.e. $$Y = \frac{\text{Tensile stress}}{\text{Tensile strain}}$$

$$Y = \frac{F/A}{\Delta L / L_o} \quad \text{......... (5.6)}$$

It has the same unit as that of stress i.e., N m^{-2}. The value of Young's modulus of different material is given in Table 5.1.

There are various methods to determine the Young's modulus of a wire. One of the method is Searle's method.

Table 5.1: Elastic constants for some materials

Material	Young's Modulus 10^9 N m^{-2}	Bulk Modulus 10^9 N m^{-2}	Shear Modulus 10^9 N m^{-2}
Aluminium	70	70	30
Bone	15	-	80
Brass	91	61	36
Concrete	25	-	-
Copper	110	140	44
Diamond	1120	540	450
Glass	55	31	23
Ice	14	8	3
Lead	16	7.7	5.6
Mercury	0	27	0
Steel	200	160	84
Tungsten	390	200	150
Water	0	2.2	0

5.4 DETERMINATION OF YOUNG'S MODULUS OF A WIRE

Experimentally, the magnitude of Young's modulus for a material in the form of wire can be found out mostly with the help of Searle's apparatus as shown in Fig. 5.4.

It consists of two wires, reference wire and test wire of equal lengths of same material having same diameters attached to a rigid support. Both wires are connected to horizontal bars (frames F_1 and F_2) at the other ends. Hang a constant weight to the hook of horizontal bar of reference wire and hanger on test wire so that wire remains stretched and free from kinks.

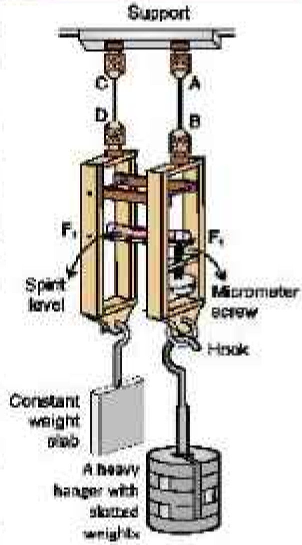


Fig. 5.4: Searle's apparatus

Procedure

The following procedure is adopted for finding Young's modulus of a wire experimentally.

1. Measure the initial length L_o of the wire using a metre scale.
2. Measure the diameter 'd' of the wire using a screw guage. The diameter should be measured at several

different points along the wire and take average.

3. Adjust the spirit level so that it is in horizontal position by turning the micrometer. Record the micrometer reading to use it as the reference reading.
4. Load the test wire with a further weight, the spirit level tilts due to elongation of the test wire.
5. Adjust the micrometer screw to restore the spirit level in the horizontal position. Subtract the first micrometer reading from the second micrometer reading to obtain the extension of the test wire.
6. Calculate stress and strain from the following formula:

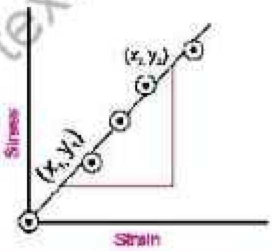
$$\text{Stress} = \frac{\text{Weight}}{\text{Area of wire}} = \frac{F}{A} = \frac{mg}{\pi r^2}$$

$$\text{Strain} = \frac{\Delta L}{L_o} = \frac{\text{Change in length}}{\text{Original length}}$$

7. Repeat the above steps by increasing load on test wire to obtain more values of stresses and strains.
8. Plot the above values on stress strain graph, it should be straight line. Now determine the value of slope Y. The value of slope is equal to Young's modulus of wire.

Brain teaser

A steel rod and a rubber band are subjected to a same force. Which one will be stretched more?



5.5 ELASTIC DEFORMATION, PLASTIC DEFORMATION AND ELASTIC LIMIT

In a tensile test machine, metal wire is extended at a specified deformation rate, and stresses generated in the wire during deformation are continuously measured by a suitable electronic device fitted in the mechanical testing machine. Force-elongation diagram or stress-strain curve is plotted automatically on X-Y chart recorder. A typical stress-strain curve for a ductile material is shown in Fig. 5.5.

In the initial stage of deformation, stress is increased linearly with the strain till we reach point A on the stress-strain curve. This is called proportional limit (σ_p). It is defined as the greatest stress that a material can withstand without losing straight line proportionality between stress and strain. Hooke's law which states that the strain (deformation) is directly proportional to stress (force or load) is obeyed in the region OA. From A to B, stress and strain are not proportional.

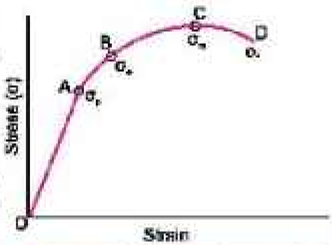


Fig. 5.5: Stress-strain curve of a typical ductile material.

Elastic limit

If the load is removed at any point between O and B, the curve will be retraced and the material will return to its original state. In the region OB, the material is said to be elastic. The point B is called the yield point. The value of stress at B is known as elastic limit σ_e.

Plastic Deformation

If the stress is increased beyond the yield stress or elastic limit of the material, the specimen becomes permanently changed and does not recover its original shape or dimension after the stress is removed. This kind of behaviour is called plasticity. The region of plasticity is represented by the portion of the curve from B to C, the point C in Fig. 5.5 represents the ultimate tensile strength (UTS) σ_u of the material. The UTS is defined as the maximum stress that a material can withstand, and can be regarded as the nominal strength of the material. Once point C corresponding to UTS is crossed, the material breaks at point D, responding the fracture stress (σ_f).

Ductile substances

Substances which undergo plastic deformation until they break, are known as ductile substances. For example, Lead, copper and wrought iron are ductile substances.

Brittle Substances

The substances which break just after the elastic limit is reached, are known as brittle substances. For example, glass and high carbon steel are brittle. Moreover, Beryllium, Bismuth, Chromium are also brittle metals.

Example 5.1 A steel wire 12 mm in diameter is fastened to a log and is then pulled by a tractor. The length of steel wire between the log and the tractor is 11 m. A force of 10,000 N is required to pull the log. Calculate: (a) the stress and strain in the wire. (b) how much does the wire stretch when the log is pulled? ($E = 200 \times 10^9$ N m^{-2})

Solution (a) Tensile stress $\sigma = \dfrac{F}{A} = \dfrac{10{,}000\ \text{N}}{3.14(6\times10^{-3}\,\text{m})^2} = 88.46\times10^6\ \text{N m}^{-2}$

Tensile strain $\varepsilon = \dfrac{\Delta L}{L_o}$, also $E = \dfrac{\text{Stress}}{\text{Strain}} = \dfrac{88.46\times10^6\ \text{N m}^{-2}}{\text{Strain}} = 200\times10^9\ \text{N m}^{-2}$

$$\text{Strain} = \frac{88.46\times10^6\ \text{N m}^{-2}}{200\times10^9\ \text{N m}^{-2}} = 4.4\times10^{-4}$$

(b) Using the relation; Strain $= \dfrac{\Delta L}{L_o}$ or $\Delta L = \text{Strain}\times L_o = 4.4\times10^{-4}\times 11\ \text{m} = 4.84\times10^{-3}$ m

5.6 STRAIN ENERGY IN DEFORMED MATERIALS

When a body is deformed by a force, work is done against elastic restoring force. It is stored in it as its potential energy and is equal to the gain in potential energy of the molecules of a body due to the displacement of these molecules from their mean positions.

Derivation of Expression for Energy Stored in a Stretched Material

Consider a material in the form of a spring as shown in Fig. 5.6. It is stretched by a force F through extension x. As the extension is directly proportional to the stretching force within the elastic limit, therefore the force increases uniformly from zero to F as shown in Fig. 5.7. Thus, the average force that stretches the spring through x is1/2F. Hence work done by the stretching force will be given as:

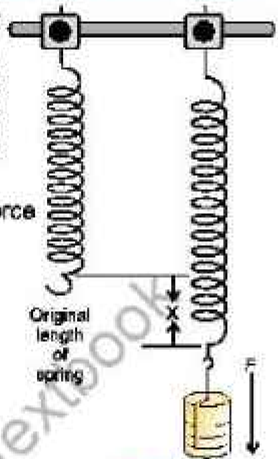


Fig. 5.6

Work done = Average force × Distance in the direction of the force

$$W = \frac{1}{2}F \times x \quad .... \quad (5.7)$$

From Hooke's law $\quad F = k(x)$

Therefore, $\quad W = (\frac{1}{2}kx).(x) \quad = \frac{1}{2}kx^2$

or $\quad W$ = Area of OPQ

The work done by the stretching force is stored in the spring as its strained energy and is equal to the potential energy stored in its molecules.

Strained energy stored in the body $E = \frac{1}{2}F \cdot x = \frac{1}{2}kx^2 \cdots (5.8)$

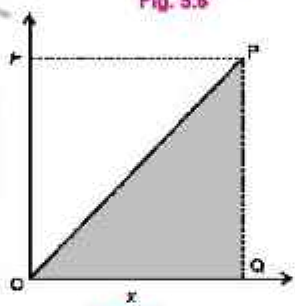


Fig. 5.7

For your information

The amount of work done in stretching a material is equal to the average force applied multiplied by the distance moved.
Therefore, the area under a force-extension graph represents the work done to stretch the material. Work done to stretch the material is also equal to elastic *P.E.* stored in the material.

5.7 ARCHIMEDES' PRINCIPLE AND FLOATATION

An air-filled balloon immediately shoots up to the surface when released under the surface of water. The same would happen if a piece of wood is released under water. We might have noticed that a mug filled with water feels light under water but feels heavy as soon as we take it out of water.

More than two thousand years ago, the Greek scientist, Archimedes noticed that there is an upward force which acts on an object which is kept inside a liquid. As a result, an apparent reduction in weight of the object is observed. This upward force acting on the object is called the upthrust of the liquid. Archimedes' principle states that:

> When an object is totally or partially immersed in a liquid, an upthrust acts on it equal to the weight of the fluid it displaces.

Consider a solid cylinder of cross-sectional area A and height h immersed in a liquid as shown in Fig. 5.8. Let h_1 and h_2 be the depths of the top and bottom faces of the cylinder respectively from the surface of the liquid. Then

$$h_2 - h_1 = h \quad \text{..........................(5.9)}$$

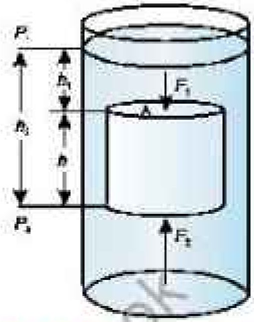


Fig. 5.8. Upthrust on a body immersed in a liquid is equal to the weight of the liquid displaced.

If P_1 and P_2 are the liquid pressures at depths h_1 and h_2 respectively and ρ is its density, then using equation $P = \rho gh$ of liquid pressure at height h:

$$P_1 = \rho g h_1$$

and $$P_2 = \rho g h_2$$

Let the force F_1 be exerted at the top of cylinder by the liquid due to pressure P_1 and the force F_2 be exerted at the bottom of the cylinder by the liquid due to P_2.

Then $F_1 = P_1 A = \rho g h_1 A$

and $F_2 = P_2 A = \rho g h_2 A$

F_1 and F_2 are the forces acting on the opposite faces of the cylinder. Therefore, the net force F will be equal to the difference of these forces. This net force F on the cylinder is called the upthrust of the liquid. Hence

$$F_2 - F_1 = \rho g h_2 A - \rho g h_1 A$$

$$= \rho g A (h_2 - h_1) \quad \text{..........(5.10)}$$

or Upthrust of liquid $= \rho g A h$

$$\text{Upthrust} = \rho g V \quad \text{....................(5.11)}$$

For your information

Archimedes was born about 287 BC, in Syracuse on the Island of Sicily. He was killed by a Roman soldier after he refused to leave his mathematical work.

Here Ah is the volume V of the cylinder and is equal to the volume of the liquid displaced by the cylinder, therefore, ρgV is the weight of the liquid displaced. This equation shows that an upthrust acts on a body immersed in a liquid and is equal to the weight of liquid displaced, which is Archimede's principle.

Example 5.2 A wooden cube of sides 10 cm each has been dipped completely in water. Calculate the upthurst of water acting on it.

Solution **Given**

Length of side $L = 10\text{ cm} = 0.1\text{ m}$

Volume $V = L^3 = (0.1\text{ m})^3 = 1 \times 10^{-3}\text{ m}^3$

Density of water $\rho = 1000\text{ kg m}^{-3}$

Upthrust $F = ?$

Using Archimede's principle

Upthrust of water $= \rho g V$

$= 1000\text{ kg m}^{-3} \times 9.8\text{ m s}^{-2} \times 1 \times 10^{-3}\text{ m}^3 = 9.8\text{ N}$

Thus, upthrust of water acting on the wooden cube is 9.8 N.

Brain teaser

Why does a ship made of heavy steel float on water, while a small rock sink?

Floatation

An object sinks into a fluid if its weight is greater than the upthrust acting on it. However, an object floats if its weight is equal or less than the upthrust. When an object floats in a fluid, the upthrust acting on it is equal to the weight of the object. In case of floating object, the object may be partially immersed. The upthrust is always equal to the weight of the fluid displaced by the object. This is the principle of floatation. It states that:

A floating object displaces a fluid having weight equal to the weight of the object.

Archimedes' principle is applicable on liquids as well as on gases. We find numerous applications of this principle in our daily life.

Applications

Following are some important applications of Archimedes' principle.

1. Hot-air balloon

The reason why hot-air balloons (Fig. 5.9) rise and float in mid-air is because of the density of the hot-air balloon is less than the surrounding air. When the upthrust of the surrounded air is more, it starts to rise. This is done by varying the quantity of hot air in the balloon.

Fig. 5.9

2. Wooden block floating on water

A wooden block floats on water. It is because the weight of an equal volume of water is greater than the weight of the block. According to the principle of floatation, a body floats if its displaced water is equal to the weight of the body when it is partially or completely immersed in water.

Fig.5.10 (a): A ship floating over water

3. Ships and boats

Ships and boats are designed on the same principle of floatation. They carry passengers and goods over water. It would sink in water if its total weight becomes greater than the upthrust of water.

4. Submarine

A submarine can travel over as well as under water using the same principle of floatation.

It floats over water when the weight of water equal to its volume is greater than its weight. Under this condition, it is similar to a ship and remains partially above water level. It has a system of tanks which can be filled with and emptied from seawater. When these tanks are

Fig.5.10 (b): Submarine

filled with seawater, the weight of the submarine increases. As soon as its weight becomes greater than the upthrust, it dives into water and remains under water. To come up on the surface, the tanks are made empty from seawater.

Example 5.3 An empty meteorological balloon weighs 80 N. It is filled with 10 cubic metres of hydrogen. How much maximum contents the balloon can lift besides its own weight? The density of hydrogen is 0.09 kg m^{-3} and the density of air is 1.3 kg m^{-3}.

Solution **Given:**

Weight of the balloon w = 80 N

Volume of hydrogen V = 10 m^3

Density of hydrogen ρ_1 = 0.09 kg m^{-3}

Density of air ρ_2 = 1.3 kg m^{-3}

Weight of hydrogen w_1 = ?

Weight of the contents w_2 = ?

Upthrust F = Weight of air displaced

$= \rho_2 gV$

$= 1.3\ kg\ m^{-3} \times 9.8\ m\ s^{-2} \times 10\ m^3$

$= 127.4\ N$

Weight of hydrogen $w_1 = \rho_1 Vg$

$= 0.09\ kg\ m^{-3} \times 10\ m^3 \times 9.8\ m\ s^{-2}$

$= 8.82\ N$

Total weight lifted $F = w + w_1 + w_2$

To lift the contents, the total weight of the balloon should not exceed F.

Thus $w + w_1 + w_2 = F$

$80\ N + 8.82\ N + w_2 = 127.4\ N$

or $w_2 = 38.58\ N$

Thus, the maximum weight of 38.58 N can be lifted by the balloon in addition to its own weight.

5.8 STEADY, NON-VISCOUS AND IDEAL FLUID

Moving fluids have great importance. In order to find the behaviour of the fluid in motion, we consider their flow through the pipes. When a fluid is in motion, its flow can take place in two ways, either streamline or turbulent.

Streamline or Laminar Flow

The flow is said to be streamline or laminar flow.

If every particle that passes a particular point, moves along exactly the same path, as followed by particles which passed that points earlier.

In a steady flow of a fluid, the motion of the particles is smooth and regular, as shown in Fig. 5.11. The smooth path followed by fluid particles in laminar flow is called a streamline. The streamline may be the straight or curved and tangent to any point gives the direction of flow of a fluid. The different streamlines cannot cross each other.

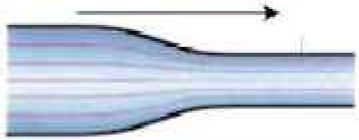


Fig. 5.11: Streamlines (laminar flow)

Example: A fluid flowing in a pipe as shown in Fig. 5.12 will have certain velocity v_1 at P, a velocity v_2 at Q and so on. If the velocity of a particle of the fluid at P, Q and R does not change with the passage of time, then the flow is said to be steady flow or streamline flow.

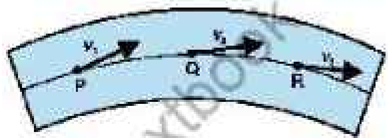


Fig. 5.12: The velocities of the particles at different points on streamline.

The line PQR which represents the path followed by the particle is called a streamline. It represents the fixed path followed by orderly processing particles. In streamline flow, all the particles passing through P also pass through Q and R. It means that two streamlines cannot cross each other.

Fig. 5.13: Turbulent flow

Turbulent Flow

The irregular or unsteady flow of the fluid is called turbulent flow.

Above a certain velocity of the fluid flow, the motion of the fluid becomes unsteady and irregular. Under this condition, the velocity of the fluid changes abruptly as shown in the Fig. 5.13. In this case, the exact path of the particles cannot be considered.

If two streamlines cross each other, then the particles will go in one or in the other directions and flow will not be a steady flow. Such a flow is a turbulent flow. When the flow is unsteady or turbulent, there are eddies and whirlpools in the motion and the paths of the particles are continuously changing.

For your information

Formula One racing cars have a streamlined design.

Dolphins have streamlined bodies to assist their movement in water.

Ideal Fluid

The behaviour of the fluid which satisfies the following conditions is called ideal fluid:

1. The fluid is non-viscous i.e., there is no frictional force between adjacent layers of the fluid.

2. The fluid is incompressible i.e., its density is constant.
3. The fluid motion is steady.

Rate of Flow

The rate of flow of a fluid through a pipe is the volume of the fluid passing through any section of pipe per unit time.

Formula For Rate of Flow

Consider a fluid flowing through a pipe of area of cross-section A as shown in Fig. 5.14. Let the velocity of the fluid be v and it flows through the pipe for time t, then the distance covered by the fluid in time is:

Fig. 5.14: Rate of flow of a liquid

$$\ell = v\,t$$

where ℓ is the length of the pipe through which the fluid passes in time t. Volume of the fluid passing through the pipe in time t, is:

$$A \times \ell = Avt$$

Thus The rate of flow of the liquid $= \dfrac{\text{Volume}}{\text{Time}}$

$$= \frac{Avt}{t} = Av$$

$$\text{Rate of flow} = Av \quad \ldots\ldots\ldots\ldots\ldots\ldots(5.12)$$

In SI units, it is measured in cubic metre per second ($m^3\,s^{-1}$). Sometimes, it is also measured in litres per second ($L\,s^{-1}$).

Steady Flow

If the overall flow pattern does not change with time, the flow is called steady flow.

In steady flow, every particle of the fluid follows the same flow line as its previous particle.

5.9 EQUATION OF CONTINUITY

Statement

The product of cross-sectional area of the pipe and the fluid speed (i.e., Av) at any point along the pipe is a constant. This constant is equal to the volume flow per second of the fluid or simply the flow rate.

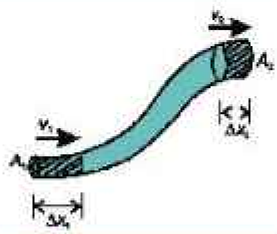


Fig. 5.15: Steady flow of a fluid

Thus $\quad Av = \text{Constant} = \dfrac{\text{Volume}}{\text{Time}}$

Consider a fluid flowing through a pipe of non-uniform size.

The particles in the fluid move along the same lines in a steady state flow as shown in Fig 5.15.

If we consider the flow for a short interval of time Δt, the fluid at the lower end of the tube covers a distance Δx, with a velocity v_1, then distance covered by the fluid is:

$$\Delta x_1 = v_1 \Delta t \quad \text{.................(5.13)}$$

Scientific Fact

Euler obtained the continuity equation for a incompressible fluid with a large number of terms in 1752. Later, it was translated by C. Truesdall from English in 1954.

Let A_1 be the area of cross-section of the lower end, then volume of the fluid that flows into the tube at A_1 is:

$$V = A_1 \Delta x_1$$

or $$V = A_1 v_1 \Delta t$$

If ρ_1 is the density of the fluid, then the mass of the fluid contained in the shaded region (through A_1) is:

$$\Delta m_1 = \text{Volume} \times \text{Density}$$

or $$\Delta m_1 = A_1 v_1 \Delta t \times \rho_1$$

Similarly, the mass of the fluid that moves with velocity v_2 through the upper end of the pipe having cross-sectional area A_2 in the same time Δt is given by

$$\Delta m_2 = A_2 v_2 \Delta t \times \rho_2$$

where ρ_2 is the density of the fluid flowing out through A_2 and Δm_2 indicates small mass.

If the fluid is incompressible and the flow is steady, the mass of the fluid is conserved. That is the mass flowing into the bottom of the pipe through A_1 in a time Δt must be equal to the fluid flowing out though A_2 in the same time. Therefore,

$$\Delta m_1 = \Delta m_2 \quad \text{..............................(5.14)}$$

So $$A_1 v_1 \Delta t \times \rho_1 = A_2 v_2 \Delta t \times \rho_2$$

or $$A_1 v_1 \rho_1 = A_2 v_2 \rho_2 \quad \text{..........................(5.15)}$$

Equation (5.15) is called the equation of continuity. Since density is constant for the steady flow of incompressible fluid, therefore, the equation of continuity becomes:

$$A_1 v_1 = A_2 v_2 \quad \text{..............................(5.16)}$$

Equation (5.16) states that in steady flow, the rate of flow of the fluid inward is equal to the rate of flow of the fluid outward.

This equation justifies the conservation of mass of the fluid which is flowing through a pipe.

Example 5.4 A water hose with an internal diameter of 20 mm at the outlet discharge 30 kg of water in 60 s. Calculate the water speed at the outlet. Assume the density of water is 1000 kg m^{-3} and its flow is steady.

Internal diameter of water hose $D = 20\text{ mm} = 0.02\text{ m}$

Solution Radius $r = \dfrac{D}{2} = \dfrac{0.02\text{ m}}{2} = 0.01\text{m}$

Mass of water $m = 30\ \text{kg}$

Time taken $t = 60\ \text{s}$

Density of water $\rho = 1000\ \text{kg m}^{-3}$

Speed of water $v = ?$

Mass flow per second $m/t = 30\ \text{kg} / 60\ \text{s}$

$= 0.5\ \text{kg s}^{-1}$

Cross-sectional area $A = \pi r^2$

$= 3.14 \times (0.01\ \text{m})^2$

$= 3.14 \times 10^{-4}\ \text{m}^2$

From equation of continuity, the mass of water discharging per second through area A is:

$$\rho A v = \text{Mass / Second}$$

$$v = \frac{\text{Mass / Second}}{\rho A}$$

$$v = \frac{0.5\ \text{kg s}^{-1}}{1000\ \text{kg m}^{-3} \times 3.14 \times 10^{-4}\ \text{m}^2}$$

$$= 1.6\ \text{m s}^{-1}$$

Tidbits

As the water falls, its speed increases and so its cross sectional area decreases as mandated by the continuity equation.

For your information

The equation of continuity is applied to:

(i) blood flow in arteries and veins

(ii) water flow in rivers and pipes

(iii) air flow in duct and ventilation systems.

5.10 INCREASE IN FLOW VELOCITY

We can increase the flow velocity of water in a rubber pipe by squeezing it. When we squeeze the rubber pipe, we decrease the cross-sectional area through which the water flows. According to the equation of continuity,

$$A_1 v_1 = A_2 v_2$$

where A is the cross-sectional area and v is the flow velocity. By decreasing the cross-sectional area ($A_2 < A_1$), the velocity of the water (v_2) must increase to maintain the same flow rate. Therefore, squeezing the rubber pipe increases the flow velocity of fluid.

5.11 BERNOULLI'S EQUATION

The sum of pressure, *K.E.* per unit volume and *P.E.* per unit volume of an ideal fluid throughout its steady flow remains constant.

As the fluid moves through a pipe of varying cross-section and height, the pressure will change along the pipe. Bernoulli's equation is the fundamental equation in fluid dynamics that relates pressure to fluid speed and height.

Fig. 5.16: The speed of water spraying from the end of a garden hose increases as the hose is squeezed with the thumb.

In deriving Bernoulli's equation, we assume that the fluid is incompressible, non-viscous and flows in a steady state manner. Let us consider the flow of the fluid through the pipe in time t, as shown in Fig. 5.17.

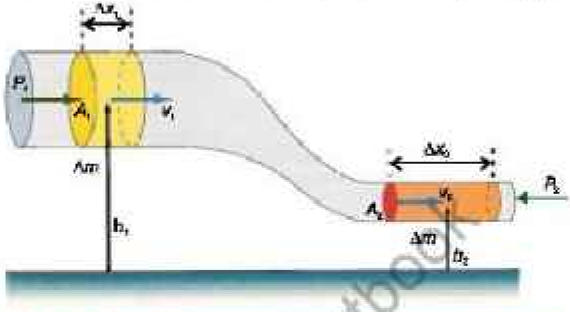


Fig. 5.17: An ideal flow of fluid through a non-uniform cross-section pip at different heights.

The force on the upper end of the fluid is P_1A_1 where P_1 is the pressure and A_1 is the area of cross-section at the upper end. The work done on the fluid, by the fluid behind it, in moving it through a distance Δx_1, will be:

$$W_1 = F_1\Delta x_1 = P_1A_1\Delta x_1$$

Similarly, the work done on the fluid at the lower end is:

$$W_2 = -F_2\Delta x_2 = -P_2A_2\Delta x_2$$

where P_2 is the pressure, A_2 is the area of cross-section of lower end and Δx_2 is the distance moved by the fluid in same time interval t. The work W_2 is taken to be –ve as this work is done against the fluid force. The net work done will be:

$$W = W_1 + W_2$$

or $$W = P_1A_1\Delta x_1 - P_2A_2\Delta x_2 \quad \text{.........(5.17)}$$

If v_1 and v_2 are the velocities at the upper and lower ends respectively, then

$$W = P_1A_1v_1t - P_2A_2v_2t$$

Brain teaser

How does the shape of a curveball in baseball relate to Bernoulli's principle?

From equation of continuity;

$$A_1v_1 = A_2v_2$$

Hence $$A_1v_1t = A_2v_2t = V \text{ (volume)}$$

So $$W = (P_1 - P_2)V \quad \text{..............................(5.18)}$$

If m is the mass and ρ is the density, then $V = \frac{m}{\rho}$. So, Eq. (5.18) becomes:

$$W = (P_1 - P_2)\frac{m}{\rho} \quad \text{..............................(5.19)}$$

A part of this work is utilized by the fluid in changing its $K.E.$ and a part is used in changing its gravitational $P.E.$

$$\text{Change in } K.E. = \Delta K.E. = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 \quad \text{..........................(5.20)}$$

$$\text{Change in } P.E. = \Delta P.E. = mgh_2 - mgh_1 \quad \text{..........................(5.21)}$$

where h_1 and h_2 are the heights of the upper and lower ends respectively.

Applying the law of conservation of energy to this volume of the fluid, we have

$$(P_1 - P_2)\frac{m}{\rho} = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 + mgh_2 - mgh_1 \quad \text{......(5.22)}$$

Rearranging Eq. (5.22), we have

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

This is Bernoulli's equation and is often expressed as:

$$P + \frac{1}{2}\rho v^2 + \rho g h = \text{constant}$$

5.12 USES OF BERNOULLI'S EQUATION

A number of devices operate by means of pressure difference that results from changes in the speed of the fluid.

1. Aeroplane Wings

Uplifting of an aeroplane is due to the designing of its wings, which deflect the air so that streamlines are closer together above the wing than below it as illustrated in Fig.5.18. We have seen that where the streamlines are forced closer together, the speed is faster. Thus, air is travelling faster on the upper side of the wing than on the lower. The pressure will be lower at the top of the wing, and the wing will be forced upward and the lift of an aeroplane is due to this effect.

Fig. 5.18: Lift of an aeroplane

2. Swing of a Ball

When a ball is thrown or kicked with spin or the ball is made smoother on one side by the bowler and remains rough on the other side, the air moves faster over rough side and slows over the smoother (Fig. 5.19). According to Bernoulli's equation, the faster moving air creates lower pressure, while the slower moving air creates higher pressure, this pressure difference generates a sideways force, known as Magnus effect which causes the ball to curves in the air.

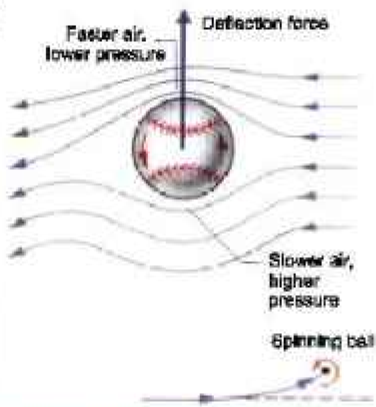


Fig. 5.19: Turbulent flow

3. Filter Pump

A filter pump has a constriction in the centre, so that a jet of water from the tap flows faster here. This causes a drop in pressure near it and air, therefore, flows in from the side tube. The air and water together are expelled through the lower part of the pump (Fig. 5.20).

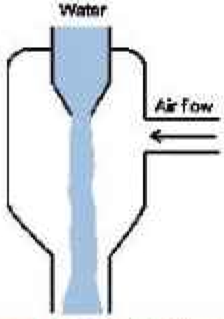


Fig. 5.20: Turbulent flow

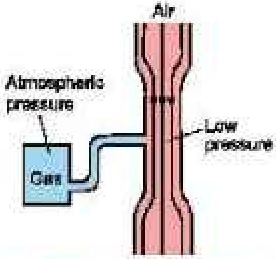


Fig. 5.21: Carburetor of an engine

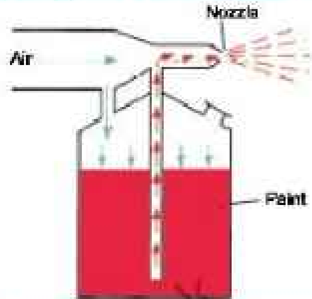


Fig. 5.22: A stream of air passing over a tube dipped in a liquid.

4. Carburetor

The carburetor of a car engine uses a Venturi duct to feed the correct mixture of air and petrol to the cylinders. Air is drawn through the duct and along a pipe to the cylinders (Fig. 5.21). A tiny inlet at the side of duct is fed with petrol.

The air through the duct moves very fast, creating low pressure in the duct, which draws petrol vapours into the air stream.

5. Paint Sprayer

A stream of air passing over a tube dipped in a liquid will cause the liquid to rise in the tube as shown in Fig. 5.22. This effect is used in perfume bottles and paint sprayers. Actually when the rubber ball of atomizer is squeezed, the air is blown through tube and it rushes out through the narrow aperture with high speed and it causes fall of pressure. So, the atmospheric pressure pushes the perfume up leading to the narrow aperture.

6. Venturi Relation

Consider a pipe within which a fluid of density ρ is flowing through different areas of cross-section as shown in the Fig. 5.23.

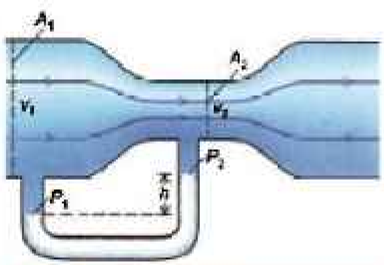


Fig. 5.23: Venturi meter

Let A_1 be the cross-sectional area at wide end and A_2 be the cross-sectional area at narrow portion.

Suppose that v_1 and v_2 be the flow speeds at the wide and narrow portions respectively. Pressure P_1 and P_2 indicate the liquid pressure at both the portions by connecting the limbs of the manometer.

As the pipe is placed horizontally, therefore, we consider that average potential energy is the same at both places while using Bernoulli's equation.

Thus, Bernoulli's equation can be written as:

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 - \frac{1}{2}\rho v_2^2$$

or $$P_1 - P_2 = \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2$$

or $$P_1 - P_2 = \frac{1}{2}\rho\left(v_2^2 - v_1^2\right) \quad \text{..................(5.23)}$$

From the equation of continuity:

$$A_1 v_1 = A_2 v_2$$

or $$v_1 = \frac{A_2 v_2}{A_1}$$

As the cross-sectional area A_2 is small as compared to the area A_1 as is clear from the figure, i.e. $A_2 < A_1$. So, v_1 will be small as compared to v_2. Thus, the speed of the fluid is very slow in wider portion of the pipe as compared to the narrow portion, So, we can neglect v_1 on the right-hand side of Eq.(5.23). Hence

$$P_1 - P_2 = \frac{1}{2}\rho v_2^2 \quad \text{.................. (5.24)}$$

This is known as Venturi relation, which is used in venturi meter, a device used to measure speed of liquid flow.

Interesting Information

It is clear from the result of Bernoulli's Equation for horizontal pipe that "where speed is high, the pressure will be low".
Mathematically,

$$P + \frac{1}{2}\rho v^2 = \text{constant}$$

7. Torricelli's Theorem

A simple application of Bernoulli's equation is shown in Fig. 5.24. Suppose a large tank of fluid has two small orifices A and B on it. Let us find the speed with which the water flows from the orifice A.

Since the orifices are so small, the efflux speeds v_2 and v_3 will be much larger than the speed v_1 of the top surface of water. We can therefore, take v_1 as approximately zero. Hence, Bernoulli's equation can be written as:

$$P_1 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

But $P_1 = P_2$ = Atmospheric pressure

Therefore, the above equation becomes:

$$v_2 = \sqrt{2g(h_1 - h_2)} \quad \text{..................(5.25)}$$

This is Torricelli's theorem which states that:

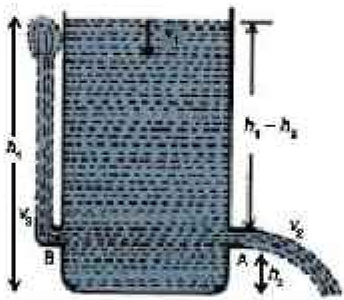


Fig.5.24:
A tank containing fluid with a orifice.

> The speed of efflux is equal to the velocity gained by the fluid in falling through the distance $(h_1 - h_2)$ under the action of gravity.

Notice that the speed of the efflux of liquid is the same as the speed of a ball that falls through a height $(h_1 - h_2)$. The top level of the tank has moved down a little and the *P.E.* has been transferred into *K.E.* of the efflux of fluid. If the orifice had been pointed upward at B as shown in Fig.6.4, this *K.E.* would allow the liquid to rise to the level of water tank. In practice, viscous-energy losses would alter the result to some extent.

5.13 VISCOUS DRAG AND STOKES' LAW

The frictional effect between different layers of a flowing fluid is described in terms of viscosity of the fluid. Viscosity measures, how much force is required to slide one layer of the liquid over another layer. Substances that do not flow easily, such as thick tar and honey, etc., have large coefficients of viscosity, usually denoted by Greek letter 'η'. Substances which flow easily, like water, have small coefficient of viscosities. Since liquids and gases have non zero viscosity, therefore, a force is required if an object is to be moved through them. Even the small viscosity of the air causes a large retarding force on a car as it travels at high speed. If you stick out your hand out of the window of a fast moving car, you can easily recognize that considerable force has to be exerted on your hand to move it through the air. These are typical examples of the following fact.

For Your Information

Viscosities of Liquids and Gases at 30°C

Material	Viscosity 10^{-3} (N s m^{-2})
Air	0.019
Acetone	0.295
Methanol	0.510
Benzene	0.564
Water	0.801
Ethanol	1.000
Plasma	1.6
Glycerin	6.29

An object moving through a fluid experiences a retarding force called a drag force. The drag force increases as the speed of the object increases.

Even in the simplest cases, the exact value of the drag force is difficult to calculate. However, the case of a sphere moving through a fluid is of great importance.

The drag force F on a sphere of radius r moving slowly with speed v through a fluid of viscosity η is given by Stokes' law as under:

$$F = 6\pi\eta r v \quad \text{.......................} \quad (5.26)$$

However, at high speeds the force is no longer simply proportional to speed.

Do you know?

A chimney works best when it is tall and exposed to air currents, which reduces the pressure at the top and forces the upward flow of smoke.

5.14 TERMINAL VELOCITY

Consider a water droplet having radius r such as that of fog falling vertically, the air drag on the water droplet increases with speed. The droplet accelerates rapidly under the over powering force of gravity which pulls the droplet rapidly downward due to force of gravity. However, the upward drag force on it increases as the speed of the droplet increases. The net force on the droplet is

$$\text{Net force} = \text{Weight} - \text{Drag force} \quad \ldots\ldots\ldots\ldots \quad (5.27)$$

As the speed of the droplet continues to increase, the drag force eventually approaches the weight in the magnitude. Finally, when the magnitude of the drag force becomes equal to the weight, the net force acting on the droplet is zero. Then the droplet will fall with constant speed called terminal velocity.

To find the terminal velocity v_t in this case, we use Stokes' law for the drag force. Equating it to the weight of the drop, we have

$$0 = mg - 6\pi\eta r v_t$$

$$v_t = \frac{mg}{6\pi\eta r} \quad \ldots\ldots\ldots\ldots \quad (5.28)$$

The mass of the droplet is ρV, where $V = \frac{4}{3}\pi r^3$ is the volume of the sphere.

Substituting above values in the Eq. (5.28), we have

$$v_t = \frac{2gr^2\rho}{9\eta} \quad \ldots\ldots\ldots\ldots \quad (5.29)$$

Example 5.5 A tiny water droplet of radius 0.010 cm descends through air from a high building. Calculate its terminal velocity. Given that η for air $= 19 \times 10^{-6}\ \text{kg m}^{-1}\ \text{s}^{-1}$ and density of water $\rho = 1000\ \text{kg m}^{-3}$.

Solution

$$r = 1.0 \times 10^{-4}\ \text{m}, \quad \rho = 1000\ \text{kg m}^{-3}, \quad \eta = 19 \times 10^{-6}\ \text{kg m}^{-1}\ \text{s}^{-1}$$

Putting the above values in Eq. (5.29)

$$v_t = \frac{2 \times 9.8\ \text{m s}^{-2} \times (1.0 \times 10^{-4}\ \text{m})^2 \times 1000\ \text{kg m}^{-3}}{9 \times 19 \times 10^{-6}\ \text{kg m}^{-1}\ \text{s}^{-1}}$$

Terminal velocity = $1.1\ \text{m s}^{-1}$.

5.15 REAL FLUIDS ARE VISCOUS FLUIDS

Ideal fluid

It is a fluid that does not have viscosity and cannot be compressed. This type of fluid cannot exist practically.

Real fluid

All types of fluids that possess viscosity are termed as real fluids.

Examples: Kerosene oil, castor oil and honey etc.

Comparison of Ideal and Real Fluids

An example of ideal fluid cannot be provided because it does not exist in the real world.

However, every fluid that we see around us like water, diesel, petrol, honey, etc. are real fluids. Moreover, differences in viscosity can be found in real life, for example, honey is more viscous than water. Bernoulli's equation states that the speed of fluid flow is increased as a result of a simultaneous decrease in the potential energy of the fluid or a decrease in the static pressure on the fluid. When a fluid is viscous, it essentially refers to the thickness of the fluid or the friction, the fluid faces while fluid flows. Therefore, ideal fluids do not face the opposing force and have a non-viscous flow, while real fluids have a viscous flow. Ideal fluids are incompressible. It is not also subjected to surface tension.

5.16 SUPERFLUIDS

Superfluidity is the characteristic property of fluids with zero viscosity i.e., flow is frictionless. A substance exhibiting this property is a superfluid. Superfluids flow without loss of kinetic energy. They can flow through incredibly narrow spaces without any friction.

Superfluidity is achieved in some substances at extremely low temperature. For example, in fluid dynamics, a vortex is region in a fluid in which the flow revolves around an axial line, which may be straight or curved. The vortices are generally created at a moving boundary due to frictionless conditions. Vortices move with the fluid and dispersed by the action of viscosity.

Superfluid helium-4 is the most studied example of superfluidity. It changes from a liquid to a superfluids just a few degree below its boiling point of –452°F (-269°C or 4 K). Superfluids helium-4 moving as a normal clear liquid, but it has no viscosity. This means that once it starts to flow, it keeps moving past any obstacles.

Tidbit

Parachutes increase air resistance (drag) by creating a large surface area, which counteracts the force of gravity. This slows down the persons fall, allowing them to land safely.

Tidbit

Superfluids can "climb" up walls and over edges of containers because they do not experience friction like normal fluids do.

Superfluidity Applications

Currently, there are few practical uses for superfluids. Superfluid helium-4 serves as a coolant for high-field magnets. Both helium-3 and helium-4 are utilized in advanced particle detectors. Researching superfluidity also helps us learn more about superconductivity.

Liquid helium is recognized for its great thermal conductivity and is used in cryogenic applications, including cooling superconducting magnets, scientific research, and medical uses. Additionally, it is employed in industry for leak testing and in the production of electronic and optical products.

QUESTIONS

Multiple Choice Questions

Tick (✓) the correct answer.

5.1 The region of stress-strain curve which obeys Hooke's law is:
(a) proportional limit (b) elastic region (c) plastic region (d) yield limit

5.2 Which of the following is more elastic?
(a) Rubber (b) Wood (c) Sponge (d) Steel

5.3 Which of the following is polymer solid?
(a) Wool (b) Glass (c) Sodium chloride (d) Copper

5.4 The effect of decrease of pressure with the increase in speed of a fluid in horizontal pipe is:
(a) Torricelli's effect (b) Bernoulli's effect (c) Venture's effect (d) Doppler's effect

5.5 The pressure will be low when speed of a fluid is:
(a) zero (b) high (c) low (d) constant

5.6 As per law of fluid friction for steady streamline flow, the friction:
(a) varies proportionally to velocity of fluid
(b) varies inversely proportional to pressure
(c) does not depend on pressure
(d) first increases then decreases

5.7 If a stone is submerged in water and it weighs less in water than in air, this phenomenon is due to:
(a) the reduction of mass in water (b) increase of density in water
(c) buoyant force acting upwards (d) the gravitational force acting upward

5.8 The principle of floatation is a direct application of:
(a) Pascal's law (b) Bernoulli's principle
(c) Archimedes' principle (d) Newton's third law

5.9 An ideal flow of any fluid must satisfy:
(a) Pascal law (b) Bernoulli's equation
(c) Continuity equation only (d) Both (b) and (c)

5.10 The lift force experienced by an aeroplane wings is primarily due to:
(a) viscosity of air (b) density of air
(c) pressure difference above and below the wing (d) gravitational force

5.11 In medical field, a venture mask, used to deliver a known oxygen concentration to patients operates is based on:

(a) Newton's third law
(b) Archimedes' principle
(c) Pascal's law
(d) Bernoulli's principle

5.12 Which of the following is a defining characteristic of a superfluid?

(a) Zero viscosity
(b) Infinite density
(c) Zero temperature
(d) Infinite thermal conductivity

Short Answer Questions

5.1 What is meant by (i) cohesive force (ii) viscosity?

5.2 Differentiate between streamline and turbulent flow of a fluid.

5.3 How does pressure changes with depth in fluids?

5.4 How is variation in pressure related to speed of a fluid?

5.5 How is the flow rate related to the cross-sectional area and velocity of the fluid?

5.6 How do you study the variation in velocity of a fluid at different points in a hose with varying diameter?

5.7 How does an object float or sink according to Archimedes Principle?

5.8 How does Archimedes reportedly discover the principle that bears his name?

5.9 Why standing near fast moving train is dangerous? Explain briefly.

5.10 What are some potential applications of superfluidity?

5.11 Differentiate between stress, strain and Young's modulus. Write down their SI units.

Constructed Response Questions

5.1 The ratio stress/strain remains constant for small deformation. What will be effect on this ratio when the deformation made is very large?

5.2 When pure water falls on a flat glass plate, it spreads on the plate while the mercury, when falls on the same plate gets converted into small globules. Why?

5.3 According to Bernoulli's theorem, the pressure of a fluid should remain uniform in a pipe of uniform radius. But actually, it goes on decreasing. Why is it so?

5.4 Why wings of an aeroplane are rounded outward while flattened inward?

5.5 What is the difference in real fluid, ideal fluid and superfluid? Which of these really exits in the world? Explain.

5.6 Why is the study of superfluids important for advancing our knowledge of low temperature physics?

Comprehensive Questions

5.1 Explain in detail the classification of solids with respect to atomic arrangements.

5.2 What is Archimedes' principle? Explain it in detail for finding upthrust.

5.3 Justify that mass remains conserved when a fluid flows through a pipe.

5.4 Explain the term superfluidity.

5.5 State and derive equation of continuity.

5.6 State and prove Bernoulli's equation.

5.7 Give some practical applications of Bernoulli's equation.

5.8 Define terminal velocity of a body and show that terminal velocity is directly proportional to the square of radius of the body.

Numerical Problems

5.1 A steel wire of length 2 metres and cross-sectional area of $2 \times 10^{-6}\ m^2$ is stretched by a force of 400 N. If the Young's modulus of steel is $2 \times 10^{11}\ N\ m^{-2}$, calculate the extension of the wire. (Ans: 0.002 m)

5.2 A spring with a spring constant 200 $N\ m^{-1}$ is stretched by 0.5 m. Find the elastic *P.E.* stored in the spring. (Ans: 25 J)

5.3 A copper wire of length 3 metres and cross-sectional area of $1 \times 10^{-6}\ m^2$ is subjected to a force of 500 N. Calculate the stress and strain produced in the wire. (Young's modulus of copper $Y = 1.1 \times 10^{11}\ N\ m^{-2}$) (Ans: $5 \times 10^{8}\ N\ m^{-2}$, 0.00455)

5.4 A block of wood of mass 10 kg and density of 600 $kg\ m^{-3}$ is floating in water. Calculate the buoyant force acting on the block. (Density of water = 1000 $kg\ m^{-3}$.)

(Ans: 98 N)

5.5 Water flows through a pipe with a diameter of 0.05 m at a velocity of 2 $m\ s^{-1}$. If the pipe narrows to a diameter of 0.03 m, calculate the velocity of water at narrow section. (Ans: 5.56 $m\ s^{-1}$)

5.6 Water flows through a horizontal pipe with a velocity of 3 $m\ s^{-1}$ and pressure of 200,000 Pa at point 1. At the nozzle (point 2), the pressure decreases to atmospheric pressure 101,300 Pa and the velocity increases to 14 $m\ s^{-1}$. Calculate the velocity of the water exiting the nozzle. (Density of water = 1000 $kg\ m^{-3}$)

(Ans: 14.37 $m\ s^{-1}$)

5.7 A tank filled with water has a hole at a depth of 5 m from the water surface. Calculate the velocity of water flowing out of the hole. (Ans: 9.9 $m\ s^{-1}$)

5.8 Calculate the terminal velocity of a spherical raindrop with a radius 0.5 mm falling through the air. (η for air = $19 \times 10^{-6}\ kg\ m^{-1}\ s^{-1}$, ρ = 1000 $kg\ m^{-3}$ for water)

(Ans: 28.65 $m\ s^{-1}$)

Chapter 6

Heat and Thermodynamics

Learning Objectives

After studying this chapter, the students will be able to:

- Describe the basic assumptions of the kinetic theory of gasses. [Including understanding the temperature, pressure and density conditions under which an ideal gas is a good approximation of a real gas.]
- State that regions of equal temperature are in thermal equilibrium
- Relate a rise in temperature of an object to an increase in its internal energy
- Apply the equation of state for an ideal gas[expressed as $PV = nRT$, where n = amount of substance (number of moles) and as $PV = N k_B T$, where N =number of molecules]
- State that the Boltzmann constant k is given by $k_B = R / N_A$
- Use $W = P\Delta V$ for the work done when the volume of a gas changes at constant pressure.
- Describe the difference between the work done by a gas and the work done on a gas.
- Define and use the first law of thermodynamics [$\Delta U = Q - W$ expressed in terms of the increase in internal energy, the heating of the system (energy transferred to the system by heating) and the work done on the system]
- Explain qualitatively, in terms of particles, the relationship between the pressure, temperature and volume of a gas [Specifically the below case:
  - (a) pressure and temperature at constant volume
  - (b) volume and temperature at constant pressure
  - (c) pressure and volume at a constant temperature
- Use the equation, including a graphical representation of the relationship between pressure and volume for a gas at constant temperature.
- Justify how the first law of thermodynamics expresses the conservation of energy.
- Relate a rise in temperature of a body to an increase in its internal energy.
- State the working principle of a heat engine.
- Describe the concept of reversible and irreversible processes.
- State and explain the second law of thermodynamics.
- State the working principle of Carnot's engine
- Describe that refrigerator is a heat engine operating in reverse as that of an ideal heat engine.
- Explain that an increase in temperature, increases the disorder of the system.
- Explain that increase in entropy means degradation of energy.
- Explain that energy is degraded during all natural processes.
- Identifying that system tends to become less orderly over time.
- Explain that Entropy, S is a thermodynamic quantity that relates to the degree of disorder of the particles in a system.
- State that the Carnot cycle sets a limit for the efficiency of a heat engine at the temperatures of its heat reservoir given by; Efficiency = $1 - \frac{T_{cold\ reservoir}}{T_{hot\ reservoir}}$

Thermodynamics is the branch of physics that deals with the relationships and conversions between heat and other forms of energy. It encompasses principles governing the behaviour of systems at macroscopic scales, such as temperature, pressure, and volume. Thermodynamics thus plays a key role in technology, since almost all the raw energy available for our use is liberated in the form of heat. In this chapter, we shall study the bebaviour of gases and laws of thermodynamics, their significance and applications.

6.1 ASSUMPTIONS OF THE KINETIC THEORY OF GASES

The kinetic theory of gases is a fundamental theory in physics and chemistry that explains the behaviour of gases based on the motion of their constituent particles. This theory provides a macroscopic understanding of gas properties such as pressure, temperature, and volume. Here are the key assumptions of the kinetic theory of gases.

1. Gas Particles are in Constant, Random Motion

Gas molecules are in perpetual, random motion. They move in straight lines until they collide with either another molecule or the walls of the container.

2. Negligible Volume of Gas Particles

The volume of the individual gas molecules is negligible as compared to the total volume of the gas. This means that the particles are considered point masses with no significant volume.

3. No Intermolecular Forces

There are no attractive or repulsive forces between the gas molecules. The particles do not exert any force on each other except during collisions.

4. Elastic Collisions

The collisions between gas molecules, and with the walls of the container are perfectly elastic. This means that there is no net loss of kinetic energy during collisions. The total kinetic energy is conserved.

5. Large Number of Particles

A gas contains a large number of particles. This large number allows for the use of statistical methods to describe the properties of the gas.

6. Average Kinetic Energy is Proportional to Temperature

The average kinetic energy of gas particles is directly proportional to the absolute temperature of the gas. This implies that as the temperature increases, the speed of the gas particles also increases.

7. Pressure due to Particle Collisions

The pressure exerted by a gas on the walls of its container is due to the collisions of gas particles with the walls. The force exerted by the particles during collisions generates pressure.

8. Time of Collisions is Negligible

The time interval of a collision between gas particles is extremely short compared to the time between collisions. This assumption simplifies the analysis of particle dynamics.

Limitations of Kinetic Molecular Theory

Real Gases

The assumptions of the kinetic theory hold true for ideal gases, but real gases exhibit deviations due to intermolecular forces and finite molecular volume, especially at high pressures and low temperatures.

In summary, the kinetic theory of gases provides a macroscopic view of gas behaviour, linking macroscopic properties like pressure and temperature to the motion of gas particles, and serves as a foundational concept in understanding thermodynamics and statistical mechanics.

Equation of State for an Ideal Gas

A gas that obeys kinetic theory of gases is termed as an ideal gas. Ideal gas equation is given by

$$PV = nRT \qquad \ldots\ldots (6.1)$$

Here P represents pressure, V is volume, n is number of moles of the gas, R is universal gas constant ($R = 8.3145\ \text{J mol}^{-1}\ \text{K}^{-1}$) and T is the absolute temperature.

Equation (6.1) implies that product of pressure and volume is directly proportional to the absolute temperature for an ideal gas.

Real Gas to Behave Like an Ideal Gas

According to kinetic theory of gases, a gas has no intermolecular interaction and molecules are far apart from each other. For a real gas to behave like an ideal gas, some conditions must be satisfied. $P.E.$ of the gas molecules is negligible and this have only $K.E.$

Number of moles 'n' can be given by

$$n = \frac{\text{Mass of gas}}{\text{Molar mass of gas}} = \frac{m}{M}$$

So, Eq. (6.1) becomes: $PV = \frac{m}{M}RT$ or $PM = \left(\frac{m}{V}\right)RT$

We know that density; $\rho = \frac{m}{V}$, So, $\rho = \frac{PM}{RT}$ as $\frac{M}{R}$ is constant $\therefore \rho \propto \frac{P}{T}$

The density of a gas will be low at low pressure and high temperature due to which molecules of the gas will be at large distance from each other and the intermolecular forces will be negligible. So, the real gas behaves like an ideal gas at low pressure and high temperature.

Ideal Gas Equation in Terms of Boltzmann Constant

From ideal gas equation:

$$PV = nRT \quad \ldots\ldots\ldots\ldots \text{(i)}$$

A mole can be defined as the number of atoms or molecules per unit Avogadro's number ($N_A = 6.02 \times 10^{23}$).

Mathematically;

$$n = \frac{N}{N_A} \quad \ldots\ldots\ldots\ldots \text{(ii)}$$, N is number of atoms or molecules

Substituting Eq. (ii) in Eq. (i), we have

$$PV = \frac{N}{N_A} RT \quad \ldots\ldots\ldots\ldots \text{(iii)}$$

For your information

Real gases approach ideal behaviour under:

(i) low pressure

(ii) high temperature

The term $\frac{R}{N_A}$ is termed as Boltzmann constant k_B, i.e.,

$$k_B = \frac{R}{N_A} \quad \ldots\ldots\ldots\ldots (6.2)$$

Substituting the values of R and N_A, we have

$$k_B = 1.38 \times 10^{-23}\ \text{J K}^{-1}$$

Substituting Eq. (6.2) in Eq. (iii),

$$PV = N k_B T \quad \ldots\ldots\ldots\ldots (6.3)$$

Equation (6.3) gives ideal gas equation in terms of Boltzmann constant k_B.

Example 6.1 One mole of an ideal gas is at a temperature of 300 K. If the Boltzmann constant is 1.38×10^{-23} J K^{-1}, calculate the volume of the gas at a pressure of 1 atom. [1atm = 101325 Pa]

Solution We know that:

$$PV = nRT$$

$$R = N_A \times k_B \quad \text{where} \quad k_B = \frac{R}{N_A}$$

Here $$V = \frac{nN_A k_B T}{P}$$

$$V = \frac{1\ \text{mol} \times 6.02 \times 10^{23}\ \text{mol}^{-1} \times 1.38 \times 10^{-23}\ \text{J K}^{-1} \times 300\ \text{K}}{101325\ \text{Pa}}$$

$$V \approx 0.0245\ \text{m}^3$$

Thus, volume of gas would be $0.0245\ \text{m}^3$.

Gas Laws

There are some variables (state functions) which describe quantity of gas which includes pressure, volume, and temperature (P, V, and T) with change in one variable, the second variable changes while the third is kept constant. The laws that relate these variables mutually for an ideal gas are termed as gas laws.

Boyle's Law

This law was introduced by Robert Boyle in 1662, and it provides a relationship between pressure and volume of a gas at constant temperature. It sates that for a fixed mass of an ideal gas, the pressure P exerted by a gas varies inversely with volume V occupied by the gas at constant temperature.

Mathematically;

$$P \propto \frac{1}{V} \quad \text{at constant } T$$

$$P = \text{constant}\ \frac{1}{V} \quad \text{or} \quad PV = \text{constant}$$

or $\quad P_1V_1 = P_2V_2$

Graphical representation of Boyle's law is shown in Fig. 6.1.

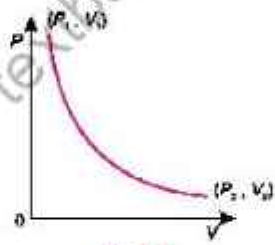


Fig. 6.1

Charles' Law

Charles' law relates volume and temperature of an ideal gas for a fixed mass at constant pressure. This law was formulated in 1870 by a French Physicist Jacques Charles. It states that the volume of given mass of gas at constant pressure is directly proportional to the absolute temperature.

Mathematically;

$$V \propto T \quad \text{at constant } P$$

or $$\frac{V}{T} = \text{constant}$$

or $$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

Graphically, it can be shown in Fig. 6.2.

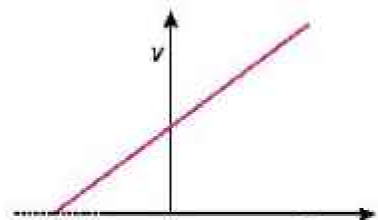


Fig. 6.2

Joseph lussac's Law

It states that for a fixed mass of an ideal gas, the pressure exerted by a gas varies directly with absolute temperature of the gas at constant volume.

Mathematically;

$$P \propto T \quad \text{at constant } V$$

or $$P = \text{constant} \times T$$

or $$\frac{P}{T} = \text{constant}$$

or $$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

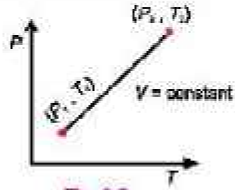


Fig. 6.3

Graphically, Joseph lussac's law is shown in Fig. 6.3.

Thermal Equilibrium

When two bodies are at the same temperature, the thermal energy (which is related to the kinetic energy of particles) of each body is equal. As a result, there is no driving force for heat transfer between them, and thus they remain in thermal equilibrium.

Example

When we put a metal spoon into a hot cup of coffee:

(i) initially, the coffee is hotter than the spoon.

(ii) over time, heat flows from the coffee to the spoon.

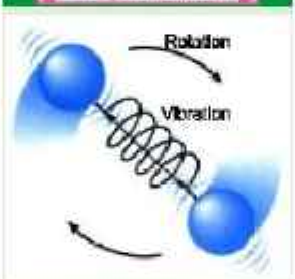
(iii) eventually, the coffee and spoon reach at the same temperature.

Thermal equilibrium is achieved at this point, there is no net heat flow between the coffee and the spoon, and they are said to be in thermal equilibrium.

6.2 INTERNAL ENERGY

The sum of all forms of molecular energies (kinetic and potential) of a substance is termed as its internal energy. In the study of thermodynamics, usually ideal gas is considered as a working substance. The molecules of an ideal gas are mere mass points which exert no forces on one another. So, the internal energy of an ideal gas system is generally the translational *K.E.* of its molecules. Since the temperature of a system is defined as the average *K.E.* of its molecules, thus for an ideal gas system, the internal energy is directly proportional to its temperature.

Do you know?



A diatomic gas molecule has both translational and rotational energy. It also has vibrational energy associated with the spring like bond between its atoms.

According to the kinetic theory of gases, the average kinetic energy of gas molecules is given by

$$<K.E> = \frac{3}{2}k_B T \text{ or } <\frac{1}{2}mv^2> = \frac{3}{2}k_B T$$

where k_B is Boltzmann constant.

Tidbit

Different processes can lead to changes in internal energy and temperature, such as heating (adding heat), adiabatic compression or expansion (no heat exchange), or phase changes (where heat energy changes the state of matter without changing temperature).

Therefore, the rise in temperature of an object reflects an increase in the internal kinetic energy of its particles. This increase in internal energy can occur due to the absorption of heat energy, which raises the average kinetic energy of the particles and thus increases the temperature of the object.

6.3 HEAT AND WORK

We know that both heat and work correspond to transfer of energy by some means. The idea was first applied to the steam engine where it was natural to transfer heat in and get work out. Consequently, it made a sense to define both heat in and work out as positive quantities. Hence, work done by the system on its environment is considered positive while work done on the system by the environment is taken as negative. If an amount of heat Q enters the system, it could manifest itself as either an increase in internal energy or as a resulting quantity of work performed by the system on the surrounding or both.

For your information

Internal energy is a function of state. Consequently, it does not depend on path but depends on initial and final states of the system. Thus, internal energy is similar to the gravitational P.E. So, like the potential energy, it is the change in internal energy and not its absolute value, which is important.

We can express the work in terms of directly measurable variables. Consider the gas enclosed in the cylinder with a moveable, frictionless piston of cross-sectional area A as shown in Fig.6.4(a). In equilibrium, the system occupies volume V and exerts a pressure P on the walls of the cylinder and its piston. The force F exerted by the gas on the piston is PA.

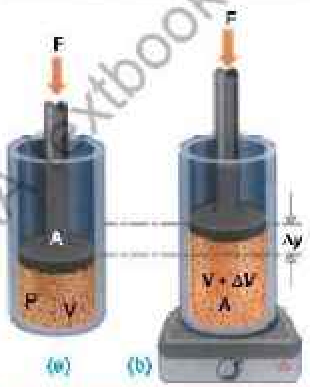


Fig. 6.4

A gas is sealed in a cylinder by a weightless, frictionless piston. The constant downward applied force F equals PA, and when the piston is displaced, downward work is done on the gas.

We assume that the gas expands through ΔV very slowly, so that it remains in equilibrium as shown in Fig. 6.4(b). As the piston moves up through a small distance Δy, the work W done by the gas is:

$$W = F\Delta y = PA\Delta y$$

Since $A\Delta y = \Delta V$ (Change in volume)

Hence $W = P\Delta V$ .... (6.4)

The work done can also be calculated by area of the curve under P-V graph as shown in Fig. 6.5.

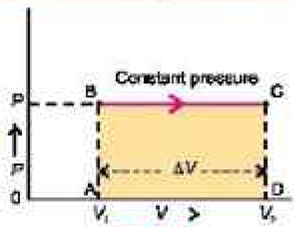


Fig. 6.5

Knowing the details of the change in internal energy and the mechanical work done, we are in a position to describe the general principles which deal with heat energy and its transformation into mechanical energy. These principles are known as laws of thermodynamics.

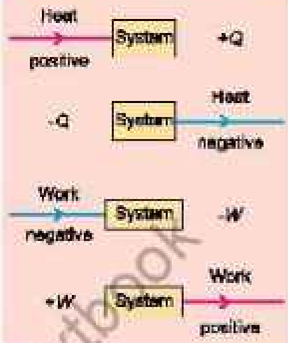
6.4 FIRST LAW OF THERMODYNAMICS

When heat is added to a system there is an increase in the internal energy due to the rise in temperature, an increase in pressure or change in the state. If at the same time, a substance is allowed to do work on its environment by expansion, the heat Q required will be the heat necessary to change the internal energy of the substance from U_1 in the first state to U_2 in the second state plus the work W done on the environment.

Thus $\quad Q = (U_2 - U_1) + W$

or $\quad Q = \Delta U + W \quad \ldots\ldots\ldots \quad (6.5)$

For your information



Thus the change in internal energy $\Delta U = U_2 - U_1$ is defined as $Q - W$. Since it is the same for all processes concerning the state, the first law of thermodynamics, thus can be stated as,

> In any thermodynamic process, when heat Q is added to a system, this energy appears as an increase in the internal energy ΔU stored in the system plus the work W done by the system on its surroundings.

1. **Conservation Principle:** The underlying principle of the first law of thermodynamics is the conservation of energy. It asserts that while energy can change from one form to another (such as from chemical potential energy to thermal energy), the total energy in an isolated system remains constant over time.
2. **Wider Applicability:** Beyond mechanical systems, the first law of thermodynamics applies universally to all forms of energy and all types of processes, including chemical reactions, electrical systems, and nuclear reactions. It provides a foundational understanding that allows scientists and engineers to predict and understand energy transformations in various contexts.

The first law of thermodynamics expresses the law of conservation of energy by affirming that energy is a conserved quantity in isolated systems. It provides a framework to understand how energy is transferred and transformed within systems without violating the fundamental principle that energy neither be created nor be destroyed.

A bicycle pump is a good example of first law of thermodynamics. When we pump on the handle rapidly, it becomes hot due to mechanical work done on the gas, raising thereby its internal energy. One such simple arrangement is shown in Fig. 6.6. It consists of a bicycle pump with a blocked outlet.

Fig. 6.6

A thermocouple connected through the blocked outlet allows the air temperature to be monitored. Thermocouple thermometer can detect a minute variation of the temperature. When the piston is rapidly pushed, thermometer shows a temperature rise due to increase of internal energy of the air. The push force does work on the air, thereby, increasing its internal energy, which is shown, by the increase in temperature of the air.

Human metabolism also provides an example of energy conservation. Human beings and other animals do work when they walk, run, or move. Work requires energy. Energy is also needed for growth to make new cells and to replace old cells that have died. Energy transforming processes that occur within an organism are named as metabolism. We can apply the first law of thermodynamics ($\Delta U = Q - W$), to an organism of the human body. Work done will result in the decrease in internal energy of the body. Consequently, the body temperature or in other words internal energy is maintained by the food we eat.

Example 6.2 A gas is enclosed in a container fitted with a piston of cross-sectional area 0.10 m². The pressure of the gas is maintained at 8000 N m⁻². When heat is slowly transferred, the piston is pushed up through a distance of 4.0 cm. If 42 J heat is transferred to the system during the expansion, what is the change in internal energy of the system?

Solution The work done by the gas is

$$W = P\Delta V = PA\Delta y = 8000\ \text{N m}^{-2} \times 0.10\ \text{m}^2 \times 4.0 \times 10^{-2}\ \text{m}$$
$$= 32\ \text{N m} = 32\ \text{J}$$

The change in internal energy is found from first law of thermodynamics

$$\Delta U = Q - W = 42\ \text{J} - 32\ \text{J} = 10\ \text{J}$$

Isothermal Process

It is a process which is carried out at constant temperature and hence the condition for the application of Boyle's law on the gas is fulfilled. Therefore, when gas expands or compresses isothermally, the product of its pressure and volume during the process remains constant. If P_1, V_1 are initial pressure and volume whereas P_2, V_2 are pressure and volume after the isothermal change takes place (Fig. 6.7-a), respectively, then

$$P_1V_1 = P_2V_2$$

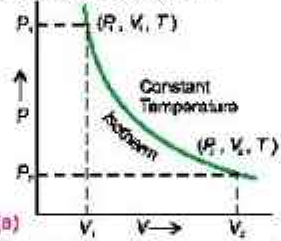


Fig. 6.7(a)

In case of an ideal gas, the P.E. associated with its molecules is zero, hence, the internal energy of an ideal gas depends only on its temperature, which in this case remains constant, therefore, $\Delta U = 0$. Hence, the first law of thermodynamics reduces to

$$Q = W$$

Thus if gas expands and does external work W, an amount of heat Q has to be supplied to the gas in order to produce an isothermal change. Since transfer of heat from one place to another requires time, hence, to keep the temperature of the gas constant, the expansion or compression must take place slowly. The curve representing an isothermal process is called an isotherm (Fig. 6.7-a).

Adiabatic Process

An adiabatic process is the one in which no heat enters or leaves the system. Therefore, $\Delta Q = 0$ and the first law of thermodynamics gives: $W = -\Delta U$

Brain teaser!

Why does the internal energy of an ideal gas remain constant during isothermal expansion?

Thus, if the gas expands and does external work, it is done at the expense of the internal energy of its molecules and, hence, the temperature of the gas falls. Conversely, an adiabatic compression causes the temperature of the gas to rise because of the work done on the gas.

Adiabatic change occurs when the gas expands or compressed rapidly, particularly when the gas is contained in an insulated cylinder. The examples of adiabatic process are:

(i) The rapid escape of air from a burst tyre.

(ii) The rapid expansion and compression of air through which a sound wave is passing.

(iii) Cloud formation in the atmosphere.

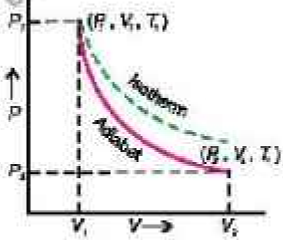


Fig. 6.7(b)

As the temperature of the gas does not remain constant, so it has been seen that;

$$PV^{\gamma} = \text{constant}$$

where γ is the ratio of the molar specific heat of the gas at constant pressure to molar specific heat at constant volume ($\gamma = \frac{C_P}{C_V}$). The curve representing an adiabatic process is called an adiabat (Fig.6.7-b).

6.5 REVERSIBLE AND IRREVERSIBLE PROCESSES

A reversible process is one which can be retraced in exactly reverse order, without producing any change in the surroundings. In the reverse process, the working substance passes through the same stages as in the direct process but thermal and mechanical effects at each stage are exactly reversed. If heat is absorbed in the direct process, it will be given out in the reverse process and if work is done by the substance in the direct process, work will be done on the substance in the reverse process. Hence, the working substance is restored to its original conditions.

A succession of events which brings the system back to its initial condition is called a cycle. A reversible cycle is the one in which all the changes are reversible.

Although no actual change is completely reversible but the processes of liquification and evaporation of a substance, performed slowly, are practically reversible. Similarly the slow compression of a gas in a cylinder is reversible process as the compression can be changed to expansion by slowly decreasing the pressure on the piston to reverse the operation.

> If a process cannot be retraced in the backward direction by reversing the controlling factors, it is an irreversible process.

All changes which occur suddenly or which involve friction or dissipation of energy through conduction, convection or radiation are irreversible. An example of highly irreversible process is an explosion.

Do you know?

First practical steam-engine was designed by John Braithwaite and John Ericsson in England around 1829.

6.6 HEAT ENGINE

A heat engine converts some thermal energy to mechanical work. Usually, the heat comes from the burning of a fuel. The earliest heat engine was the steam engine. It was developed on the fact that when water is boiled in a vessel covered with a lid, the steam inside tries to push the lid off showing the ability to do work. This observation helped to develop a steam engine.

The working principle of a heat engine is based on the conversion of heat energy into mechanical work through a cyclic process. Here is how a heat engine typically operates. The working principle of a heat engine involves the cyclic transfer of heat energy from a high temperature reservoir to a low temperature reservoir, with the objective of converting as much heat as possible into mechanical work. This process is governed by principles of thermodynamics and is essential in various applications where mechanical energy is required from heat sources.

6.7 SECOND LAW OF THERMODYNAMICS

First law of thermodynamics tells us that heat energy can be converted into equivalent amount of work, but it is silent about the conditions under which this conversion takes place. The second law is concerned with the circumstances in which heat can be converted into work and direction of flow of heat.

Before initiating the discussion on formal statement of the second law of thermodynamics, let us analyze briefly the factual operation of an engine. The engine or the system (Fig. 6.8) absorbs a quantity of heat Q_1 from the heat source at temperature T_1. It does work W and expels heat Q_2 to low temperature reservoir at temperature T_2. As the

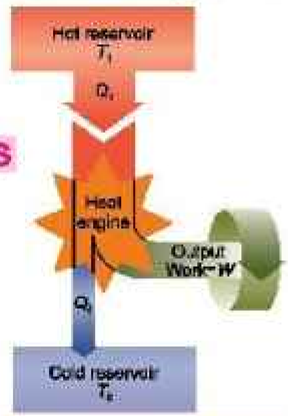


Fig. 6.8

Schematic representation of a heat engine. The engine absorbs heat Q_1 from the hot reservoir, expels heat Q_2 to the cold reservoir and does work W.

working substance goes through a cyclic process, in which the substance eventually returns to its initial state, the change in internal energy is zero. Hence, from the first law of thermodynamics, net work done should be equal to the net heat absorbed. i.e.,

$$W = Q_1 - Q_2$$

In practice, the petrol engine of a motor car extracts heat from the burning fuel and converts a part of this energy to mechanical energy or work and expels the rest to the atmosphere. It has been observed that petrol engines convert roughly 25% and diesel engines 35 to 40% available heat energy into work.

The second law of thermodynamics is a formal statement based on these observations. It can be stated in a number of different ways:

According to Lord Kelvin's statement based on the working of a heat engine:

> It is impossible to devise a process which may convert heat, extracted from a single reservoir, entirely into work without leaving any change in the working system.

Tidbits

According to the Kelvin statement of the second law of thermodynamics, the process pictured here is impossible. Heat from a source at a single temperature cannot be converted entirely into work.

This means that a single heat reservoir, no matter how much energy it contains, cannot be made to perform any work. This is true for oceans and our atmosphere which contain a large amount of heat energy but cannot be converted into useful mechanical work. As a consequence of second law of thermodynamics, two bodies at different temperatures are essential for the conversion of heat into work. Hence, for the working of heat engine there must be a source of heat at a high temperature and a sink at low temperature to which heat may be expelled. The reason for our inability to utilize the heat contents of oceans and atmosphere is that there is no reservoir at a temperature lower than any one of the two.

6.8 CARNOT ENGINE AND CARNOT'S THEOREM

Sadi Carnot in 1824 described an ideal engine using only isothermal and adiabatic processes. He showed that a heat engine operating in an ideal reversible cycle between two heat reservoirs at different temperatures, would be the most efficient engine. A Carnot cycle using an ideal gas as the working substance is shown on *PV* diagram (Fig. 6.9). It consists of following four steps:

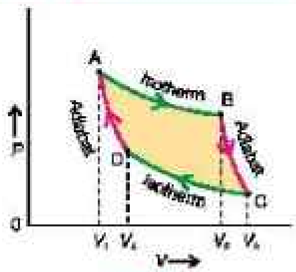


Fig. 6.9

1. The gas is allowed to expand isothermally at temperature T_1 absorbing heat Q_1 from the hot

reservoir. The process is represented by the curve AB.

2. The gas is then allowed to expand adiabatically until its temperature drops to T_2. The process is represented by the curve BC.
3. The gas at this stage is compressed isothermally at temperature T_2 rejecting heat Q_2 to the cold reservoir. The process is represented by the curve CD.
4. Finally the gas is compressed adiabatically to restore its initial state at temperature T_1. The process is represented by the curve DA.

Thermal and mechanical equilibrium is maintained all the time so that each process is perfectly reversible. As the working substance returns to the initial state, there is no change in its internal energy i.e., $\Delta U = 0$.

The net work done during one cycle equals to the area enclosed by the path ABCDA of the PV diagram. It can also be estimated from net heat ΔQ absorbed in one cycle.

$$Q = Q_1 - Q_2$$

From 1st law of thermodynamics:

$$Q = \Delta U + W$$

or

$$W = Q_1 - Q_2 \qquad (\because \Delta U = 0)$$

The efficiency η of the heat engine is defined as:

$$\eta = \frac{\text{Output (Work)}}{\text{Input (Energy)}}$$

Thus

$$\eta = \frac{Q_1 - Q_2}{Q_1} \qquad \ldots\ldots\ldots\ldots (6.6)$$

The energy transfer in an isothermal expansion or compression turns out to be proportional to kelvin temperature. So Q_1 and Q_2 are proportional to kelvin temperatures T_1 and T_2 respectively and hence,

$$\eta = \frac{T_1 - T_2}{T_1} = 1 - \frac{T_2}{T_1} \qquad \ldots\ldots\ldots\ldots (6.7)$$

Interesting information

A waterfall analogy for the heat engine.

The efficiency is usually taken in percentage, in that case:

$$\text{Percentage efficiency} = \left(1 - \frac{T_2}{T_1}\right) \times 100$$

Thus, the efficiency of Carnot engine depends on the temperature of hot and cold reservoirs. It is independent of the nature of working substance. The larger the temperature difference of two reservoirs, the greater is the efficiency. But it can never be one or 100% unless cold reservoir is at absolute zero temperature ($T_2 = 0$ K).

Such reservoirs are not available and hence the maximum efficiency is always less than one. Nevertheless, the Carnot cycle establishes an upper limit on the efficiency of all heat engines. No practical heat engine can be perfectly reversible and also energy dissipation is inevitable. This fact is stated in Carnot's theorem:

> No heat engine can be more efficient than a Carnot engine operating between the same two temperatures.

The Carnot's theorem can be extended to state that:

> All Carnot's engines operating between the same two temperatures have the same efficiency, irrespective of the nature of working substance.

In most practical cases, the cold reservoir is near room temperature. So, the efficiency can only be increased by raising the temperature of hot reservoir. All real heat engines are less efficient than Carnot engine due to friction and other heat losses.

Example 6.3 The turbine in a steam power plant takes steam from a boiler at 427 °C and exhausts into a low temperature reservoir at 77 °C. What is the maximum possible efficiency?

Solution Maximum efficiency for any engine operating between temperatures T_1 and T_2 is

$$\%\eta = \frac{T_1 - T_2}{T_1} \times 100\,\% \qquad \text{where} \qquad T_1 = 427 + 273 = 700\ \text{K}$$

and $$T_2 = 77 + 273 = 350\ \text{K}$$

So $$\eta = \frac{T_1 - T_2}{T_1} = \frac{700\ \text{K} - 350\ \text{K}}{700\ \text{K}} = \frac{350\ \text{K}}{700\ \text{K}} = \frac{1}{2} = 0.5 \quad \text{or} \quad \%\eta = 0.5 \times 100 = 50\,\%$$

6.9 REFRIGERATOR

Refrigerator is a device which maintains the temperature of a body below that of its surrounding. It operates in a cyclic process but in reverse as that of the heat engine as shown in Fig.6.10. A refrigerator absorbs heat from a cold reservoir and gives it off to a hot reservoir. This shows that in a refrigerator, the work is done on the system while in a heat engine work is done by the system.

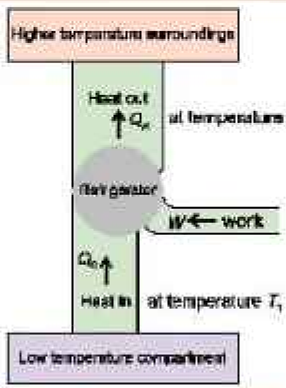


Fig. 6.10 A refrigerator transfers heat from a low-temperature compartment to higher-temperature surroundings with the help of external work. It is a heat engine operating in reverse order.

A refrigerator works on the basis of Clausius statement of second law of thermodynamics, i.e., a heat engine is operating in reverse. Heat Q_C is drawn from Low Temperature Reservoir (LTR) T_1 by compressor and is thrown into High Temperature Reservoir (HTR) T_2 with the help of external work done. The heat rejected to HTR (Q_H) is given by

$$Q_C + W = Q_H \quad \text{or} \quad W = Q_H - Q_C$$

The main purpose of refrigerator is to extract as much heat Q_C as possible from LTR with the expenditure of as little work W as possible.

Coefficient of Performance of Refrigerator

The ratio of heat removed from LTR (Q_C) to the work done (W) is called coefficient of performance of a refrigerator.

A better refrigerator will remove a greater amount of heat from inside the refrigerator for the expenditure of a smaller mechanical work or electrical energy. The coefficient of performance of a refrigerator can be given by

$$E = \frac{Q_c}{W} = \frac{Q_c}{Q_H - Q_c}$$

The coefficient of performance in terms of temperature, where $Q \propto T$ is:

$$E = \frac{T_1}{T_2 - T_1} \quad \text{..........} \quad (6.8)$$

Example 6.4

A refrigerator has a coefficient of performance 8. If temperature in the freezer is -23 °C, then what is the temperature at which it rejects the heat?

Solution

Coefficient of performance $E = 8$

Temperature of cold reservoir (freezer) $T_1 = -23°C = -23 + 273 = 250\text{ K}$

Temperature of hot reservoir (room) $T_2 = ?$

Coefficient of performance $= \dfrac{T_1}{T_2 - T_1}$

Substituting the values $8 = \dfrac{250\text{ K}}{T_2 - 250\text{ K}}$

or $8(T_2 - 250\text{ K}) = 250\text{ K}$ or $T_2 - 250\text{ K} = \dfrac{250\text{ K}}{8}$

or $T_2 = 31.25\text{ K} + 250\text{ K} = 281.25\text{ K} = 8.25\ °C$

6.10 ENTROPY

The concept of entropy was introduced into the study of thermodynamics by Rudolph Clausius in 1856 to give a quantitative basis for the second law. It provides another variable to describe the state of a system to go along with pressure, volume, temperature and internal energy. If a system undergoes a reversible process during which it absorbs a quantity of heat ΔQ at absolute temperature T, then the increase in the state variable called entropy S of the system is given by

$$\Delta S = \frac{\Delta Q}{T} \quad \text{..........} \quad (6.9)$$

Like potential energy or internal energy, it is the change in entropy of the system which is important.

The change in entropy is positive when heat is added and negative when heat is removed from the system. Suppose an amount of heat Q flows from a reservoir at temperature T_1 through a conducting rod to a reservoir at temperature T_2 when $T_1 > T_2$. The change in entropy of the reservoir, at temperature T_1, which loses heat, decreases by Q/T_1 and of the reservoir at temperature T_2, which gains heat, increases by Q/T_2. As $T_1 > T_2$, so Q/T_2 will be greater than Q/T_1, i.e. $Q/T_2 > Q/T_1$.

Hence, net change in entropy ($\frac{Q}{T_2} - \frac{Q}{T_1}$) is positive.

It follows that in all natural processes where heat flows from one system to another, there is always a net increase in entropy. This is another statement of 2nd law of thermodynamics. It states that;

> If a system undergoes a natural process, it will go in the direction that causes the entropy of the system plus the environment to increase.

It is observed that a natural process tends to proceed towards a state of greater disorder. Thus, there is a relation between entropy and molecular disorder. For example, an irreversible heat flow from a hot to a cold substance of a system increases disorder because the molecules are initially sorted out in hotter and cooler regions. This order is lost when the system comes to thermal equilibrium. Addition of heat to a system increases its disorder because of increase in average molecular speeds and therefore, the randomness of molecular motion. Similarly, free expansion of gas increases its disorder because the molecules have greater randomness of position after expansion than before. Thus, in both examples, entropy is said to be increased.

We can conclude that only those processes are probable for which entropy of the system increases or remains constant. The process for which entropy remains constant is a reversible process; whereas for all irreversible processes, entropy of the system increases.

Every time entropy increases, the opportunity to convert some heat into work is lost. For example, there is an increase in entropy when hot and cold waters are mixed. Finally, the warm water cannot be separated into a hot layer and a cold layer. There has been no loss of energy but some of the energy is no longer available for conversion into work. Therefore, increase in entropy means degradation of energy from a higher level where more work can be extracted to a lower level at which less or no useful work can be done. The energy in a sense is degraded, going from more orderly form to less orderly form, eventually ending up as thermal energy.

In all real processes where heat transfer occurs, the energy available for doing useful work decreases, eventually, the entropy increases. Even if the temperature of some system decreases, thereby decreasing the entropy, it is at the expense of net increase in entropy for some other system. When all the systems are taken together as the universe, the entropy of the universe always increases.

Example 6.5 Calculate the entropy change when 1.0 kg ice at 0 °C melts into water at 0 °C. Latent heat of fusion of ice $L_f = 3.36 \times 10^5\ J\,kg^{-1}$.

Solution

$m = 1\ kg$

$T = 0\,°C = 273\ K$

$L_f = 3.36 \times 10^5\ J\,kg^{-1}$

As $\Delta S = \frac{\Delta Q}{T}$

where $\Delta Q = mL_f$

So $\Delta S = \frac{mL_f}{T}$

Substituting the values

$$\Delta S = \frac{1.00\ kg \times 3.36 \times 10^5\ J\,kg^{-1}}{273\ K}$$

$$\Delta S = 1.23 \times 10^3\ J\,K^{-1}$$

Thus, entropy increases as it changes to water. The increase in entropy in this case is a measure of increase in the disorder of water molecules that change from solid to liquid state.

Brain teaser!

Why does a deck of cards become more disordered when shuffled?

QUESTIONS

Multiple Choice Questions

Tick (✓) the correct answer.

6.1 In an isothermal change, internal energy:

(a) decreases (b) increases (c) remains the same (d) becomes zero

6.2 First law of thermodynamics is based upon law of conservation of:

(a) mass (b) energy (c) momentum (d) charge

6.3 A device which converts thermal energy into mechanical energy is called:

(a) heat engine (b) Carnot engine (c) refrigerator (d) turbine

6.4 When two objects are made in thermal contact having same temperature, then they are at:

(a) thermal Equilibrium (b) chemical Equilibrium

(c) mechanical Equilibrium (d) physical Equilibrium

6.5 When the system is expanded by adding heat energy, then the work done will be:
(a) positive and on the system (b) negative and on the system
(c) positive and by the system (d) negative and by the system

6.6 Entropy of a system in reversible process:
(a) decreases (b) increases (c) is infinite (d) is zero

6.7 What happens to internal energy of an object when its temperature:
(a) decreases (b) remains Constant (c) increases (d) fluctuates

6.8 The value of Boltzmann constant is:
(a) 1.38×10^{-23} J K^{-1} (b) 1.38×10^{23} J K^{-1}
(c) 1.38×10^{23} J K^{-1} (d) 1.38×10^{-23} J^{-1} K^{-1}

6.9 In an adiabatic process, there is no:
(a) change in temperature (b) exchange of heat
(c) change in internal energy (d) work done

6.10 Thermodynamics mostly deals with:
(a) measurement of quantity of heat
(b) transfer of quantity of heat
(c) change of state
(d) Conversion of heat to other forms of energy

Short Answer Questions

6.1 What is meant by thermal equilibrium? Explain briefly.

6.2 What is meant by internal energy? How is it related to temperature of an ideal gas?

6.3 State 2nd law of thermodynamics in two different forms.

6.4 Is it possible to construct a heat engine of 100% efficiency? Explain.

6.5 Differentiate between reversible and irreversible processes.

6.6 Why adiabat is steeper than isotherm? Explain.

6.7 A refrigerator transforms heat from cold to hot body. Does this violate the second law of thermodynamics? Justify your answer.

6.8 Explain briefly heat death of universe in terms of entropy.

6.9 Is it possible for a cyclic reversible heat engine to absorb heat at constant temperature and transforms it completely into work without rejecting some heat at low temperature? Explain.

6.10 How does behaviour of real gases differ from ideal gas at high pressure and low temperature? Identify the reasons behind these differences based on kinetic theory of gases.

6.11 Show that area under *P-V* graph is equal to work done.

6.12 How is work done (i) by a gas (ii) on a gas?

Constructed Response Questions

6.1 Explain how thermodynamics relates to the concept of energy conservation.

6.2 Explain how thermodynamics applies to biological systems, such as human body.

6.3 A gas is expanding adiabatically. Explain what happens to temperature and pressure of the the gas.

6.4 A coffee cup is left on a table, and overtime coffee cup cools down. Explain thermodynamics processes occurring during this process.

6.5 How can we explain different weather patterns through thermodymical processes like wind, rain, etc.

Comprehensive Questions

6.1 What are the postulates of kinetic theory of gases? Derive a relation for ideal gas equation in the form $PV=Nk_BT$ from general gas equation.

6.2 State and explain various gas laws.

6.3 Explain first law of thermodynamics in detail. Give an example in support of your explanation. Give its two applications.

6.4 What is a refrigerator? Explain its working. Derive an expression for its co-efficient of performance.

6.5. What is Carnot engine? Describe Carnot cycle. State Carnot theorem and derive an expression for efficiency of Carnot engine.

6.6 Define and explain the term "Entropy".

Numerical Problems

6.1 A gas occupies 6.0 L of volume at a pressure of 12 atm. What will be the volume of gas if the pressure is increased by 2.0 atm, assuming that temperature remains constant? (Ans: 5.14 L)

6.2 In a vaccum chamber which is connected to a cryogenic pump, pressure as low as 1.00 nPa is being attained. Calculate the number of molecules in 1.00 m^3 vessel at this pressure and temperature of 300 K. (Ans: 2.41×10^{11} molecules)

6.3 A gas undergoes a thermodynamic process where it absorbs 500 J of heat energy and performs 300 J work on its surroundings. Calculate the change in internal energy of the gas. (Ans: 200 J)

6.4 A Carnot engine is operating between a high temperature reservoir at 600 K and a low temperature reservoir at 300 K. Calculate:

(i) the maximum possible efficiency

(ii) the amount of work output if the engine absorbs 500 J of heat from the high temperature reservoir. [Ans: (i) 50% (ii) 250 J]

6.5 A refrigerator extracts 1200 J of heat from its interior (the cold reservoir) and releases 1800 J of heat to the surrounding environment (the hot reservoir) during each cycle. Calculate:

(i) the work input required per cycle.

(ii) the coefficient of performance (COP) of the refrigerator.

[Ans: (i) 600 J (ii) 2]

6.6 Calculate the entropy change when 1.0 mole of ice at 0°C melts to form liquid water at the same temperature. (Heat of fusion of ice per mole = 6.01×10^3 J) (Ans: 22.0 J K^{-1})

6.7 A gas occupies 400 mL at 20 °C. What volume will it occupy at 80 °C, assuming constant pressure? (Ans: 482 mL)

6.8 A gas has a pressure of 2 atm at 300 K. What pressure will it have at 450 K, assuming constant volume? (Ans: 3 atm)

Chapter 7

Waves and Vibrations

Learning Objectives

After studying this chapter, the students will be able to:

- Use the principle of superposition of waves to solve problems.
- Differentiate between constructive and destructive interference.
- Apply the principle of superposition to explain the working of noise canceling headphones.
- Illustrate experiments that demonstrate stationary waves [using microwaves, stretched strings and air columns (it will be assumed that end corrections are negligible; knowledge of the concept of end corrections is not required)]
- Explain the formation of a stationary wave using graphical representation.
- Explain the formation of harmonics in stationary waves.
- Describe an experiment that demonstrates diffraction [including the qualitative effect of the gap width relative to the wavelength of the wave; for example diffraction of water waves in a ripple tank]
- Explain beats [as the pulsation caused by two waves of slightly different frequencies interfering with each other]
- Illustrate examples of how beats are generated in musical instruments.
- Use intensity= power/area to solve problems. Use intensity ∝ (amplitude)^2 for a progressive wave to solve problems.
- Explain that when a source of sound waves moves relative to a stationary observer, the observed frequency is different from the source frequency [describing the Doppler effect for a stationary source and a moving observer is not required].
- Use the expression $f_o = \frac{f_s \cdot v}{v \pm u_s}$ for the observed frequency when a source of sound waves moves relative to a stationary observer.
- Explain the applications of the Doppler effect [such as radar, sonar, astronomy, satellite, radar speed traps and studying cardiac problems in humans].

We are well familiar with various types of waves such as water waves in the ocean and circular ripples formed on a still pool of water by rain drops. When a musician plucks a guitar string, sound waves are generated in air which reach our ears and produces sensation of music. The vast energy of the Sun, millions of kilometres away, is transferred to the Earth by light waves. In this chapter, we will discuss, formation, propagation and applications of different types of waves.

7.1 WAVES

A wave is a regular disturbance or variation that carries energy, which spreads out from the source. For example, energy is transferred from the Sun to the Earth in the form of light waves called electromagnetic waves. These waves can even travel through vacuum. However, in a medium, energy is transferred due to the regular and repeated disturbances that travel through the medium, making its particles to move up and down or back and forth (to and fro). Imagine a stone thrown into a pond of water (Fig. 7.1).

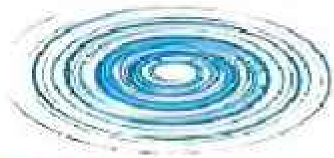


Fig. 7.1: Representing a travelling disturbance (ripple)

- The stone produces a disturbance (ripple) that travels through the water (medium).
- The water particles move up and down about their mean positions, creating a repeating pattern known as wave that spreads out.

The displacement of a particle of a wave is its distance in a specified direction from its rest / equilibrium position. If the displacement is plotted along the *y*-axis and the time in the direction of energy travel along the *x*-axis, we get a waveform as shown in Fig. 7.2.

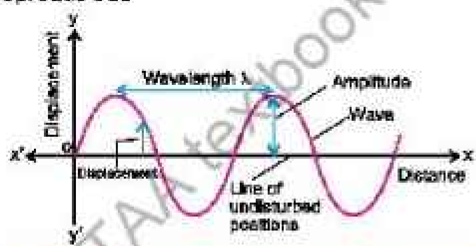
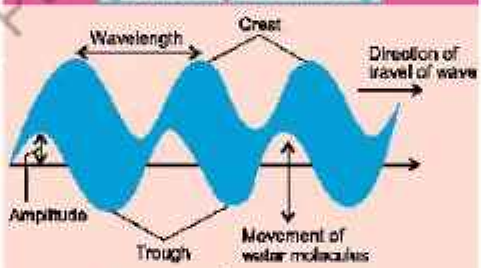


Fig. 7.2: Graphical description of a wave

The waves can be described by the following parameters:

Interesting Information



Wavy shape of water wave in 3D

1. **Amplitude (A):** The maximum displacement of the wave (or particles of the medium) from its equilibrium position.
3. **Frequency (*f*):** The number of oscillations or vibrations or cycles per second.
3. **Wavelength (λ):** The distance between two consecutive similar points on the wave that are in phase.
4. **Period (*T*):** The time taken by the wave to complete one oscillation or cycle. It is the reciprocal of the frequency $T = 1/f$.
5. **Speed (*v*):** The speed at which the wave travels. If a wave crest moves one wavelength λ in one period of oscillation T, the speed v is given by $v = \lambda / T$

   as $1/T = f$, so $$v = f\lambda \quad \text{.................} \quad (7.1)$$
6. **Phase (θ):** The relative position of a point on the wave at a given time.

Types of Waves

Waves have various forms, each with unique characteristics. A brief detail of different types of waves is given below:

1. Mechanical Waves

These waves require a physical medium (solid, liquid, or gas) to propagate. Examples are water waves (ocean, lake, or pond ripples), sound waves (audible vibrations in air, water, or solids), seismic waves (earthquakes), etc.

2. Electromagnetic Waves

They do not require a medium to propagate and therefore, can travel through vacuum. Examples are radiowaves (wireless communication), Microwaves (cooking and heating), Infrared waves (IR or heat radiation), Visible light (sunlight, lamp light), Ultraviolet waves (UV radiation), X-rays (medical imaging), Gamma rays (high-energy radiation), etc.

3. Quantum Waves

Quantum waves are associated with particles like electrons and photons. Examples are matter waves/particle waves (electron waves in atoms) or de-Broglie waves, photon waves (light quanta), etc.

Do you know?

These wave types are essential to understand various phenomena in physics, engineering, and everyday life.

4. Surface Waves

Surface waves propagate along surfaces or interfaces between two mediums. Examples are ocean surface waves (wind-driven waves), seismic surface waves, etc.

Transverse and Longitudinal Waves

There are two main types of waves which are named as **transverse waves** and **longitudinal waves**. A transverse wave is one in which the vibrations of the particles are at right angles to the direction in which the energy of the wave is travelling whereas a longitudinal wave is one in which the direction of the vibration of the particles is along or parallel to the the direction in which the energy of the wave is travelling.

The transverse wave and longitudinal wave are illustrated in Figs. 7.3 (a and b), respectively.

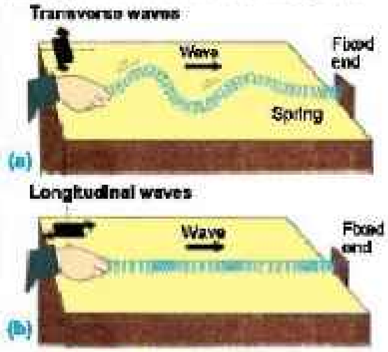


Fig. 7.3: Main types of waves

7.2 PRINCIPLE OF SUPERPOSITION OF WAVES

If a particle of the medium is simultaneously acted upon by two waves, then the resultant displacement of the particle is the algebraic sum of their individual displacements. This is called principle of superposition.

In other words, the displacements of the individual waves are added together to form a new wave pattern as shown in Fig.7.4 (a and b), respectively.

(i) If two waves, which overlap each other, have same phase, their resultant displacement will be:

$$y = y_1 + y_2$$

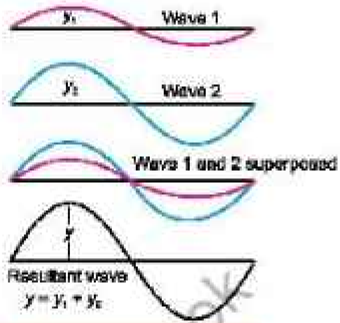
where y_1 = amplitude of wave 1

y_2 = amplitude of wave 2

and y = resultant amplitude

Particularly, if $y_1 = y_2$, then resultant displacement will be:

$$y = 2y_1 \text{ or } y = 2y_2$$



7.4 (a): Superposition of two waves of the same frequency which are exactly in phase.

(ii) If two waves, which cross each other, have opposite phase, their resultant displacement will be:

$$y = y_1 + (-y_2)$$

$$y = y_1 - y_2$$

Particularly, if $y_1 = y_2$, then resultant displacement will be $y = 0$.

Thus, if a particle of a medium is simultaneously acted upon by n waves such that its displacement due to each of the individual n waves be $y_1, y_2, \ldots\ldots, y_n$ then the resultant displacement y of the particle, under the simultaneous action of these n waves is the algebraic sum of all the displacements, i.e.,

$$y = y_1 + y_2 + \ldots\ldots\ldots\ldots + y_n$$

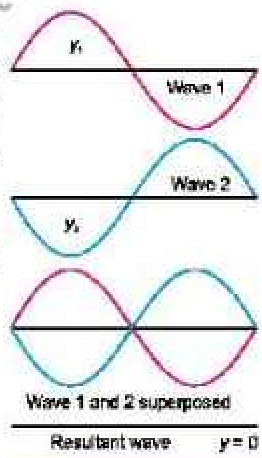
This is called principle of superposition of waves. Mathematically, this can also be represented as:

$$y(x,t) = y_1(x,t) + y_2(x,t) + \ldots\ldots\ldots + y_n(x,t)$$

where $y(x,t)$ is the resultant wave, whereas $y_1(x,t)$, $y_2(x,t), \ldots\ldots\ldots, y_n(x,t)$ are the individual waves.

In the context of waves, $y(x,t)$ represents the wave function or wave displacement at a given point x and time t. It describes the shape of the wave and its evolution over time.

The principle of superposition applies to linear waves or small amplitude waves.



7.4 (b): Superposition of two waves of the same frequency which are exactly out of phase.

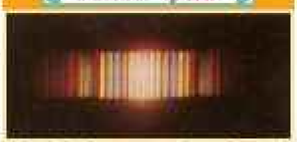
Principle of superposition of waves leads to many interesting phenomena:

(i) Two waves having same frequency and travelling in the same direction (Interference).

(ii) Two waves of slightly different frequencies and travelling in the same direction (Beats).

(iii) Two waves of equal frequency travelling in opposite direction (Stationary waves).

Ponder upon!



An interference pattern formed with white light.

Applications of the Principle of Superposition

By applying the principle of superposition of waves, noise-cancelling headphones effectively eliminate unwanted noise, providing a more immersive and peaceful listening experience.

1. The headphones contain one or more microphones that capture ambient noise (like background chatter or engine rumble or any environmental noise).
2. The microphone sends the sound signals to an amplifier and a processing unit in the headphones.
3. The processing unit generates an "anti-noise" signal, which is the exact opposite of the ambient noise (in terms of amplitude and phase).
4. The anti-noise signal is then played through the headphones, along with the desired audio (like music or voice).
5. When the anti-noise signal meets the ambient noise, the two waves cancel each other out resulting in a much quieter listening experience.

Though the above example is an oversimplification of the situation, as noise-cancelling headphones use complex algorithms and multiple microphones to achieve optimal noise cancellation, the basic principle of superposition remains a fundamental concept in understanding how they work.

7.3 INTERFERENCE AND ITS TYPES

Superposition of two waves having the same frequency and travelling in the same direction results in phenomenon called interference.

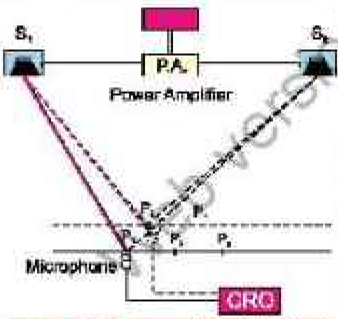


Fig. 7.5(a): An experimental setup to observe interference of sound waves.

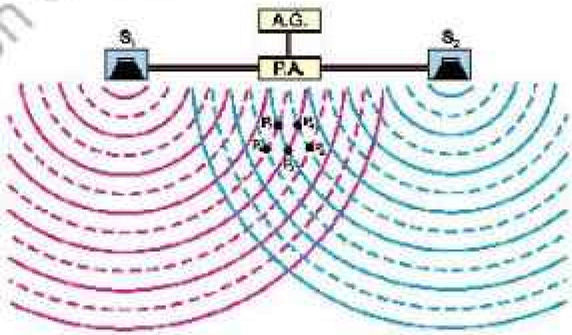


Fig. 7.5 (b): Interference of sound waves

An experimental setup to observe interference effect of sound waves is shown in Fig. 7.5(a). Two loud speakers S_1 and S_2 act as two sources of harmonic sound waves of a fixed frequency produced by an Audio Generator (AG). Since the two speakers are driven from the same generator, therefore, they vibrate in phase. Such sources of waves are called coherent sources. A microphone attached to a sensitive Cathode Ray Oscilloscope (CRO) acts as a detector of sound waves. The CRO is a device to display the input signal into waveform on its screen. The microphone is placed at various points, turn by turn, in front of the loud speakers as shown in Fig.7.5(b).

Constructive Interference

At points P_1, P_3 and P_5, we find that a compression meets a compression and a rarefaction meets a rarefaction. So, the displacement of two waves are added up at these points according to the principle of superposition and a large resultant displacement is seen on the CRO screen (Fig.7.5-c).

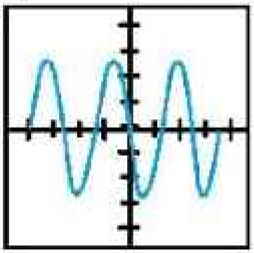


Fig. 7.5(c):
Constructive Interference: Large displacement is displayed on the CRO screen

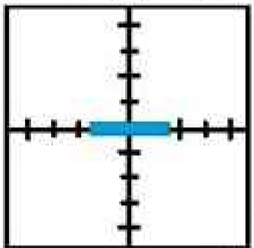
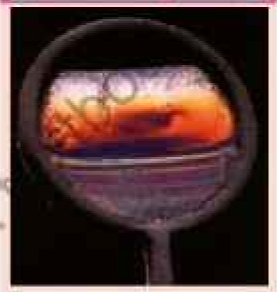


Fig. 7.5(d):
Destructive Interference: Zero displacement is displayed on the CRO screen

Interesting fact!



Interference pattern produced by a thin soap film illuminated by white light.

From Fig. 7.5 (b), we find that the path difference ΔS between the waves at the point P_1, is:

$$\Delta S = S_2P_1 - S_1P_1$$

$$\Delta S = 4\frac{1}{2}\lambda - 3\frac{1}{2}\lambda = \lambda$$

Similarly, at points P_3 and P_5, path difference is zero and λ, respectively. Here, λ is the wavelength which is the distance between any two successive solid or dashed lines.

> Whenever the path difference is an integral multiple of wavelength, the two waves are added up. This effect is called constructive interference.

Therefore, the path difference (condition) for constructive interference can be written as:

$$\Delta S = n\lambda \qquad \text{where} \quad n = 0, \pm1, \pm2, \pm3, \ldots\ldots$$

Destructive Interference

At points P_2 and P_4, a compression meets a rarefaction, so that they cancel each other's effect according to the principle of superposition. The resultant displacement becomes zero, as shown in Fig. 7.5(d).

The path difference ΔS between the waves at points P_2 and P_4 is:

$$\Delta S = S_2P_2 - S_1P_2$$

$$\Delta S = 4\lambda - 3\frac{1}{2}\lambda = \frac{1}{2}\lambda$$

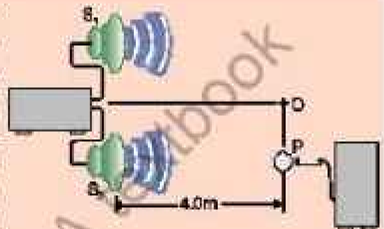
Similarly, at P_4 the path difference is $\frac{1}{2}\lambda$.

> So, at points where the displacements of two waves cancel each other's effect, the path difference is an odd integral multiple of half the wavelength. This effect is called destructive interference.

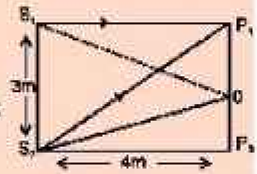
Therefore, the path difference (condition) for destructive interference can be written as:

$$\Delta S = (2n + 1)\,\lambda/2 \qquad \text{where } n = 0, \pm1, \pm2, \pm3, \ldots\ldots\ldots$$

Example 7.1 Two speakers are arranged as shown in the figure. The distance between them is 3.0 m and they emit a constant tone of 344 Hz. A microphone P is moved along a line parallel to and 4.0 m from the line connecting the two speakers. It is found that tone of maximum loudness is heard and displayed on the CRO when microphone is on the centre of the line and directly opposite to each speaker. Calculate the speed of sound.



Solution



Distance between speakers S_1S_2 = 3.0 m

Tone frequency f = 344 Hz

Distance between speakers and line of motion of P S_2P = 4.0 m

Speed of sound v = ?

For tone of maximum loudness or the condition for constructive interference, the path difference must be 0, $\pm1\lambda$, $\pm2\lambda$, $\pm3\lambda$,...

At middle point 'O' the path difference between two sound waves is zero ($S_1O = S_2O$), thus at that point 'O' constructive interference takes place.

For the next point P of constructive interference, the path difference between waves should be λ. So, λ = Path difference = $S_2P_1 - S_1P_1$

Now, we calculate values of S_2P_1 from right angle triangle $S_1S_2P_1$

$$S_2P_1 = \sqrt{(S_1S_2)^2 + (S_1P_1)^2} \qquad \text{(By Pythagoras Theorem)}$$

$$S_2P_1 = \sqrt{(3\text{ m})^2 + (4\text{ m})^2} = \sqrt{(9+16)}\text{ m} = \sqrt{25}\text{ m} = 5\text{ m}$$

Therefore $\lambda = S_2P_1 - S_1P_1$ or $\lambda = 5\text{ m} - 4\text{ m} = 1\text{ m}$

This is the path difference for constructive interference.

As $v = f\lambda$

Putting the values, we have $v = 344\text{ m s}^{-1} \times 1\text{ m} = 344\text{ m s}^{-1}$

Example 7.2 The wavelength of a signal from a radio transmitter is 1500 m and the frequency is 200 kHz. What is the wavelength for a transmitter operating at 1000 kHz and with what speed the radiowaves travel?

Solution

$\lambda_1 = 1500\ m = 1.5 \times 10^3\ m$, $f_1 = 200\ kHz = 2.0 \times 10^5\ Hz$

$f_2 = 1000\ kHz = 1 \times 10^6\ Hz$, $\lambda_2 = ?$, $v = ?$, As $v = f\lambda$.

Since, the speed of both the signals is same, so

$$v = f_1\lambda_1$$

$$v = 2.0 \times 10^5\ Hz \times 1500\ m$$

$$v = 3.0 \times 10^8\ m\ s^{-1}$$

Also

$$v = f_2\lambda_2$$

$$\lambda_2 = \frac{v}{f_2}$$

$$\lambda_2 = \frac{3 \times 10^8\ m\ s^{-1}}{1 \times 10^6\ Hz}$$

$$\lambda_2 = 3 \times 10^2\ m$$

For your information

Monochromatic Light
Sodium chloride in a flame gives out pure yellow light. This light is not a mixture of red and green.

7.4 STATIONARY WAVES & THEIR FORMATION

Stationary waves, also known as standing waves, are the waves that oscillate in a fixed position, without moving or propagating. They are formed by the superposition of two waves with the same frequency and amplitude, travelling in opposite directions. The resulting wave pattern remains stationary, with nodes (points of zero amplitude) and antinodes (points of maximum amplitude) at fixed positions. Examples include waves on a string, and sound waves in a pipe. The term "standing wave" describes that the wave pattern remains fixed in space, oscillating between positive and negative values, without moving forward or backward.

Let us consider the superposition of two waves moving along a string in opposite directions. Figures 7.6 (a) and (b) show the profile of two such waves at instants $t = 0, T/4, T/2, 3/4\ T$ and T, where T is the time period of the wave. We are interested in finding out the displacements of the points 1, 2, 3, 4, 5, 6 and 7 at these instants as the waves superpose. It is obvious that the points 1,2,...,7 are distant $\lambda/4$ apart, λ being the wavelength of the waves. We can determine the resultant displacement of these points by applying the principle of superposition.

Figure 7.6 (c) shows the resultant displacement of the points 1,3,5 and 7 at the instants $t = 0, T/4, T/2, 3T/4$ and T. It can be seen that the resultant displacement of these points is always zero. These points of the medium are known as nodes. Here, the distance between two consecutive nodes is $\lambda/2$.

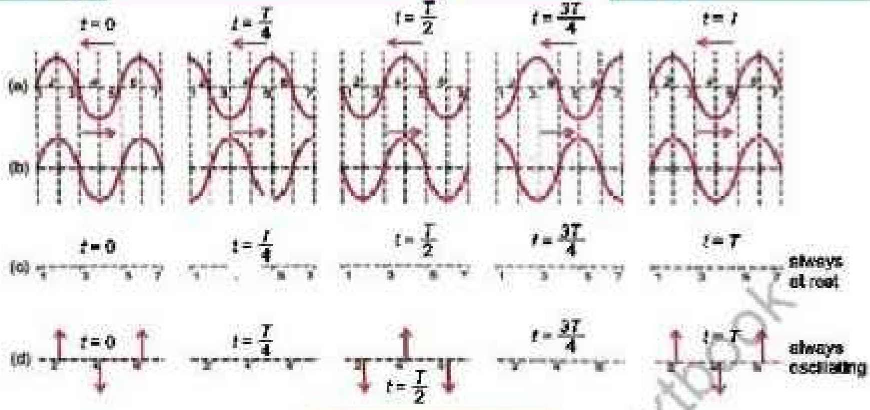


Fig. 7.6: Stationary waves

Figure 7.6 (d) shows the resultant displacement of the points 2,4 and 6 at the instants t = 0, $T/4$, $T/2$, $3T/4$ and T. The figure shows that these points are moving with an amplitude which is the sum of the amplitudes of the component waves represented by arrows. These points are known as antinodes. They are situated midway between the nodes and are $\lambda/2$ apart. The distance between a node and the next antinode is $\lambda/4$. Such a pattern of nodes and antinodes is known as a stationary or standing wave.

Energy in a wave transfers because of the motion of the particles of the medium. The nodes always remain at rest, so energy cannot flow past these points. Hence, energy remains "standing" in the medium between nodes, although it alternates between potential and kinetic forms at the antinodes. When the antinodes are all at their extreme displacements, the energy stored is wholly potential and when they are simultaneously passing through their equilibrium positions, the energy is wholly kinetic.

7.5 STATIONARY WAVES ON A STRETCHED STRING

Consider a string of length ℓ which is kept stretched by clamping its ends so that the tension in the string is F.

(a) String Plucked at its Middle Point

If the string is plucked at its middle point, two transverse waves will originate from this point. One of them will move towards the left end of the string and the other towards the right end. When these waves reach the two clamped ends, they are reflected back thus giving rise to stationary waves. As the two ends of the string are clamped, no motion will take place there. So, nodes will be formed at the two ends and one mode of vibration of the string will be as shown in Fig.7.7(a) with the two ends as nodes with one antinode in between. Visually, the string seems to vibrate in one loop. As the distance between two

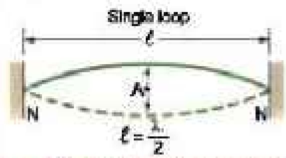


Fig. 7.7(a): First mode of vibration

consecutive nodes is one half of the wavelength of the waves set up in the string, so in this mode of vibration, the length ℓ of the string is

$$\ell = \lambda_1/2$$

$$\lambda_1 = 2\ell \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (7.2)$$

where λ_1 is the wavelength of the waves set up in this mode. The speed v of the waves in the string depends upon the tension F of the string and m, the mass per unit length of the string. It is independent of the point from where string is plucked to generate wave. It is given by

$$v = \sqrt{\frac{F}{m}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7.3)$$

Knowing the speed v and wavelength λ_1, the frequency f_1 of the waves is:

$$f_1 = \frac{v}{\lambda_1} = \frac{v}{2\ell}$$

Substituting the value of v,

$$f_1 = \frac{1}{2\ell}\sqrt{\frac{F}{m}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (7.4)$$

Thus, in the first mode of vibration shown in Fig.7.7 (a), waves of frequency f_1 only will be set up in the given string.

(b) String Plucked at Quarter Length

If the same string is plucked from one quarter of its length, again stationary waves will be set up with nodes and antinodes as shown in Fig. 7.7 (b). Note that now the string vibrates in two loops. This particular configuration of nodes and antinodes has developed because the string was plucked from the position of an antinode. As the distance between two consecutive nodes is half the wavelength, so the length ℓ of string is equal to the wavelength of the waves set up in this mode. If λ_2 is the measure of wavelength of these waves, then,

$$\ell = \frac{\lambda_2}{2} + \frac{\lambda_2}{2}$$

$$\ell = 2\frac{\lambda_2}{2}$$

$$\lambda_2 = \ell \quad \ldots\ldots\ldots\ldots\ldots\ldots (7.5)$$

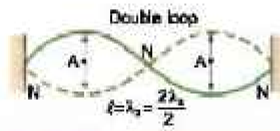


Fig. 7.7(b): Second mode of vibration

Comparison of Eq. (7.5) with Eq. (7.1) shows that the wavelength in this case is half of that in the first case. Equation (7.2) shows that the speed of the waves depends on tension and mass per unit length of the string, it is independent of the point where the string is plucked. So, speed v remains the same in both cases.

If f_2 is frequency of vibration of string in its second mode, then

$$f_2 = \frac{v}{\lambda_2}$$

Since $\lambda_2 = \ell$

Therefore $f_2 = \frac{v}{\ell}$

Multiplying and dividing by 2, we have

$$f_2 = \frac{2v}{2\ell}$$

We know that; $f_1 = \frac{v}{2\ell}$

So $f_2 = 2f_1$

Brain teaser

A guitar string is plucked at its centre. What harmonic is produced?

Thus, when the string vibrates in two loops, its frequency becomes double than when it vibrates in one loop.

(c) String Plucked at an Arbitrary Point

Let the string resonates in n number of loops with $(n + 1)$ nodes and n antinodes. Thus, we can say that if the string is made to vibrate in n loops, the frequency of stationary waves set up on the string will be:

$$f_n = n\left(\frac{v}{2\ell}\right)$$

$$f_n = nf_1 \quad \text{..............................(7.6)}$$

and the corresponding wavelength is, $\lambda_n = \frac{2\ell}{n}$ ....(7.7)

where $n = 1,2,3,\ldots$

It is clear that as the string vibrates in more than one loop, its frequency f goes on increasing and the wavelength λ gets correspondingly shorter. However, the product of the frequency f and wavelength λ is always equal to v, the speed of waves.

The above discussion clearly establishes that;

1. The stationary waves have a discrete set of frequencies f_1, $2f_1$, $3f_1$, ...,nf_1 which is known as harmonic series. The lowest characteristic frequency of vibration is the fundamental frequency f_1 corresponds to the first harmonic. The frequency $f_2 = 2f_1$ corresponds to the second harmonic and so on.
2. In other words, quantum jumps in frequency exist between the resonance frequencies. This phenomenon is known as the Quantization of frequencies. It means $f_n = nf_1$ where $n = 1, 2, 3, \ldots$(Integral multiples). The stationary waves can be set up on the string only with the frequencies of harmonic series determined by the tension, length and mass per unit length of the string. Waves which are not in harmonic series are quickly damped out.
3. The frequency of a string on a musical instrument can be changed either by varying the tension or by changing the length. For example, the tension in guitar and violin

strings is varied by tightening the pegs on the neck of the instrument. Once the instrument is tuned, the musicians vary the frequency by moving their fingers along the neck, thereby changing the length of the vibrating portion of the string.

Harmonics

In the above example, the set of all the possible standing waves, having frequencies f_1, $2f_1$, $3f_1$, ..., nf_1, are called harmonics of the system. The lowest or fundamental frequency of all the harmonics is called the fundamental or first harmonic. Subsequent frequencies are called as second harmonic, third harmonic, etc.

Example 7.3 A stationary wave is established in a string which is 120 cm long and fixed at both ends. The string vibrates in four segments; at a frequency of 120 Hz. Determine its wavelength and the fundamental frequency?

Solution $\ell = 120\text{ cm} = \frac{120}{100}\text{ m} = 1.2\text{ m}$

$n = 4$

$f_4 = 120\text{ Hz}$

$f_1 = ?$

$\lambda = ?$

(i) As the string vibrates in four segments and the distance between two consecutive nodes is $\lambda/2$, so the wavelength of the string is:

$$\ell = n\frac{\lambda_n}{2}$$

$$\lambda_n = \frac{2\ell}{n}$$

$$\lambda_4 = \frac{2 \times 1.2\text{ m}}{4}$$

$$\lambda_4 = 0.6\text{ m}$$

(ii) If string vibrates in n loops, then frequency of stationary waves will be:

$$f_n = nf_1$$

$$f_4 = 4f_1$$

$$120\text{ Hz} = 4f_1$$

$$f_1 = \frac{120\text{ Hz}}{4}$$

$$f = 30\text{ Hz}$$

7.6 STATIONARY WAVES IN AIR COLUMNS

Stationary waves can be set up in other media also, such as air column inside a pipe or tube. A common example of vibrating air column is in the organ pipe.

Organ pipe

An organ pipe is a wind instrument in which sound is produced, due to setting up of stationary waves in air column. It consists of a hollow long tube with both ends open or with one end open and the other closed. The relationship between the incident wave and the reflected wave depends upon whether the reflecting end of the pipe is open or closed.

(i) If the reflecting end is open, the air molecules have complete freedom of motion and this behaves as an antinode.

(ii) If the reflecting end is closed, then it behaves as a node because the movement of the molecules is restricted.

Modes of Vibrations

Stationary longitudinal waves occur in a pipe as discussed by the following two cases:

Case (1): Modes of vibrations in an organ pipe open at both ends

Let us consider an organ pipe of length ℓ which is open at both ends. As at the open end, an air molecule has complete freedom of motion so it acts as antinode as shown in Fig.7.8. In this figure, longitudinal waves set up inside the pipe have been represented by transverse curved lines which represent the displacement and amplitude of vibration of air particles at various points along the axis of pipe.

(a) Fundamental mode of vibration

In this case, as shown in Fig.7.8 (a), there is only one node N at the middle of the pipe. As both ends of pipe are open, there are two antinodes at both the ends. If λ_1 is the wavelength of sound, then

$$\ell = \frac{\lambda_1}{4} + \frac{\lambda_1}{4}$$

$$\ell = \frac{\lambda_1}{2}$$

or $\lambda_1 = 2\ell$ ....(7.8)

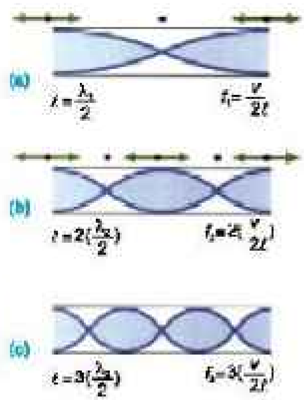


Fig. 7.8: Stationary longitudinal waves in a pipe open at both ends.

If f_1 is the frequency of sound, then the velocity of sound is:

$$v = f_1 \lambda_1$$

$$f_1 = v / \lambda_1$$

Putting the value of λ_1, we have

$f_1 = v / 2\ell$ ....(7.9)

This frequency is called fundamental frequency or first harmonic.

(b) Second mode of vibration

If there are three antinodes and two nodes, frequency will be twice that of fundamental frequency. It is second mode of vibration as shown in Fig.7.8 (b). In this case, there are three antinodes and two nodes.

If λ_2 is the wavelength of sound, then

$$\ell = \frac{\lambda_2}{4} + \frac{\lambda_2}{2} + \frac{\lambda_2}{4}$$

$$\ell = (1+2+1)\ \frac{\lambda_2}{4}$$

or $$\lambda_2 = \ell$$

If f_2 is the frequency of sound, then speed v of sound becomes:

$$v = f_2 \lambda_2$$

or $$f_2 = \frac{v}{\lambda_2}$$

Putting the value of λ_2, we have

$$f_2 = \frac{v}{\ell}$$

or $$f_2 = 2f_1 \qquad (\because \frac{v}{2\ell} = f_1)$$

Brain teaser

A stationary wave is formed on a string with a frequency of 100 Hz. If the string is 2 m long, how many nodes and antinodes are formed?

(c) nth mode of vibration

Similarly, frequency for air column vibrating in n loops is:

$$f_n = n(v/2\ell)$$

$$f_n = nf_1$$

and wavelength is

$$\lambda_n = \frac{2\ell}{n} \quad .... (7.10)$$

where $n = 1,2,3,4,5, ...$

So, the longitudinal stationary waves have a discrete set of frequencies f_1, $2f_1$, $3f_1$,...,nf_1, which is known as harmonic series. The frequency f_1 is known as fundamental frequency and the others are called harmonics.

Case (2): Modes of vibration in an organ pipe closed at one end

Let us consider an organ pipe of length ℓ which is closed at one end. Then at the closed end, we get a node while at the open end, we get an antinode as shown in the Fig. 7.9.

(a) Fundamental mode of vibration:

Fundamental mode of vibration has one node and one antinode as shown in Fig.7.9 (a). If λ_1 is the wavelength of fundamental mode, then length of the pipe is:

$$\ell = \frac{\lambda_1}{4}$$

or $$\lambda_1 = 4\ell \quad ....(7.11)$$

So, the speed v becomes:

$$v = f_1\lambda_1$$

$$f_1 = \frac{v}{\lambda_1}$$

$$f_1 = \frac{v}{4\ell} \qquad (\because \lambda_1 = 4\ell)$$

The frequency f_1 is called fundamental frequency.

(b) Second Mode of Vibration:

Second mode of vibration contains two nodes and two anti-nodes as shown in Fig.7.9 (b).

If λ_3 is the wavelength, then length of the pipe is:

$$\ell = \frac{\lambda_3}{4} + \frac{\lambda_3}{2}$$

$$\ell = \frac{3}{4}\lambda_3$$

$$\lambda_3 = \frac{4\ell}{3} \quad \text{...............(7.12)}$$

$$f_3 = \frac{v}{\lambda_3}$$

Putting value of λ_3, we have $v = f_3\lambda_3$

$$f_3 = \frac{v}{4\ell/3}$$

or

$$f_3 = \frac{3v}{4\ell}$$

so

$$f_3 = 3f_1 \qquad [\because \frac{v}{4\ell} = f_1]$$

This is called second harmonic.

(c) nth mode of vibration

If air column vibration in n loops, then frequency f_n is:

$$f_n = n\left(\frac{v}{4\ell}\right)$$

$$f_n = nf_1 \qquad \text{where } n = 1, 3, 5, \ldots$$

and the wavelength λ_n is:

$$\lambda_n = \frac{4\ell}{n} \quad \text{..............................(7.13)}$$

By studying both cases, we conclude that the pipe which is open at both ends is richer in harmonics than that of closed at one end.

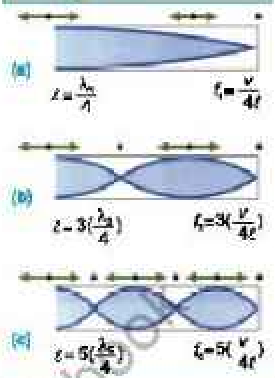
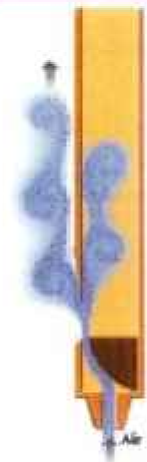


Fig. 7.9: Stationary longitudinal waves in a pipe closed at one end. Only odd harmonics are present.

Scientific Fact



In an organ pipe, the primary driving mechanism is wavering, sheet like jet of air from flute-slit, which interacts with the upper lip and the air column in the pipe to maintain a steady oscillation.

Example 7.4 An organ pipe has a length of 50 cm. Find the frequency of its fundamental note and the next harmonic, when it is:

(a) open at both ends

(b) closed at one end

(Speed of sound = 350 m s^{-1})

Ponder upon!

Open pipes produce all harmonics, while closed pipes produced only odd harmonics.

Solution $\ell = 50\,\text{cm} = \frac{50}{100}\,\text{m} = 0.5\,\text{m}$

$v = 350\,\text{m s}^{-1}$

a) When pipe is open at both ends:

Fundamental frequency $f_1 = ?$

Next harmonic frequency $f_2 = ?$

The frequency for *n*th harmonic in an open organ pipe is:

$$f_n = n\frac{v}{2\ell} \quad \text{when } n = 1,2,3,\ldots\ldots\ldots$$

so the fundamental frequency is,

$$f_1 = \frac{1 \times 350\,\text{m s}^{-1}}{2 \times 0.5\,\text{m}} \quad \text{put } n = 1$$

$$f_1 = 350\,Hz$$

Next harmonic frequency i.e., n = 2 is:

$$f_2 = \frac{2v}{2\ell}$$

$$f_2 = \frac{v}{\ell} = \frac{350\,\text{m s}^{-1}}{0.5\,\text{m}} = 700\,\text{s}^{-1}$$

$$f_2 = 700\,\text{Hz}$$

b) When pipe is closed at one end:

Fundamental frequency $f_1 = ?$

Next harmonic frequency $f_3 = ?$

When the pipe is closed at one end, then frequency for nth harmonic is

$$f_n = n\frac{v}{4\ell} \quad \text{when } n = 1,3,5,7,\ldots\ldots\ldots$$

So fundamental frequency is:

$$f_1 = \frac{1 \times 350\,\text{m s}^{-1}}{4 \times 0.5\,\text{m}} \quad \text{putting } n = 1$$

$$f_1 = 175\,\text{Hz}$$

Next harmonic frequency i.e., $n = 3$ is:

$$f_3 = \frac{3v}{4\ell}$$

$$f_3 = \frac{3\times 350 \text{ m s}^{-1}}{4\times 0.5 \text{ m}}$$

$$f_3 = 525 \text{ Hz}$$

Example 7.5 A church organ consists of pipes, each open at one end, of different lengths. The minimum length is 30 mm and the longest is 4 m. Calculate the frequency range of the fundamental notes. (Speed of sound = 340 m s^{-1})

Solution $$\ell_{min} = 30 \text{ mm} = \frac{30}{1000} \text{ m} = 30\times 10^{-3} \text{ m}$$

$$\ell_{max} = 4 \text{ m}$$

$$v = 340 \text{ m s}^{-1}$$

Frequency range = ?

For an organ pipe open at one end only:

$$f_n = \frac{nv}{4\ell}$$

(i) Minimum length

For fundamental frequency, put n =1

$$f_{1,max} = \frac{nv}{4\ell_{min}}$$

$$f_{1,max} = \frac{1\times 340 \text{ m s}^{-1}}{4\times 30\times 10^{-3} \text{ m}}$$

$$f_{1,max} = 2833.33 \text{ Hz}$$

(ii) Maximum length:

For fundamental frequency, put n = 1

$$f_{1,min} = \frac{nv}{4\ell_{max}}$$

$$f_{1,min} = \frac{1\times 340 \text{ m s}^{-1}}{4\times 4 \text{ m}}$$

$$f_{1,min} = 21.25 \text{ Hz}$$

So, the fundamental frequency range is approximately from 21 Hz to 2833 Hz.

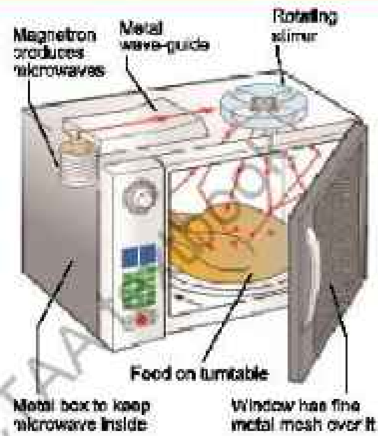
Brain teaser

A flute player notices that the flute is producing a pitch which is slightly sharp. What could be the cause of this problem?

7.7 EXPERIMENT DEMONSTRATING STATIONARY WAVES USING MICROWAVES

Microwaves are a form of electromagnetic radiations. They are called "micro" waves because their wavelengths are typically of the order of millimetres or centimetres, much shorter than radiowaves. Stationary waves, also known as standing waves, can be produced by microwaves when they are confined to a specific region or cavity such as wave guides or resonant chambers. In these structures, microwaves can bounce back and forth, creating a standing wave pattern with nodes and antinodes. It occurs when the microwave frequency matches the resonant frequency of the cavity.

Structure of Microwave Oven



The stationary waves can be created using microwaves by the following simple method as shown in Fig. 7.10.

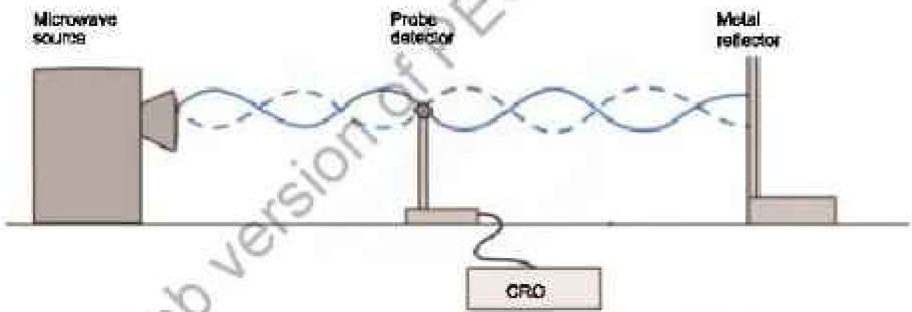


Fig. 7.10: Experimental setup for stationary waves using microwave

The experiment setup consists of a microwave source (transmitter), a probe detector and a metal reflector (a metallic plate for the reflection of microwave). Three of the mentioned are placed in line.

The waves coming out of microwave source are moving towards the metal plate and then reflected back. The reflected wave and incident wave superpose and create a stationary wave pattern. This can be detected by a probe detector placed between transmitter and metallic plate. The intensity of the signal can be observed by the detector. You can move the plate or detector to observe antinode and node. By finding the distance from one antinode to the next antinode, the wavelength of stationary wave can be found.

7.8 DIFFRACTION OF WAVES

Diffraction of waves is the bending of waves around the sharp edges or corners of obstacles or the spreading of waves beyond a barrier. It occurs when a wave encounters a physical barrier or an opening (a slit) that is comparable in size to the wavelength of the wave. The longer the wavelength, the greater the spreading and vice versa.

Diffraction can be observed in various types of waves, including water waves, sound waves, light waves and electromagnetic waves.

Some examples of the phenomenon of diffraction include:

- Hearing of sound waves around corners or through door way from where they were generated as sound waves bend around the corners.
- Diffraction of X-rays by crystals as the spacing between the regular arrays of atoms is of the order of X-rays wavelength.

Diffraction is a fundamental aspect of wave behaviour and has many practical applications in various fields.

The Ripple Tank

Figure 7.11 shows a ripple tank. It is a very useful apparatus not only to generate water waves, but also to demonstrate wave properties (such as reflection, diffraction and refraction).

Fig. 7.11: A ripple tank

Ripple tank contains water, vibrator (e.g. a motorized oscillating needle), obstacles (e.g. a small rectangular block or a semicircular barrier) and gap widths of different sizes. It creates a series of concentric circles

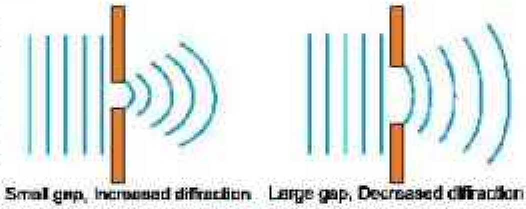


Fig. 7.12

or parallel waves using the vibrator. An obstacle is placed for creating a gap with a specific width. The experiment can be repeated with different gap widths.

It is observed that when the gap width is small compared to the wavelength, diffraction is significant and the waves bend around the obstacle, creating a semicircular pattern. As the gap width increases relative to wavelength, diffraction decreases and the waves pass through the gap with less bending.

For your information

Diffraction of white light is shown by a fine diffraction grating.

This experiment demonstrates the qualitative effect of gap width on diffraction in a ripple tank, illustrating how the relationship between gap width and wavelength affects wave behaviour.

Example 7.6

In a ripple tank, a wave generator produces 500 pulses in 10 s. Find the frequency at time period of the pulses produced?

Solution $n = 500$ pulses, $t = 10\,s$, $f = ?$

As Frequency $= \dfrac{\text{Number of pulses}}{\text{Time}}$

$$f = \frac{500}{10\,s} = 50\text{ Hz}$$

We know that: $T = \dfrac{1}{f} = \dfrac{1}{50\text{ Hz}}$

$$T = 0.02\,s$$

Interesting information

The fine rulings, each 0.5 μm wide, on a compact disc function as a diffraction grating. When a small source of white light illuminates a disc, the diffracted light forms coloured lanes that are composite of the diffraction patterns from the rulings.

7.9 BEATS

When two waves of slightly different frequencies, travelling in the same direction overlap each other then there is a periodic variation of sound between maximum and minimum loudness which is called as beats.

Tuning forks give out pure notes (single frequency). If two tuning forks A and B of the same frequency say 32 Hz are sounded separately, they will give out pure notes. If they are sounded simultaneously, it will be difficult to differentiate the notes of one tuning fork from that of the other. The sound waves of the two will be superposed on each other and will be heard by the human ear as a single pure note.

If the tuning fork B is loaded with some wax or plasticine, its frequency will be lowered slightly, say it becomes 30 Hz. If now the two tuning forks are sounded together, a note of alternately increasing and decreasing intensity will be heard. This note is called **beat note** or a beat which is due to interference between the sound waves from tuning forks A and B.

Fig. 7.13 (a) shows the waveform of the note emitted from a tuning fork A. Similarly, Fig. 7.13(b) shows the waveform of the note emitted by tuning fork B. When both the tuning forks A and B are sounded together, the resultant waveform is shown in Fig. 7.12(c). It shows how do the beat note occur. At some instant X, the displacement of the two waves is in the same direction. The resultant displacement is large and a loud sound is heard.

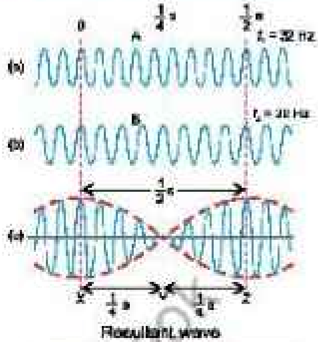


Fig. 7.13: Formation of beats

After 1/4 s the displacement of the wave due to one tuning fork is opposite to the displacement of the wave due to the other tuning fork resulting in minimum displacement at Y, hence, faint sound or no sound is heard.

Another 1/4 s later, the displacements are again in the same direction and a loud sound is heard again at Z.

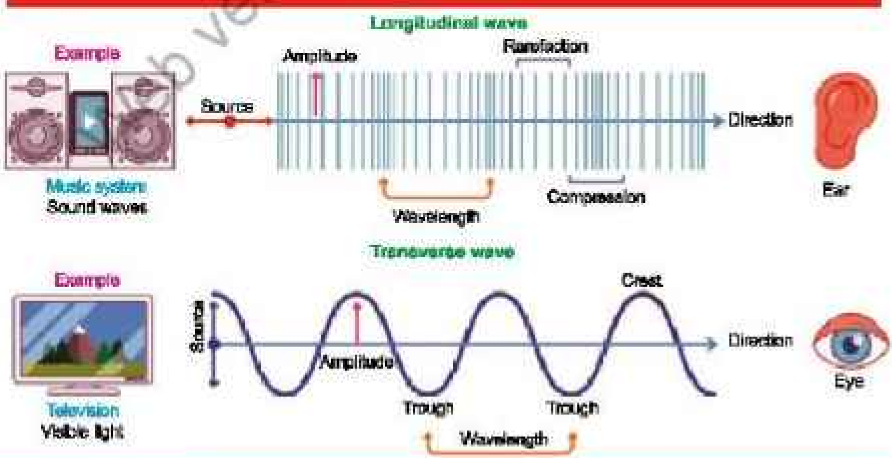
Thus, during one second, we observe two faintest sounds or two loudest. As the difference of the frequency of the two tuning forks is also 4 Hz so, we find that the number of beats per second is equal to the difference between the frequencies of the tuning forks.

$$f_A = 32\text{ Hz}, \quad f_B = 30\text{ Hz} \qquad f_A > f_B$$

$$\text{No. of beats} = f_A - f_B = 32\text{ Hz} - 30\text{ Hz} = 2$$

However, when the difference between the frequencies of the two sounds is more than 10 Hz, it becomes difficult to recognize the beats.

Pictorial Comparison



The difference between the frequencies of the two waves is termed as beat frequency f_{beat}.

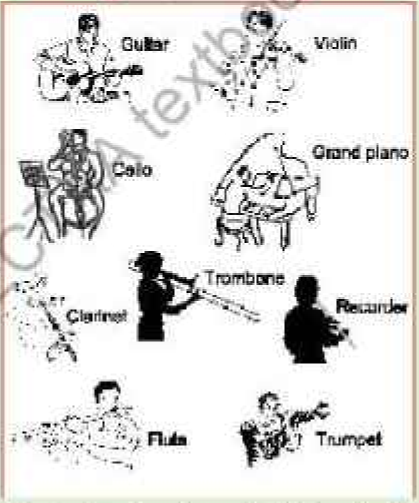
One can use beats to tune a string instrument, such as piano or violin, by beating a note against a note of known frequency. The string can then be adjusted to the desired frequency by tightening or loosening it until no beats are heard.

Tuning Musical Instruments

Here are some examples of how beats are generated in musical instruments:

1. **Guitar:** When playing two strings with slightly different tunings, beats are created. For example, playing a standard tuned string and a string tuned a few cents higher or lower.
2. **Piano:** Playing two keys white and black, adjacent to each other, creates beats.
3. **Violin:** When playing two strings with slightly different bow pressures or speeds, beats are generated.
4. **Drums:** When two drums with slightly different tunings are played simultaneously, beats are created.
5. **Flute:** When playing two notes with slightly different embouchure (lip and facial muscles) positions, beats are generated.
6. **Organ:** When playing two pipes with slightly different tunings, beats are created.
7. **Synthesizer:** Generating two oscillators with slightly different frequencies creates beats.

Do you know?



The sound produced by most of string and wind musical instruments is due to the formation of stationary waves or standing waves in these instruments.

In each of these examples, the slight difference in frequency between the two sound sources creates a periodic increase and decrease in amplitude, resulting in a "beat" or pulsation effect.

Musicians often use beats intentionally to create interesting rhythmic effects, add texture, or produce a sense of tension and release. However, in some cases, beats can be unwanted and may require adjustments to tuning, pitch, or playing technique to minimize their impact.

Example 7.7 Two tuning forks exhibit beats at a beat frequency of 3 Hz. The frequency of one fork is 256 Hz. Its frequency is then lowered slightly by adding a bit of wax to one of its prongs. The two forks then exhibit a beat frequency of 1 Hz. Determine the frequency of the second tuning fork.

Solution Frequency of first tuning fork $= f_1 = 256$ Hz

Beat frequency before loading = 3 Hz

Beat frequency after loading = 1 Hz

Frequency of second tuning fork $= f_2 = ?$

As $f_1 - f_2 = \pm n$

Then $f_2 = f_1 \pm n$

Putting the values, we have

$f_2 = 256\text{ Hz} \pm 3\text{ Hz}$

Either $f_2 = 256\text{ Hz} + 3\text{ Hz}$;

or $f_2 = 256\text{ Hz} - 3\text{ Hz}$

$f_2 = 259\text{ Hz}$ or 253 Hz

Let us consider 259 Hz as correct answer (i.e., frequency of second tuning fork). When first fork is loaded with wax, the frequency of first fork must fall below 256 Hz i.e., 255 Hz, 254 Hz and thus the number of beats produced per second will increase and will be greater than 3 beats. Since the number of beats per second decreases on loading first fork is one, therefore 259 Hz is not correct frequency of second tuning fork.

Thus, Correct frequency $= f_2 = 253$ Hz

Do you know?

In 1711, F. J. Shore, who was a royal trumpeter and lutenist invented tuning forks.

7.10 INTENSITY (I) OF A WAVE

Intensity is defined as the amount of energy transmitted per unit area per unit time in the direction of propagation of progressive wave.

It is a measure of the power of a wave and is usually denoted by the symbol "I". It is measured in units of watts per square metre ($W\,m^{-2}$).

A progressive wave or travelling wave is one that travels through a medium in a consistent direction and transferring energy from one point to another. It is a wave that propagates or moves forward, as opposed to a stationary or standing wave. Examples of progressive waves include water waves, sound waves, light waves, etc.

By definition, the intensity of a wave is:

$$I = \frac{E}{A \times t}$$

$$I = \frac{E/t}{A}$$

$$I = \frac{P}{A} \quad (\because E/t = P)$$

Here

I = Intensity of wave in (W m^{-2})

E = Energy in joules (J)

t = Time in seconds (s)

P = Power in watts (W)

We know that in mechanical waves, such as sound waves, water waves, or waves on a vibrating string, energy is stored as kinetic energy and potential energy of the medium's particles. How much energy is stored depends upon the displacement (amplitude) of the particles from the mean position. Therefore, the intensity I of waves is proportional to the square of the amplitude A, i.e.,

$$I \propto A^2$$

or $$I = kA^2 \quad \text{...... (7.14)}$$

Here k is the constant of proportionality and depends upon the physical properties of the wave and the medium.

Example 7.8

(a) A wave has an intensity of 0.5 W m^{-2} at a distance of 3.0 m from the source. What is the power of the wave?

(b) Two progressive waves have intensities of 0.5 W m^{-2} and 0.25 W m^{-2}. Find total intensities of two waves.

Solution

(a)
$$I = 0.5 \text{ W m}^{-2}$$
$$r = 3.0 \text{ m}$$
$$P = ?$$
$$\because I = \frac{P}{A}$$
$$\because I = \frac{P}{4\pi r^2}$$

Putting the values, $$0.5 \text{ W m}^{-2} = \frac{P}{4 \times 3.14(3.0 \text{ m})^2}$$

$$P = 0.51 \times 13.04$$
$$P = 56.52 \text{ W}$$

(b)
$$I_1 = 0.5 \text{ W m}^{-2}$$
$$I_2 = 0.25 \text{ W m}^{-2}$$

Tidbits

Frequency and amplitude of a travelling wave are independent of each other. That is why you can turn up the volume of a song (increase amplitude) without changing its pitch (which depends on frequency).

$I = ?$

$I = I_1 + I_2$

$I = 0.5\ \text{W m}^{-2} + 0.25\ \text{W m}^{-2}$

$I = 0.75\ \text{W m}^{-2}$

Brain teaser

Can you find the decibel level of a traveling wave whose intensity is 10 W m^{-2}?

Example 7.9 A speaker is emitting sound waves with a power of 50 watts. If the sound waves are spreading out evenly in all directions and the intensity of the sound waves is measured at a distance of 5 m from the speaker, what is the intensity of the sound waves if the area of the sphere (the surface area of a sphere) at that distance is approximately 314 m^2?

Solution Power $P = 50\ \text{W}$

Area $A = 314\ \text{m}^2$

$$\text{Intensity} = \frac{\text{Power}}{\text{Area}}$$

$$I = \frac{P}{A}$$

$$I = \frac{50\ \text{W}}{314\ \text{m}^2}$$

$$I \approx 0.159\ \text{W m}^{-2}$$

Tidbit

Sound cannot travel through vacuum, as there are no particles to transmit the sound wave.

7.11 DOPPLER EFFECT

The apparent change in the frequency (or pitch) of waves due to the relative motion between the source and observer (listener) is called Doppler Effect.

This effect was first observed by John Doppler while he was observing the frequency of light emitted from a star. In some cases, the frequency of emitted light was found to be slightly different from that emitted from a similar source on the Earth. He found that the change of frequency of light depends upon motion of star relative to Earth.

This effect can be observed with sound waves also. For example, when an observer is standing on a railway platform, the pitch of whistle of an engine coming towards the platform appears to become higher to an observer standing on the platform. However, the pitch of whistle of an engine going away from the platform appears to become lower to an observer standing on the platform.

Consider a source of sound S at rest which emits sound waves having wavelength λ. Let speed of the sound for a stationary observer (i.e., listener) is v then the number of waves received by observer in one second i.e., frequency f is:

$$f = \frac{v}{\lambda}$$

Case 1: When source of sound moves towards the stationary observer

When the source moves towards the stationary observer A with velocity u_s then waves are compressed and their wavelength is decreased as shown in Fig. 7.14. In this case, the waves are compressed by an amount given as

$$\Delta\lambda = \frac{u_s}{f}$$

The compression of the waves is due to the fact that same number of waves are contained in a shorter space depending upon the velocity of the source. The wavelength observed by the observer A is then,

$$\lambda_A = \lambda - \Delta\lambda$$

or

$$\lambda_A = \frac{v}{f} - \frac{u_s}{f}$$

$$\lambda_A = \frac{v - u_s}{f}$$

Here $\Delta\lambda$ is the decrease in wavelength in one second and is called Doppler shift.

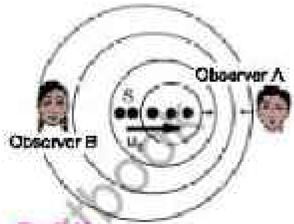


Fig. 7.14

A source moving with velocity u_s towards a stationary observer A and away from stationary observer B respectively. Observer A hears an increased and observer B hears a decreased frequency.

Thus, the number of waves received by observer A in one second (i.e., changed or apparent frequency) is

$$f_A = \frac{v}{\lambda_A}$$

Putting the value of λ_A, we have

$$f_A = \left[\frac{v}{(v - u_s)/f}\right]$$

$$f_A = \left[\frac{v}{v - u_s}\right] f$$

As

$$\frac{v}{v - u_s} > 1$$

Therefore

$$f_A > f$$

Thus, the apparent frequency of sound heard by the observer increases which in turn will increase the pitch of sound.

Case 2: When source of sound moves away from the stationary observer

When the source moves away from the stationary observer B with velocity u_s, then waves are expanded and their wavelength is increased. In this case, the waves expanded by an amount:

$$\lambda_s = \frac{u_s}{f}$$

The expansion of the waves is due to the fact that same number of waves are now contained in a large distance. The wavelength observed by the observed B is then,

$$\lambda_B = \lambda + \Delta\lambda$$

where $\Delta\lambda$ is the increase in wavelength in one second and is called Doppler shift.

Thus, the number of waves received by observer B in one second (i.e., changed or apparent frequency) is:

$$f_B = \frac{v}{\lambda_B}$$

Putting the value of λ_B, we have

$$f_B = \left[\frac{v}{(v+u_s)/f}\right]$$

$$f_B = \left[\frac{v}{(v+u_s)}\right]f$$

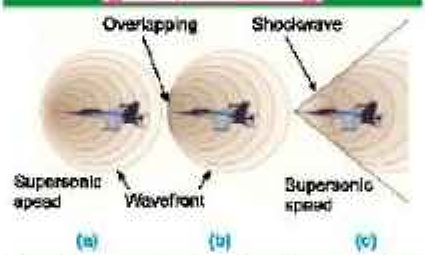
As $$\frac{v}{v+u_s} < 1$$

Therefore $$f_B < f$$

Thus, the apparent frequency of sound heard by the observer decreases which in turn will decrease the pitch of sound.

Info Corner	
Hearing Ranges	
Organisms	Frequencies in Hz
Dolphin	150 – 150,000
Bat	1000 – 120,000
Cat	60 – 70,000
Dog	15 – 50,000
Human	20 – 20,000

Do you know?



As a plane accelerates, it builds up a front of air pressure by pushing air in front of it. When it passes the speed of sound, the pressure trails behind like a boat's wake, forming a sonic shockwave.

Example 7.10 Two trucks P and Q travelling along a motorway in the same direction. The loading truck P travels at a steady speed of 12 m s^{-1}, the other truck Q, travelling at a steady speed of 20 m s^{-1}, sound its horn to emit a steady note which P's driver estimate, has a frequency of 830 Hz. What frequency does Q's own driver hear?

(Speed of sound = 340 m s^{-1})

Solution

u_P = 12 m s^{-1}

u_Q = 20 m s^{-1}

v = 340 m s^{-1}

f_P = 830 Hz

f_Q = ?

Speed of Q relative to P = $u_s = u_Q - u_P$ = 20 m s^{-1} – 12 m s^{-1} = 8 m s^{-1}

$$\because \quad f = \left(\frac{v}{v-u_s}\right) f'$$

$$\therefore \quad f_P = \left(\frac{v}{v-u_s}\right) f_Q$$

Putting the value, we have

$$830\text{ Hz} = \left(\frac{340\text{ m s}^{-1}}{340\text{ m s}^{-1} - 8\text{ m s}^{-1}}\right) f_Q$$

$$830\text{ Hz} = \left(\frac{340\text{ m s}^{-1}}{332\text{ m s}^{-1}}\right) f_Q$$

$$\text{or} \quad f_Q = \left(\frac{830\text{ Hz} \times 332\text{ m s}^{-1}}{340\text{ m s}^{-1}}\right)$$

$$f_Q = 810.47\text{ Hz}$$

Example 7.11 A train sounds its horn before it sets off from the station and an observer waiting on the platform estimates its frequency at 1200 Hz. The train then moves off and accelerates steadily. Fifty seconds after departure, the driver sounds the horn again and the platform observer estimates the frequency at 1140 Hz. Calculate the train speed 50 s after departure. How far from the station is the train after 50 s.

(Speed of sound = 340 m s^{-1})

Solution

Original frequency of horn = f = 1200 Hz

Apparent frequency = f' = 1140 Hz

Speed of sound = v = 340 m s^{-1}

Time $= t = 50\ s$

Speed of source (i.e., train) $= u_s = ?$

Distance covered by the train $= S = ?$

(i)
$$f' = \left(\frac{v}{v + u_s}\right) f$$

Putting the values, we have
$$1140\ Hz = \left(\frac{340\ m\ s^{-1}}{340\ m\ s^{-1} + u_s}\right) \times 1200\ Hz$$
$$340\ m\ s^{-1} + u_s = \frac{340\ m\ s^{-1} \times 1200\ Hz}{1140\ Hz}$$
$$u_s = 357.89 - 340$$
$$u_s = 17.89\ m\ s^{-1}$$

(ii)
$$S = v_{av} t$$
$$S = \left(\frac{v_i + v_f}{2}\right) t$$
$$S = \left(\frac{0 + 17.89\ m\ s^{-1}}{2}\right) 50\ s$$
$$S = 448\ m$$

Do you know?

Bats navigate and find food by echo location.

7.12 APPLICATIONS OF DOPPLER EFFECT

Doppler effect is also applicable to electromagnetic waves. One of its important applications is the radar system, which uses radio waves to determine the elevation and speed of an aeroplane. **RADAR (RAdio Detection And Ranging)** is a device, which transmits and receives radio waves. If an aeroplane approaches towards the radar, then the wavelength of the wave reflected from aeroplane would be shorter and if it moves away, then the wavelength would be larger as shown in Fig.7.15 (a) & Fig.7.15 (b), respectively. Similarly, speed of satellites moving around the Earth can also be determined by the same principle.

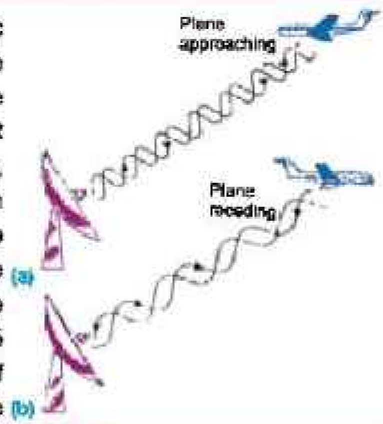


Fig. 7.15: A frequency shift is used in a radar to detect the motion of an aeroplane.

SONAR is an acronym derived from "Sound Navigation And Ranging". It is the general name for sonic or ultrasonic underwater echo-ranging and echo-sounding system. Sonar is the name of a technique for detecting the presence of objects under water by acoustical echo. In **Sonar**, "Doppler detection" relies upon the relative speed of the target and the detector to provide an indication of the target speed. It employs the Doppler effect, in which an apparent change in frequency occurs when the source and the observer are in relative motion to one another. Its known military applications include the detection and location of submarines, control of antisubmarine weapons, mine hunting and depth measurement of sea.

In Astronomy, astronomers use the Doppler effect to calculate the speeds of distant stars and galaxies. By comparing the line spectrum of light from the star with light from a laboratory source, the Doppler shift of the star's light can be measured. Then, the speed of the star can be calculated.

(i) Stars moving away from the Earth show a red shift as shown in Fig.7.16(b). The emitted waves have a longer wavelength than if the star had been at rest. So, the spectrum is shifted towards longer wavelength, i.e., towards the red end of the spectrum. Astronomers have also discovered that all the distant galaxies are moving away from us and by measuring their red shifts, they have estimated their speeds.

(ii) Stars moving towards the Earth show a blue shift as shown in Fig.7.16(c). This is because the wavelength of light emitted by the star are shorter than if the star had been at rest. So, the spectrum is shifted towards shorter wavelength, i.e., to the blue end of the spectrum.

Another important application of the Doppler shift using electromagnetic waves is the **radar speed trap**. Microwaves are emitted from a transmitter in short bursts. Each burst is reflected off by any car in the path of microwaves in between sending out bursts. The transmitter is open to detect reflected microwaves. If the reflection is caused by a moving obstacle, the reflected microwaves are Doppler shifted. By measuring the Doppler shift, the speed at which the car moves is

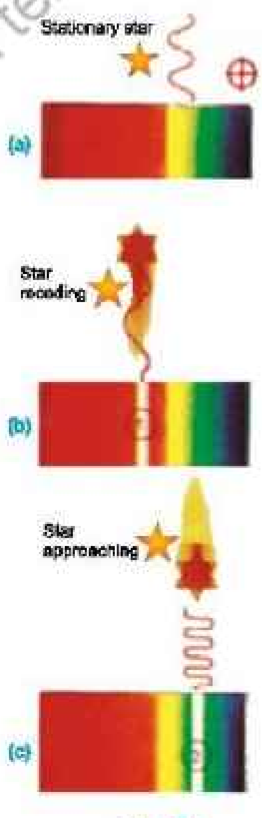


Fig. 7.16

calculated by computer programme.

Satellite Navigation uses Doppler shift to determine satellite velocity and position, enabling accurate location tracking.

Satellite Communication also uses Doppler shift compensation ensuring stable communication signals.

Doppler radar detects wind velocity and precipitation patterns. **Doppler shift** helps measure Earth's surface velocity and deformation.

Doppler echocardiography measures blood flow velocity and detects cardiac abnormalities, such as valve stenosis or regurgitation. Doppler echocardiography optimizes pacemaker settings.

Doppler ultrasound measures blood flow and calculates cardiac output. Doppler ultrasound detects vascular stenosis or occlusion.

Example 7.12 The wavelength of one of its lines of the absorption spectrum of a faint galaxy is identified as Ca-α line found to be 478 nm. The wavelength of same line is observed and measured as 397 nm in the laboratory.

(a) Is the galaxy moving towards or away from the Earth?

(b) Compute the speed of the galaxy relative to the Earth.

Solution

Laboratory measured original wavelength

$$\lambda = 397\ \text{nm} = 397 \times 10^{-9}\ \text{m}$$

Changed or Apparent wavelength $\lambda' = 478\ \text{nm} = 478 \times 10^{-9}\ \text{m}$

Speed of light $c = 3 \times 10^{8}\ \text{m s}^{-1}$

(a)

$$\because \quad v = f\lambda$$

$$\therefore \quad c = f\lambda \qquad (\because v = c)$$

$$\Rightarrow \quad f = \frac{c}{\lambda}$$

$$f = \frac{3 \times 10^{8}\ \text{m s}^{-1}}{397 \times 10^{-9}\ \text{m}}$$

$$f = 7.56 \times 10^{-3} \times 10^{8} \times 10^{9}\ \text{s}^{-1}$$

Laboratoy frequency $f = 7.56 \times 10^{14}\ \text{Hz}$

Apparent frequency $f' = \frac{c}{\lambda'}$

$$= \frac{3 \times 10^{8}\ \text{m s}^{-1}}{478 \times 10^{-9}\ \text{m}}$$

$$f' = 6.28 \times 10^{14}\ \text{Hz}$$

$\because \quad \lambda' > \lambda \quad$ or $\quad f' < f$

$\therefore$ The galaxy is moving away from the Earth.

(b)
$$f' = \left(\frac{v}{v + u_s}\right) f$$

$$f' = \left(\frac{c}{c + u_s}\right) f \qquad (\because v = c)$$

Putting the values, we have

$$6.28 \times 10^{14}\,\text{Hz} = \left(\frac{3 \times 10^8\,\text{m s}^{-1}}{3 \times 10^8\,\text{m s}^{-1} + u_s}\right) 7.56 \times 10^{14}\,\text{Hz}$$

$$3 \times 10^8 + u_s = \frac{3 \times 10^8\,\text{m s}^{-1} \times 7.56 \times 10^{14}\,\text{Hz}}{6.28 \times 10^{14}\,\text{Hz}}$$

$$u_s = \frac{22.68 \times 10^8}{6.28} - 3 \times 10^8\,\text{m s}^{-1}$$

$$u_s = 6.12 \times 10^7\,\text{m s}^{-1}$$

Galactic Motion

A galaxy is moving away from us at 20% of the speed of light (i.e., 0.2 c). We observe a spectral line from this galaxy that is normally emitted at a wavelength of 500 nm.

QUESTIONS

Multiple Choice Questions

Tick (✓) the correct answer.

7.1 The simple wave speed equation is represented by:

(a) $v = f\lambda$ (b) $v = S/t$ (c) $v = r\omega$ (d) $v = \Delta d/\Delta t$

7.2 The principle of superposition in waves is stated as:

(a) the displacement of a wave is the sum of the displacements of its individual components

(b) the velocity of a wave is the product of its individual components

(c) the frequency of a wave is the difference of its individual components

(d) the amplitude of a wave is the ratio of its individual components

7.3 A node in a stationary wave is:

(a) a point of maximum displacement (b) a point of intermediate displacement

(c) a point of zero displacement (d) a point of infinite displacement

7.4 An antinode in a stationary wave is:

(a) a point of maximum displacement (b) a point of minimum displacement

(c) a point of zero displacement (d) a point of infinite displacement

7.5 Stationary waves are defined as:

(a) waves that move with a constant velocity
(b) waves that move with a changing velocity
(c) waves that oscillate in a fixed position
(d) waves that propagate through a medium

7.6 Harmonics are:

(a) integer multiples of a fundamental frequency
(b) Integer submultiples of a fundamental frequency
(c) random frequencies
(d) non-integer multiples of a fundamental frequency

7.7 The result of constructive interference between two waves is represented as:

(a) a decrease in amplitude (b) an increase in amplitude
(c) no change in amplitude (d) a shift in phase

7.8 If the amplitude of the wave is tripled, then the amount of energy is increased by:

(a) 3 times (b) 6 times (c) 9 times (d) 12 times

7.9 What type of waves do headphones use to produce sound?

(a) Electromagnetic waves (b) Mechanical waves
(c) Pressure waves (d) Longitudinal waves

7.10 The typical frequency range of microwaves is:

(a) $10^3 - 10^5$ Hz (b) $10^5 - 10^7$ Hz (c) $10^7 - 10^9$ Hz (d) $10^9 - 10^{11}$ Hz

7.11 The banding of waves around an obstacle is called as:

(a) refraction (b) reflection (c) diffraction (d) interference

7.12 The Doppler Effect used in astronomy is for:

(a) measuring the diameters of stars
(b) determining velocity of galaxies
(c) analyzing properties of black holes
(d) studying behaviour of electromagnetic waves

Short Answer Questions

7.1 What are the conditions for interference to occur?

7.2 Differentiate between constructive and destructive interference of waves.

7.3 What are coherent waves and coherent sources? Give examples.

7.4 Distinguish between longitudinal and transverse waves.

7.5 Is it possible for two identical waves travelling in the same direction along a string to give rise to a stationary wave? How is it so?

7.6 How would you apply Doppler effect in studying cardiac problems in humans?

7.7 What is meant by diffraction of waves? For what purpose, the ripple tank is used?

Constructed Response Questions

7.1 Which measurement of a wave is the most important when determining the wave's intensity?

7.2 Can you apply Doppler effect to light waves? Describe briefly.

7.3 Can you compare the compressions and rarefactions of the longitudinal wave with the peaks and troughs of the transverse wave? Discuss.

7.4 How should a source of sound move with respect to an observer so that the frequency of its sound does not change? Write two examples.

7.5 Why is it difficult to recognize beats when the frequency difference is greater than 10 Hz? Exemplify.

Comprehensive Questions

7.1 State and explain the principle of superposition of waves. Apply this principle to elaborate the working of noise canceling headphones.

7.2 What are standing waves? Illustrate a detailed experiment that demonstrates the standing waves using stretched strings.

7.3 Find the frequencies of the harmonics produced in an organ pipe when it is open at both ends and when it is closed at one end.

7.4 Define and exemplify diffraction of waves. Describe this phenomenon by ripple tank experiment.

7.5 What is meant by the term beats? Prove that number of beats per second is equal to the difference between the frequencies of vibrating tuning forks.

7.6 What do you understand by progressive waves? Discuss the intensity of progressive waves.

7.7 Keeping in mind "Doppler effect", analyze the following cases:

(a) when source of sound moves away from the stationary observer.

(b) when source of sound moves towards the stationary observer.

Numerical Problems

7.1 The speed of a wave on a typical string is 24 m s^{-1}. What driving frequency will it resonate if its length is 6.0 m? (Ans: 2 Hz)

7.2 The lowest resonance frequency for a guitar string of length 0.75 m is 400 Hz. Calculate the speed of a transverse wave on the string. (Ans: 600 m s^{-1})

7.3 A tuning fork A produces 4 beats per second with another tuning fork B. It is found that by loading B with some wax, the beat frequency increases to 6 beats per second. If the frequency of A is 320 Hz, determine the frequency of B when loaded.

(Ans: 314 Hz)

7.4 A steel wire hangs vertically from a fixed point, supporting a weight of 80 N at its lower end. The diameter of the wire is 0.50 mm and its length from the fixed point to the weight is 1.5 m. Calculate the fundamental frequency emitted by the wire when it is plucked. Density of steel wire is 7.8×10^{3} kg m^{-3}. (Ans: 76.2 Hz)

7.5 Average intensity of sunlight on the surface of the Earth is nearly 500 W m^{-2}. Determine the amount of energy that falls on a solar panel having an area of 0.50 m^{2} in four hours. (Ans: 3.6×10^{6} J)

7.6 (a) If the intensity of a wave is 16 W m^{-2} and the amplitude is 2 m, what is the value of constant k? (Ans: 4 W m^{-4})

(b) If the intensity of a wave is 25 W m^{-2} and the constant k is 5 W m^{-4}, what is the amplitude? (Ans: 2.24 m)

7.7 (a) A sound system produces 200 watts of power. If the sound is directed at a crowd with an area of 150 m^{2}, what is the intensity of the sound?

(Ans: 1.33 W m^{-2})

(b) A light bulb emits 100 watts of power. If the light is spread out evenly over a sphere with a surface area of 400 m^{2}, what is the intensity of the light?

(Ans: 0.25 W m^{-2})

7.8 A radio antenna broadcasts 500 watts of power. If the signal is received at a distance of 10 km, what is the intensity of the signal? (Ans: 4×10^{-7} W m^{-2})

7.9 An organ pipe has a length of 1 m. Determine the frequencies of the fundamental and the first two harmonics:

(a) if the pipe is open at both ends and (b) if the pipe is closed at one end.

(Speed of sound in air is 340 m s^{-1})

(Ans: 170 Hz, 340 Hz, 510 Hz; 85 Hz, 255 Hz, 425 Hz, respectively)

7.10 A train is approaching a station at 90 km h^{-1}, sounding a whistle of frequency 1000 Hz. What will be the apparent frequency of the whistle as heard by a listener sitting on the platform? What will be the apparent frequency heard by the same listener if the train moves away from the station with the same speed? (Speed of sound is 340 m s^{-1}) (Ans: 1079.4 Hz, 931.5 Hz, respectively)

Chapter 8 Physical Optics and Gravitational Waves

Learning Objectives

After studying this chapter, the students will be able to:

- Explain that polarization is a phenomenon associated with transverse waves.
- Define and apply Malus's law $I = I_o \cos^2\theta$ to calculate the intensity of a plane-polarized electromagnetic wave after transmission through a polarizing filter or a series of polarizing filters. [Calculation of the effect of a polarizing filter on the intensity of an unpolarized wave is not required].
- Explain the use of polaroids in sky photography and stress analysis of materials.
- Describe qualitatively gravitational waves

  [as waves of the intensity of gravity generated by the accelerated masses of an orbital binary system that propagate as waves outward from their source at the speed of light].
- State that as a gravitational wave passes a body with mass distortion in space-time can cause the body to stretch and compress periodically.
- State that gravitational waves pass through the Earth due to far off celestial events, but they are of very minute amplitude.
- Describe the use of interferometers in detecting gravitational waves.

  [Interferometers are very sensitive detection devices that make use of the interference of laser beams (working and set up details are not required) and were used to first detect the existence of gravitational waves].

This chapter deals with two major areas of physics namely polarization of transverse waves and gravitational waves.

8.1 POLARIZATION OF LIGHT

Physical optics with reference to polarization deals with the behaviour of light waves and their interaction with matter.

Interference and diffraction effects prove the wave nature of light. These phenomena, however, do not tell us whether the light waves are longitudinal or transverse. Polarization of light suggests that the light waves are transverse in character.

In transverse mechanical waves, such as produced in a stretched string, the vibrations of the particles of the medium are perpendicular to the direction of propagation of the waves. The vibrations can be confined along vertical, horizontal or any other direction (Fig. 8.1). In each of these cases, the transverse mechanical wave is said to be

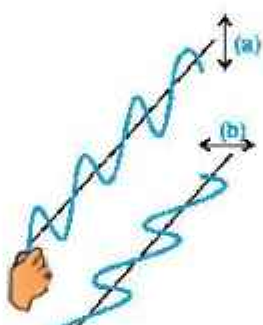


Fig. 8.1: Transverse waves on a string polarized
(a) in a vertical plane, and
(b) in a horizontal plane

polarized. The plane of polarization is the plane containing the direction of vibration of the particles of the medium and the direction of propagation of the wave.

A light wave produced by oscillating charge consists of a periodic variation of the electric field vector **E** accompanied by the magnetic field vector **B** at right angles to it. Ordinary light has components of vibration in all possible planes. Such a light is unpolarized. On the other hand, if the vibrations are confined only in one plane, the light is said to be polarized. Unpolarized light is shown in Fig. 8.2.

Fig. 8.2: An unpolarized light, due to incandescent bulb, has vibrations in all planes.

Examples of unpolarized light sources are sunlight, incandescent light bulbs, fluorescent light bulbs, light from a candle or fire.

Polarization is the process by which the electric and magnetic vibrations of light waves are restricted to a single plane of vibration. It is the property exhibited only by transverse waves such as light waves. It does not occur for longitudinal waves such as sound waves.

How an Unpolarized Light be Polarized?

An unpolarized light can be made polarized by the following methods:

1. Passing light through a polarizing filter (e.g, polaroid sheet).
2. Using a polarizing beam splitter.
3. Employing certain optical crystals or materials (e.g., calcite, quartz, etc.).

The most common method by which an unpolarized light can be polarized is by passing it through a polarizing filter, such as a polarizing beam splitter or a polaroid sheet. When an unpolarized light passes through the polarized filter, only that electric field vector which is parallel to the axis of polarized filter can pass through it, while all other vectors are blocked. The resultant light then becomes polarized as shown in Fig. 8.3.

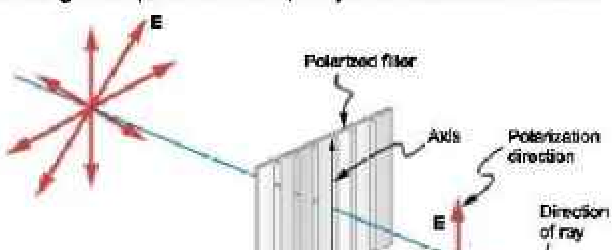


Fig. 8.3: Polarization of light using a polarized filter

Thus, in simple words, the process of transforming an unpolarized light into a polarized light is said to be polarization.

The orientation of the electric field vector E of light waves in a specific direction is the basis of Polarization.

8.2 TYPES OF POLARIZATION

Here are the basic types of polarization of light.

1. Linear Polarization

When the electric field vector oscillates in a single plane, light is said to be linearly polarized as shown in Fig.8.4 (a). Example is the light passing through a polarizing filter, like sunglasses.

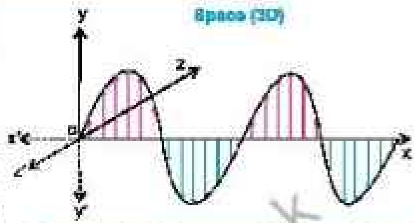


Fig. 8.4 (a): Linear polarization of light

2. Circular Polarization

When the electric field rotates circularly, either clockwise (right-handed), or counterclockwise (left-handed), the light is said to be circularly polarized as shown in Fig.8.4 (b). Example is the light reflected off a CD (Compact Disc) or DVD (Digital Versatile Disc).

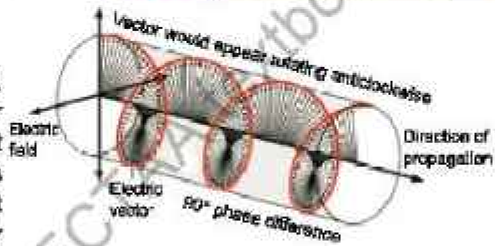


Fig. 8.4 (b): Circular polarization of light

3. Elliptical Polarization

A combination of linear and circular polarization, where the electric field vector traces an elliptical path is called elliptical polarization. In elliptical polarization as shown in Fig.8.4(c), the two components of electric field E_x and E_y are not equal or they differ in phase by an arbitrary angle θ. Example is the light passing through a stress plate or a waveplate.

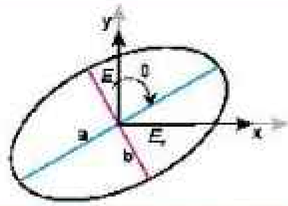


Fig. 8.4 (c): Elliptical polarization of light

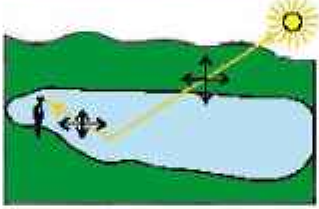
8.3 PRODUCTION AND DETECTION OF PLANE POLARIZED LIGHT

The light emitted by an ordinary incandescent bulb is unpolarized, because its electrical vibrations are randomly oriented in space as shown in Fig.8.2.

If unpolarized light is made incident on a sheet of polaroid (polarizer), the transmitted light will be plane polarized. If a second sheet of polaroid (analyzer) is placed in such a way that the axes of the two polaroids shown by straight lines drawn on them are parallel (Fig. 8.5-a), the light is transmitted through the second polaroid. If the second polaroid (analyzer) is slowly rotated about the beam of light as axis of rotation, the light emerging from the second polaroid gets dimmer and dimmer and ultimately disappears when the axes become mutually perpendicular as shown in Fig. 8.5(b), the light reappears on further rotation and becomes brightest when the axes are again parallel to each other.

This experiment proves that light waves are transverse waves. If the light waves were longitudinal, they would never disappear even if the two polaroids were mutually perpendicular.

Ponder Upon!



Light reflected from smooth surface of water is partially polarized parallel to the surface.

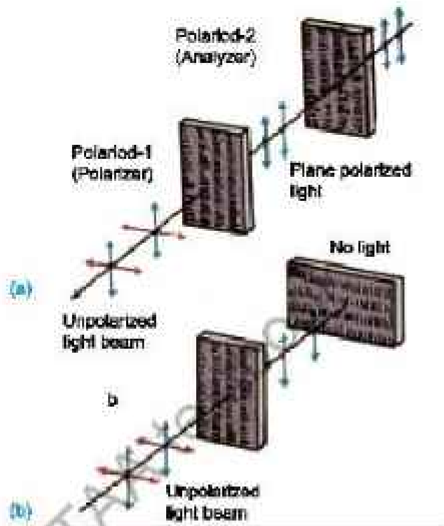


Fig. 8.5: Experimental arrangement to show that light waves are transverse. The lines with arrows indicate electric vibrations of light waves.

8.4 POLARIZATION OF LIGHT BY THE METHOD OF REFLECTION

In 1808, Malus discovered that polarized light is obtained when ordinary light is reflected by a plane sheet of glass. If the reflected light is viewed through a polaroid which is slowly rotated about the line of vision, the light is practically extinguished at a certain

Ray Diagram

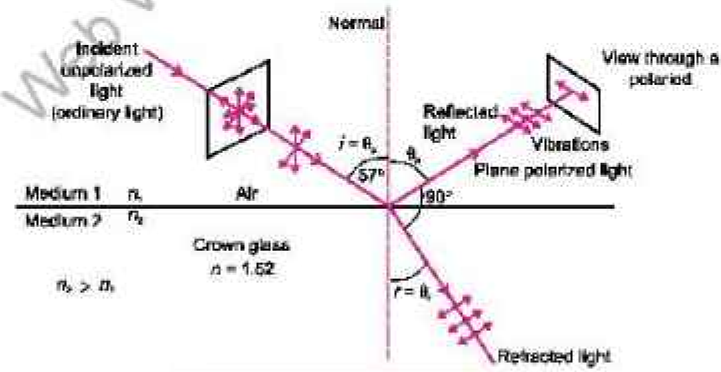


Fig. 8.6: Plane polarization by reflection

orientation of the polaroid. The most suitable angle of incidence 'i' is about 57° for glass for which the reflected ray becomes plane polarized, as illustrated by ray diagram in Fig. (8.6). This proves that the light reflected by the glass is practically plane polarized. Light reflected from the surface of a table becomes darker when viewed through a rotated polaroid, showing that it is partially plane polarized.

Brewster's Law

The particular angle of incidence on a transparent medium when the reflected light is almost plane polarized is called the polarizing angle. Let a beam of unpolarized light be made incident on the surface of medium 2 as shown in Fig.8.6. If the reflected beam of light is almost plane polarized, the reflected and refracted beams are at right angles to each other at the polarizing angle, $i = \theta_p$. Thus

$$\theta_p + \theta_r = 90^\circ$$

or

$$\theta_r = 90^\circ - \theta_p$$

From Snell's law,

$$n_1 \sin\theta_p = n_2 \sin\theta_r$$

$$n_1 \sin\theta_p = n_2 \sin(90^\circ - \theta_p)$$

$$n_1 \sin\theta_p = n_2 \cos\theta_p$$

$$\frac{\sin\theta_p}{\cos\theta_p} = \frac{n_2}{n_1}$$

> **Interesting information**
>
> Malus's law is used to study the polarization of light in biological systems, such as the polarization of light by cell membranes.

$$\tan\theta_p = \frac{n_2}{n_1} \quad \text{.................. (8.1)}$$

This equation is known as Brewster's law. In this equation, n_1 is refractive index of medium 1 and n_2 is refractive index of medium 2. If medium 1 is air, then equation becomes $\tan\theta_p = n$ because $n_1 = 1$ and $n_2 = n$. Here n is refractive index of medium on which light is incident. Hence, Brewester proved that the tangent of the angle of polarization is numerically equal to the refractive index of the medium 2 when medium 1 is air. In Brewster's law, the angle 'θ_p' for which the reflected ray and the refracted ray make an angle of 90° between them, is also called the Brewster angle θ_B. Then, $\tan\theta_B = n$ holds.

Example 8.1 A beam of light strikes the surface of a plate of glass with a refractive index of $\sqrt{3}$ at the polarizing angle. What will be the angle of refraction of the wave of light?

Solution $n = \sqrt{3}$, $\theta_r = ?$

As $\tan\theta_B = n$

or $\theta_B = \tan^{-1}(\sqrt{3})$

$\theta_B = 60^\circ$

As $\theta_r = 90^\circ - \theta_B$, $\theta_r = 90^\circ - 60^\circ$

$\theta_r = 30^\circ$

8.5 MALUS'S LAW

Malus's law states that the intensity I of plane polarized light after passing through an analyzer is directly proportional to the square of the cosine of the angle θ between the transmission axis of the analyzer and polarizer. That is;

$$I \propto \cos^2\theta$$

$$I = I_0\cos^2\theta$$

where I_0 = Intensity of the incident polarized light.

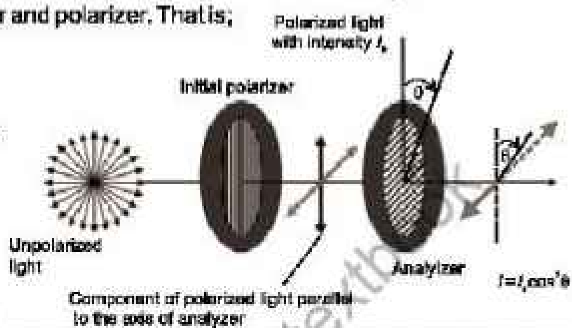


Fig 8.7 (a): Schematic representation of Malus's law

Actually, Malus's law gives a mathematical relation between the intensity of the light incident on the first polaroid (i.e., polarizer) and the intensity of light obtained after passing it through the second polaroid (i.e., analyzer). This is shown in Fig. 8.7(a). An analyzer is also a polarizer that is placed after a polarizer. Rotation of the analyzer affects the intensity of the polarized light. It is used to further reduce the intensity of light and also adjust it by adjusting the angle of the analyzer with respect to the polarizer.

Certain transparent crystalline materials, like tourmaline, calcite crystals, etc., are capable of confining vibrations of light waves in only one plane. Such materials are called polaroids which have high directionality in crystal structure. Light can also be polarized by natural phenomena like reflection, refraction and scattering.

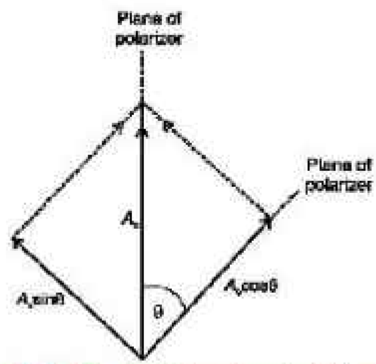


Fig.8.7(b): Resolution of amplitude into components of plane polarization light.

If a piece of polaroid is rotated in front of a polarized ray of light, it causes a variation in the intensity of the light that gets through. The reason that causes the variation of intensity is the angle between the initial polarizer and the axis of second polarizer.

When the incident polarized light of amplitude A_0 strikes the analyzer at an angle θ, it is resolved into two components $A_0\cos\theta$ and $A_0\sin\theta$ as shown in Fig. 8.7(b). The component $A_0\sin\theta$ is absorbed in the analyzer. Since, only $A_0\cos\theta$ passes through the analyzer, the amplitude A of the transmitted light is therefore,

$$A = A_0\cos\theta \qquad \text{.................... (8.2)}$$

Since intensity I is proportional to the square of the amplitude A. It can be expressed as:

$$I \propto A^2 \text{ or } I = kA^2$$

where $k=1$, we can write; $I = A^2$

or $\quad I = A_o^2 \cos^2\theta$

$$I = I_o\cos^2\theta \quad (\because I_o = A_o^2)$$

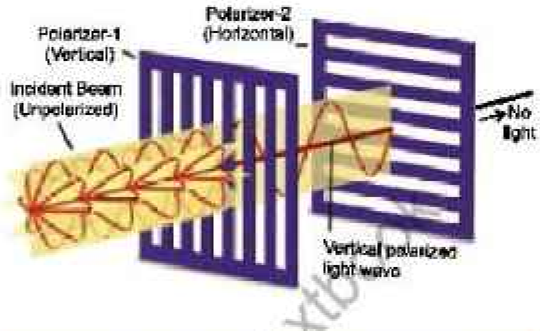
Here I_o is the intensity of the incident polarized light.

There are two extreme conditions of θ followed by the above equation given as,

i. If $\theta = 0°$, then $I = I_o$. This means the intensity transmitted through the analyzer is equal to the initial light intensity that passes through the polarizer.

ii. If $\theta = 90°$, then $I = 0$. This means the light is extinguished completely, i.e., no light is allowed to pass through the analyzer.

For your information



Light passing through crossed polarizers

Optical Activity

Optical activity is the ability of a substance to rotate the plane of polarization of light passing through it. The rotation is detected with a polarizer or analyzer as shown in Fig. 8.8.

Many crystals and solutions rotate the plane of polarization of light passing through them. Such substances are said to be optically active. Examples are quartz crystals, cinnabar (H_gS), sugar water, insulin and collagen. The amount and direction of rotation depends on following factors:

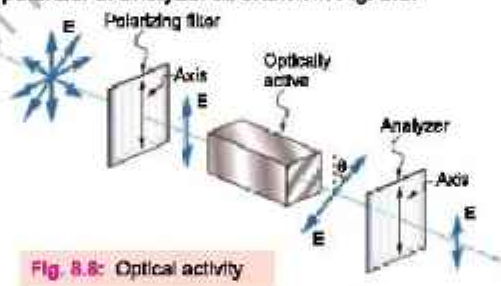


Fig. 8.8: Optical activity

(i) The type of substance

(ii) The concentration of the substance (the amount of a substance present in a given quantity of a mixture or solution).

(iii) The distance the light travels through it, and

(iv) The wavelength of light.

Optical activity occurs due to the asymmetric shape of molecules in the substance, such as being helical. A few millimetre thickness of such crystals will rotate the plane

Interesting Information

Sugar solution rotates the plane of polarization of incident light so that it is no longer horizontal but at an angle θ. The analyzer thus stops the light when rotated from the vertical (crossed) positions.

of polarization by many degrees. Certain organic substances, such as sugar and tartaric acid, show optical rotation when they are in a solution. This property of optically active substances can be used to determine their concentration in the solutions.

Example 8.2 Find the refractive index of a medium if polarizing angle is 54.5°.

Solution

$\theta_p = 54.5°, n = ?$

As $\tan\theta_p = n$

or $n = \tan\theta_p$

So $n = \tan 54.5°$

$n = 1.4$

Do you know?

Looking through two polarizers when they are "crossed", very little light passes through.

Example 8.3 Polarized light with an intensity of 75 W m^{-2} passes through an analyzer with its axis at 30° to the polarizer's axis. What is the emerging intensity?

Solution

$I_o = 75\ \text{W m}^{-2}, \theta = 30°, I = ?$

Using Malus's law:

$I = I_o\cos^2\theta$

$I = 75\ \text{N m}^{-2}\cos^2 30°$

$= 75\ \text{N m}^{-2}\,(0.866)^2$

$= 75\ \text{N m}^{-2} \times 0.75$

$I = 56.25\ \text{W m}^{-2}$

Example 8.4 A polarized light with an amplitude of 5 units passes through a polarizer with its electric field aligned at 60° to the original polarization direction. Find the amplitude of the wave after passing through the analizer?

Solution

$A_o = 5 \text{ units}, \theta = 60°, A = ?$

Using Malus's law:

$A = A_o\cos\theta$

or $A = 5\cos 60°$

$= 5 \times 0.5$

$A = 2.5 \text{ units}$

Tidbit

A beam of unpolarized light passes through a foggy atmosphere. Tell the polarization state of the scattered light.

Importance of Polarization

The immense significance of polarization of light may be justified by various fields:

1. Optics and Photonics

Polarization is essential for applications like polarized sunglasses, LCD (Liquid Crystal Display) screens, and optical communication systems.

2. Imaging and Microscopy

Polarization enhances image quality, reducing glare (unwanted light that interferes with vision) and improving contrast, especially in microscopy and medical imaging.

3. Medical Applications

Polarization is used in cancer diagnosis, tissue imaging, and laser surgery, leveraging its ability to distinguish between different tissue types.

4. Astronomy

Polarization helps us to analyze cosmic phenomena, like the polarization of light from distant stars or the cosmic microwave background radiation.

5. Miscellaneous Fields

Polarization has importance in miscellaneous fields such as optics and photonics, imaging and microscopy, biology and chemistry, communication system, etc.

Two Main Applications of Polarization

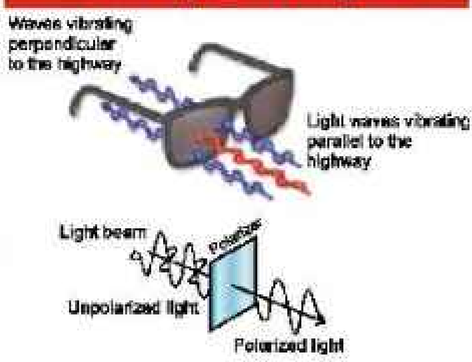
Polarizers, also known as polarizing filters, have two main applications:

1. Sky Photography

A camera which is used to photograph the clouds is fitted with a polaroid. The light coming from the sky is polarized by the polaroid.

In sky photography, polarizers are used to reduce the glare and haze which are produced by the scattering of light by small particles of molecules present in the atmosphere. Polarizers also enhance the contrast by clocking the excessive bright white light while allowing the other colours to pass through, thus creating a brighter detailed image. Thus, allows to improve the overall image quality.

Action of polarized sunglasses



2. Stress Analysis of Materials

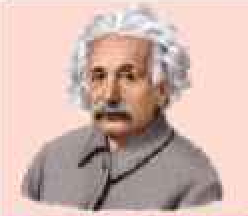
In materials science, polarizers are used to analyze the stress and strain on materials, such as plastics, metals, and glass.

When a material is stressed, its molecular structure changes, affecting the way it interacts with light, and interference patterns on fringes are formed which in turn gives qualitative information about the material. By shining polarized light through the transparent material and analyzing the changes in the light's polarization, the researchers can:

- determine the material's stress patterns
- identify potential weaknesses or defects
- analyze the material's optical properties
- understand how the material will behave under different conditions. This technique is known as **"photoelasticity"** and is widely used in fields like engineering, materials science, and quality control.

In both cases described above, polarizers play a crucial role in manipulating light to achieve specific goals, whether it is enhancing image quality or analyzing material properties.

8.6 GRAVITATIONAL WAVES (GWs)



Albert Einstein (1879-1955)
A German born Theoretical Physicist
One of the World's renowned scientists.

A gravitational wave is a stretching and compressing of space-time and can be observed by measuring the change in length between two objects.

Gravitational waves (GWs) are actually:

"Ripples in the fabric of space-time, produced by violent cosmic events, like colliding black holes or neutron stars that travel at the speed of light, carrying information about their source."

The simplest example to understand GWs is given below:

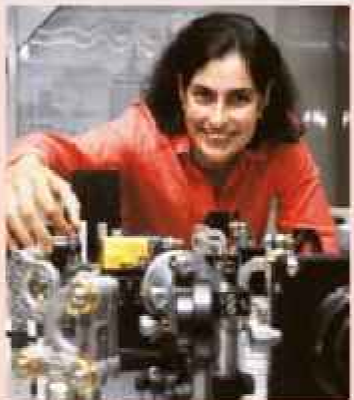
If we throw a stone into a pond, the stone creates ripples on the water surface (space-time). These ripples travel outward, carrying information about the stone (the cosmic event). They can be detected on the shore by (gravitational wave observatories), revealing the stone's presence and properties.

Prediction and Detection

Gravitational waves are a prediction of Einstein's theory of general relativity which is confirmed by observations and is opening a new window into the universe's most extreme phenomena.

According to Einstein's general theory of relativity, gravity is not a force, but a curvature of space-time caused by massive bodies. Gravity is like a dent in a mattress. Heavy things wrap the space around them, and that is why we feel gravity.

Gravitational waves, as initially predicted by Albert Einstein in 1916, are ripples in spacetime that were first detected in 2015, but announced in 2016, the first observation of its kind: the detection of gravitational waves, produced from two colliding neutron stars. In this type, there is a gradual increase in frequency and amplitude of GWs.



Dr. Nergis Mavalwala is a Pakistani-American Astrophysicists at MIT known for her work on gravitational waves

A Binary System (BS) in the context of gravitational waves refers to "a system consisting of two compact objects, such as black holes, neutron stars, or white dwarfs, which are orbiting each other and emitting gravitational waves."

Four Basic Types of GWs

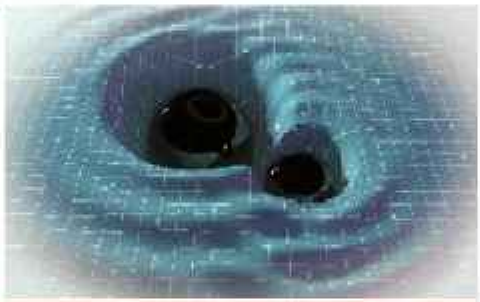
There are four basic types of gravitational waves, each with different sources and characteristics:

1. Continuous GWs

When a single massive object spins with a constant rate, such as a neutron star, continuous GWs are produced with a constant frequency and amplitude. White dwarf binary systems produce continuous GWs.

2. Compact Binary Inspirational GWs

When a binary system, such as binary neutron stars, binary black holes, or a neutron star and black hole orbiting each other, compact inspirational GWs are produced.



BS Model

3. Burst GWs

These are produced by violent events like supernovae, gamma-ray bursts, or cosmic strings.

4. Stochastic GWs

Stochastic GWs are weak, random signals of GWs which are produced by superposition of many weak gravitational wave sources, such as distant binary systems. These GWs are the most difficult to detect.

Every physical object that accelerates, produces gravitational waves. Vehicles, airplanes, etc. are included in it. The masses and acceleration of objects on the Earth are too small to make gravitational waves big enough to be detected with our instruments.

As the binary systems (also termed as binaries) orbit each other, they emit gravitational waves, which can be detected by observatories like LIGO (Laser Interferometer Gravitational wave Observatory) which is situated in USA and Virgo, a large scale gravitational wave observatory in Cascina, Italy. The waves carry information about the system's mass, spin, and merger dynamics, offering insights into these extreme cosmic objects, and move with the speed of light.

Picture of a series of concentric spheres, with the binary system at the center, radiating gravity waves outward into the cosmos.

The binary systems are significant sources of gravitational waves, and their mergers (collision and union of two massive objects resulting more massive single object) are among the most intense cosmic events.

As the masses orbit and accelerate, their gravitational intensity fluctuates, generating waves that radiate outward in all directions. These waves are not bound by the binary system's gravity; instead, they travel freely through spacetime at the speed of light. The waves propagate through the universe, weakening in intensity as they distance themselves from the source.

Tidal forces carry the mathematical signature of gravitational waves

The characteristics of GWs depend on the system's properties, such as:

- Masses of the objects
- Orbital period and frequency
- Eccentricity of the orbit: Eccentricity e is a measure of the amount by which an object deviates from a perfect circle.

$e = 0$	Circular orbit
$e = 1$	Parabolic trajectory
$e > 1$	Hyperbolic trajectory
$0 < e < 1$	Elliptical orbit

Space-time Distortion / Tidal Forces

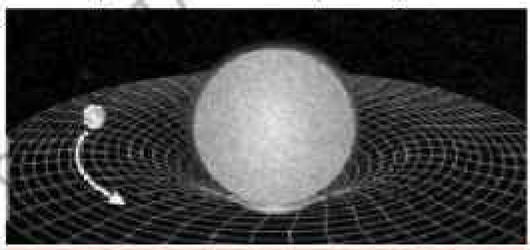
Gravitational waves passing through a body with mass can cause the body to experience periodic stretching and compressing, also known as "space-time distortion". This effect is known as "tidal forces" and is a result of the gravitational wave's oscillating nature. As the gravitational wave passes through

Spacetime curve of an artificial and a natural satellite

the body, it causes the space-time around the body to oscillate, leading to a periodic stretching and compressing of the body in the direction perpendicular to the wave's propagation. This effect is similar to how the tides on Earth are caused by the gravitational pull of the Moon and Sun.

The amount of stretching and compressing depends on the strength of the gravitational wave, as well as the mass and size of the body. This effect is an important prediction of Einstein's general theory of relativity.

Gravitational waves generated by far off celestial events, such as the merger of two black holes or neutron stars, pass through the Earth. However, the amplitude of these waves is extremely small, typically of the order of 10^{-21} to 10^{-22} metres. This means that the



Spacetime curve shown by two satellites

distortion caused by the gravitational wave is incredibly tiny, and requires extremely sensitive instruments to detect.

Despite their small amplitude, gravitational waves offer a unique window into the universe, allowing us to study strong-field gravity, to test general relativity, and to explore the universe in ways previously impossible.

8.7 INTERFEROMETER

An interferometer is an optical tool used in detecting gravitational waves. It is a very sensitive detection device that may use the interference of LASER (Light Amplification by Stimulated Emission of Radiatoin) beams. The basic LIGO interferometer can be seen in Fig. 8.9.

An interferometer that detects gravitational waves is a highly sensitive instrument that uses LASER light to measure tiny changes in distance between mirrors, caused by gravitational waves passing through the detectors. These interferometers are called

LIGO (LASER Interferometer Gravitational Wave Observatory). The main differences between LIGO and conventional interferometers are:

(a) LIGO is 1000 times larger than conventional device, and

(b) LIGO uses LASER whereas conventional interferometer has normal light source.

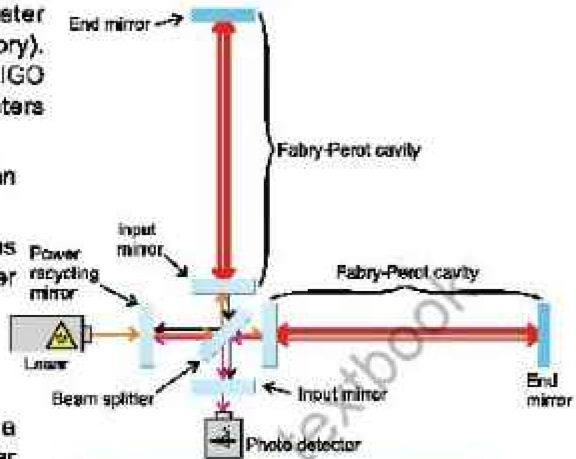


Fig. 8.9 Basic LIGO Interferometer

Basic Components of GW Interferometer

The basic components of a gravitational wave interferometer are:

1. **Laser** that produces a stable and high intensity beam of light.
2. **Power recycling mirror** continually reflects LASER light that has already travelled through the instrument back into the interferometer and hence the term recycling is used.
3. **Beam splitter** divides the LASER beam into two perpendicular beams.
4. **Mirrors** reflect the beams, creating two perpendicular arms.
5. **Fabry Perot cavity** consists of two mirrors facing each other. The purpose of the cavity is to enhance the path length.
6. **Photodetectors** measure the returning beams, detecting tiny phase shifts (if any).
7. **Arm cavities** enhance the LASER light, increasing sensitivity.

Working of Interferometer

A laser beam is split into two perpendicular beams, each travelling down two identical paths (arms) of the interferometer. The beams bounce off mirrors at the ends of each arm and return to the starting point, where they are recombined. If a gravitational wave passes through, it causes a tiny disturbance in the distance between the mirrors, resulting in a phase shift between the two beams.

When the beams recombine, they create an interference pattern, which is measured by a photodetector. The tiny phase shift caused by the gravitational wave alters the interference pattern, allowing the detector to sense the wave's presence.

Figure 8.9 is a simple figure of an interferometer, but in reality, it is much more complex. On 14 September 2015, the universe's gravitational waves were observed for the very first time by LIGO. The gravitational waves which were predicted by Albert Einstein

100 years ago, came from a collision between two black holes. It took 1.3 billion years for the waves to arrive at the LIGO detector in the USA. On their work on observation of GWs, Rainer Weiss, Barry C. Barisk and Kip S. Throne received Nobel Prize in 2017. It is interesting to note that one of the team members, Nergis Mavalwala a professor at MIT (Massachusetts Institute of Technology), belongs to Pakistan.

Virgo Detection

Similar to LIGO, there is another facility for measuring gravitational waves. This is called Virgo, which works under the European Gravitational Observatory (EGO) Cascina near Pisa, Italy. Virgo is also an interferometer with two arms of 3 km whereas LIGO has 4 km arms. The Virgo Observatory is named after the Virgo constellation, which is visible in the night sky during the months of March, April and May. The Virgo cluster is a group of about 1,500 galaxies about 50 MLYs (Million Light Years) away. Remember one Light Year (LY) is a distance which light travels in one year. The approximate value of 1LY = 9.5 billion km. Virgo has been involved in detecting gravitational wave events, with the first detection in 2017.

Example 8.5 If the gravitational waves have a wavelength of 4000 km, find their frequency.

Solution $\lambda = 4000\text{ km} = 4\times10^{6}\text{ m}$, $c = 3\times10^{8}\text{ m s}^{-1}$, $f = ?$

As $\quad v = f\lambda$

or $\quad f = \dfrac{c}{\lambda} \quad (\because v = c)$

$\because \quad f = \dfrac{3\times10^{8}\text{ m s}^{-1}}{4\times10^{6}\text{ m}}$

$f = 0.75\times10^{2}\text{ s}^{-1}$

$f = 75\text{ Hz}$

Example 8.6 A binary system emits gravitational waves with a frequency of 10^{-1} Hz. What is the wavelength of these waves?

Solution $f = 10^{-1}\text{ Hz} = 10^{-1}\text{ s}^{-1}$, $c = 3\times10^{8}\text{ m s}^{-1}$, $\lambda = ?$

As $\quad v = f\lambda$

or $\quad \lambda = \dfrac{c}{f} \quad (\because v = c)$

$\therefore \quad \lambda = \dfrac{3\times10^{8}\text{ m s}^{-1}}{10^{-1}\text{ s}^{-1}}$

$\lambda = 3\times10^{9}\text{ m}$

QUESTIONS

Multiple Choice Questions

Tick (✓) the correct answer.

8.1 The phenomenon of polarization of light is:

(a) the process of scattering of light
(b) the property of light to vibrate in a specific plane
(c) the ability of light to travel in a straight line
(d) the phenomenon of light changing colour

8.2 Malus's law states that:

(a) the intensity of light is directly proportional to the square of the cosine of the angle between the light wave and the analyzer
(b) the intensity of light is directly proportional to the square of the sine of the angle between the light wave and the analyzer
(c) the intensity of light is directly proportional to the angle between the light wave and the analyzer
(d) the intensity of light is inversely proportional to the angle between the light wave and the analyzer

8.3 The intensity of light when it passes through a polarizer:

(a) increases (b) decreases
(c) remains the same (d) becomes zero

8.4 The angle between the light wave and the analyzer is called:

(a) polarization angle (b) refraction angle
(c) reflection angle (d) azimuth angle

8.5 The key purpose of an analyzer in a polarization experiment is:

(a) to polarize the light (b) to measure the intensity of light
(c) to change the direction of light (d) to filter out unwanted light

8.6 The mathematical representation of Malus's law is:

(a) $I = I_o \cos^2\theta$ (b) $I = I_o \sin^2\theta$
(c) $I = I_o \tan^2\theta$ (d) $I = I_o \cot^2\theta$

8.7 The effect of increasing the angle between the light wave and the analyzer on the intensity of light is:

(a) the intensity increases (b) the intensity decreases
(c) the intensity remains the same (d) the intensity becomes zero

8.8 The condition for maximum intensity of light in a polarization experiment is when:

(a) the light wave and analyzer are perpendicular
(b) the light wave and analyzer are parallel
(c) the light wave and analyzer are at an angle of 45°
(d) the light wave and analyzer are at an angle of 60°

8.9 The unwanted light that interferes with vision is termed as:
(a) haze (b) glare (c) contrast (d) flare

8.10 Who predicted the existence of gravitational waves?
(a) Galileo Galilei (b) Albert Einstein
(c) Issac Newton (d) Leonardo da Vinci

8.11 What are gravitational waves?
(a) Electromagnetic waves (b) Mechanical Waves
(c) Ocean waves (d) Ripples in the fabric of spacetime

8.12 Which is the primary method used to detect gravitational waves?
(a) Optical telescopes (b) Radio telescopes
(c) LASER interferometry (d) Gravitational lensing

8.13 Which of the following is a primary source of gravitational waves?
(a) Binary black hole merger (b) Solar flares
(c) Earthquake (d) Solar wind

Short Answer Questions

8.1 Why are the polaroid sunglasses better than the ordinary sunglasses?
8.2 Is light from the sky partially polarized? How is it so?
8.3 How is Malus's law used in everyday life?
8.4 What are the applications of Brewster's angle?
8.5 What is the space-time curvature?

Constructed Response Questions

8.1 Write down some applications of plane polarized.
8.2 Would it be possible to use a polarizer as an analyzer? If yes, give at least two examples.
8.3 Explain how Malus's law is used in the design of polarized sunglasses. How do these surfaces reduce glare from reflective surface? Provide an example to illustrate your answer.
8.4 How will the sky appear if there had been no atmosphere?
8.5 What is the significance of detecting gravitational waves?
8.6 How are tidal forces formed?

Comprehensive Questions

8.1 Define the phenomenon of polarization of waves. How does polarization of electromagnetic waves occur? Also classify the polarization of waves.
8.2 How can the plane polarized light be produced and detected? What does it prove?
8.3 How can polarized light be obtained by the method of reflection? Explain.
8.4 State Malus's law. Explain the intensity formula.

8.5 What is a polaroid? Explain two main applications of polarization.

8.6 What are gravitational waves? Describe the basic types of gravitational waves.

8.7 What is an interferometer? Describe the basic LIGO interferometer in detail.

8.8 What is meant by optical activity? Discuss it.

Numerical Problems

8.1 When an unpolarized light of intensity I_o is incident on a polarizing sheet, find the intensity of light which does not get transmitted. (Ans: $\frac{I_o}{2}$)

8.2 A polarized light beam passes through a polarizer at an angle of 45°. Find the intensity of the transmitted light if the initial intensity is 100 W m^{-2}. (Ans: 50 W m^{-2})

8.3 A light wave passes through a polarizer with its electric field aligned at 30° to the horizontal. If the amplitude of the wave is 10 units, what is the amplitude of the wave passing through the polarizer? (Ans: 8.66 units)

8.4 What angle is required between the direction of polaroid light and the axis of a polaroid filter to reduce its intensity by 85%? (Ans: 67.5°)

8.5 An unpolarized light having intensity of 15 W m^{-2} is incident on a pair of polarizers. The first polaroid filter has its transmission axis at 50° from the vertical. The second polaroid filter has its transmission axis at 20° from the vertical. Calculate the intensity of light transmitted to both filters. (Ans: 7.5 W m^{-2}, 5.6 W m^{-2}, respectively)

8.6 Two polarizing sheets have their polarizing directions parallel so that intensity of emitted light is maximum. Through what angle must either sheet be rotated if the intensity is to be dropped by half? (Ans: 45°)

8.7 We wish to use a glass plate of refractive index of 1.5 in air as a polarizer. Find the polarizing angle and angle of refraction. (Ans: 56.3°, 33.7°, respectively)

8.8 At what angle of incidence, will light reflected from water be completely polarized? (Ans: 53°)

8.9 A beam of unpolarized light is incident on a stack of four polarizing sheets that are lined up so that the characteristic direction of each is rotated by 30° clockwise with respect to the preceding sheet. What fraction in percentage of the incident intensity be transmitted? (Ans: 21%)

8.10 A polarizer and an analyzer have their axes aligned at 60°. What is the fraction of the initial intensity that emerges? (Ans: 0.25)

8.11 If the gravitational waves have a wavelength of 3000 km, then find their frequency assuming it moves with the speed of light? (Ans: 100 Hz)

Chapter

9 Electrostatics and Current Electricity

Learning Objectives

After studying this chapter, the students will be able to:

- Define and calculate electric field strength.
  [Use $F = qE$ for the force on a charge in an electric field. Use $E = \frac{\Delta V}{\Delta d}$ to calculate the field strength of the uniform field between charged parallel plates]
- Describe the effect of a uniform electric field on the motion of charged particles.
- State that, for a point outside a spherical conductor, the charge on the sphere may be considered to be a point charge at its centre.
- Explain how a Faraday cage works.
  [by inducing internal electric fields that work to shield the inside from the influence of external electric fields]
- State and apply Coulomb's law.
  $F = k\frac{q_1 q_2}{r^2}$ for the force between two point charges in free space, [where $k = \frac{1}{4\pi\varepsilon_o}$]
- Use $E = k\frac{Q}{r^2}$ for the electric field strength due to a point charge in free space.
- Use, for a current-carrying conductor, the expression $I = Anvq$. [where n is the number of charge carriers per unit volume]
- State and use $V = W/Q$.
- State and use $P = IV$, $P = I^2R$ and $P = V^2/R$.
- State and use $R = \rho\frac{L}{A}$
- State that the resistance of a light dependent resistor (LDR) decreases as the light intensity increases.
- State Kirchhoff's first law and describe that it is a consequence of conservation of charge.
- State Kirchhoff's second law and describe that it is a consequence of conservation of energy.
- Use Kirchhoff's laws to solve simple circuit problems.
- State and use the principle of the potentiometer as a means of comparing potential differences.
- Explain the use of a galvanometer in null methods.
- Explain the use of thermistors and light-dependent resistors in potential dividers.
  [to provide a potential difference that is dependent on temperature and light intensity]
- Explain the internal resistance of sources and its consequences for external circuits.
- Explain how inspectors can easily check the reliability of a concrete bridge with carbon fibres as the fibres conduct electricity.

We know that magnitude of the charge on an electron is equal to that of a proton. The charge on a proton is e^+ and that on an electron is e^-. Its value is; $e = 1.6 \times 10^{-19}$ C where C (coulomb) is the SI unit of charge. This charge (e) is the smallest amount of free charge that has been discovered. Charges of larger magnitude are built up on an object by adding or removing electrons. It is the minimum amount of charge that any particle may contain. Thus, any amount of charge q is an integer multiple of e, i.e.,

$$q = Ne \qquad \text{where } N \text{ is an integer}$$

Electrostatics is the study of phenomena and properties of electric charges at rest. When charges are in motion, we call it as an electric current.

Charles de Coulomb (1736-1806)

Coulomb's major contribution to science was in the field of electrostatics and magnetism. During his lifetime, he also investigated the strengths of material and determined the forces that effect on beams, thereby, contributing to the field of structural mechanics. In the field of ergonomics, his research provided a fundamental understanding of the ways in which people can best do work.

9.1 COULOMB'S LAW

Coulomb's law is a fundamental principle in electrostatics that quantifies the force between two charged objects. The first measurement of the force between electric charges was made by a French Physicist Charles de Coulomb in 1874. Coulomb's law is essential in understanding the behaviour of charged particles and the interactions that govern many electrical phenomena. On the basis of these measurements, he deduced a law known as Coulomb's law. It states that:

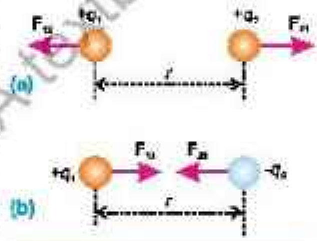


Fig. 9.1
(a) Repulsive forces between like charges and
(b) attractive forces between unlike charges.

The force between two-point charges is directly proportional to the product of the magnitudes of charges and inversely proportional to the square of the distance between them. It is mathematically expressed as

$$F \propto \frac{q_1 q_2}{r^2} \text{ or } F = k\frac{q_1 q_2}{r^2} \quad \ldots\ldots\ldots (9.1)$$

where F is the magnitude of the mutual force that acts on each of the two point charges q_1, q_2, and r is the distance between them. The force **F** always acts along the line joining the two point charges (Fig. 9.1), k is the constant of proportionality. Its value depends upon the nature of medium between the two charges and system of units in which F, q and r are measured. If the medium between the two point charges is free space and the system of units is SI, then k is represented as

$$k = \frac{1}{4\pi\varepsilon_0} \quad \ldots\ldots\ldots\ldots (9.2)$$

where ε_0 is an electrical constant, known as permittivity of free space. In SI units, its value is $8.85 \times 10^{-12}\ C^2 N^{-1} m^{-2}$. Substituting the value of ε_0 the constant

$$k = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9\ N\ m^2\ C^{-2}$$

Thus, Coulomb's force in free space is

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2} \quad \ldots\ldots\ldots\ldots (9.3)$$

Point to ponder!

Does an electrostatic force exist between a charged and an uncharged?

As stated earlier, Coulombs' force is mutual force, it means that if q_1 exerts a force on q_2, then q_2 also exerts an equal and opposite force on q_1. If we denote the force exerted on q_2 by q_1 as F_{21} and that on charge q_1 due to q_2 as F_{12} then

$$\mathbf{F}_{12} = -\mathbf{F}_{21} \quad \ldots\ldots\ldots (9.4)$$

The magnitude of both these two forces is the same and is given by Eq. 9.3. To represent the direction of these forces, we introduce unit vectors. If $\hat{r}_{21}$ is the unit vector directed from q_1 to q_2 and $\hat{r}_{12}$ is the unit vector directed from q_2 to q_1 then

$$\mathbf{F}_{21} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{\mathbf{r}}_{21} \quad \ldots\ldots\ldots (9.5)$$

$$\mathbf{F}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{\mathbf{r}}_{12} \quad \ldots\ldots\ldots (9.6)$$

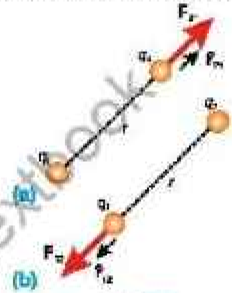


Fig. 9.2

The forces $\mathbf{F}_{21}$ and $\mathbf{F}_{12}$ are shown in Fig. 9.2 (a and b). It can be seen that $\hat{r}_{21} = -\hat{r}_{12}$, so Eqs. 9.5 and 9.6 show that;

$$\mathbf{F}_{21} = -\mathbf{F}_{12}$$

The sign of the charges in Eqs. 9.5 and 9.6 determine whether the forces are attractive or repulsive.

We shall now consider the effect of medium between the two charges upon the Coulomb's force. If the medium is an insulator, it is usually referred as dielectric. It has been found that the presence of a dielectric always reduces the electrostatic force as compared with that in free space by a certain factor which is a constant for the given dielectric. This constant is known as relative permittivity and is represented by ε_r. The values of relative permittivity of different dielectrics are given in Table 9.1.

Table 9.1

Material	ε_r
Vacuum	1
Air (atm)	1.0006
Ammonia (liquid)	22-25
Bakelite	5-15
Benzene	2.284
Germanium	16
Glass	4.8-10
Mica	3-7.5
Paraffined paper	2
Plexiglass	3.40
Rubber	2.94
Teflon	2.1
Transformer oil	7.1
Water (distilled)	78.5

Thus, the Coulomb's force in a medium of relative permittivity ε_r is given by

$$F = \frac{1}{4\pi\varepsilon_0\varepsilon_r}\frac{q_1 q_2}{r^2} \quad \ldots\ldots\ldots (9.7)$$

It can be seen in the table that ε_r, for air is 1.0006. This value is so close to one that with negligible error, the Eq. 9.3 gives the electric force in air.

Example 9.1 Three point charges q_1, q_2 and q_3 are lying in the same plane as shown in Fig. 9.3(a). Find the magnitude and direction of the net fore acting on q_1.

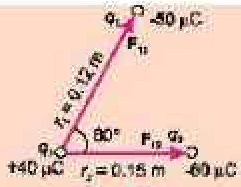


Fig. 9.3(a)

Solution

Force on q_1 exerted by q_2 is attractive. Let it be F_{12}. Its magnitude is given by

$$F_{12} = k\frac{q_1 q_2}{r_2^2} = \frac{(9\times10^9\ \text{N m}^2\ \text{C}^{-2})(40\times10^{-6}\ \text{C})(60\times10^{-6}\ \text{C})}{(0.15\ \text{m})^2} = 960\ \text{N}$$

Force on q_1 exerted by q_3 is also attractive. Let it be $\mathbf{F}_{13}$. Its magnitude is given by

$$F_{13} = k\frac{q_1 q_3}{r_2^2} = \frac{(9\times10^9\ \text{Nm}^2\ \text{C}^{-2})(40\times10^{-6}\ \text{C})(50\times10^{-6}\ \text{C})}{(0.12\ \text{m}^2)} = 1250\ \text{N}$$

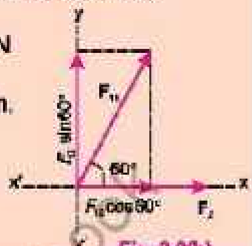


Fig. 9.3(b)

To find the resultant of $\mathbf{F}_{12}$ and $\mathbf{F}_{13}$, let us make free-body diagram, resolving $\mathbf{F}_{13}$ into its rectangular components, we have

$$F_{13x} = F_{13}\cos60° = 1250\ \text{N} \times 0.5 = 625\ \text{N}$$

$$F_{13y} = F_{13}\sin60° = 1250\ \text{N} \times 0.86 = 1075\ \text{N}$$

The x-component of resultant $\mathbf{F}$ is

$$F_x = F_{12} + F_{13}\cos60°$$

$$F_x = 960\ \text{N} + 625\ \text{N}$$

$$F_x = 1585\ \text{N}$$

y-component of $\mathbf{F}$ is:

$$F_y = F_{13}\sin60° = 1075\ \text{N}$$

Magnitude of F is given by

$$F = \sqrt{F_x^2 + F_y^2} = 1975\ \text{N}$$

For direction of $\mathbf{F}$:

$$\tan\theta = \frac{F_y}{F_x} = \frac{1075}{1585} = 0.68$$

Therefore, $\theta = 34°$ with the line joining q_1 and q_2.

Do you know?

A Van de Graaff generator is an electrostatic generator which uses a moving belt to accumulate electric charge on a hollow metal globe on the top of an insulated column, creating very high voltage direct current (DC) at low current levels. It was invented by an American physicist Robert J. Van de Graff in 1929.

The potential difference achieved in modern Van de Graaff generators can reach 5 megavolts. A tabletop version can produce of the order of 100,000 volts and can produce enough energy to produce a visible spark.

A pulley drives an insulting belt by a sharply pointed metal comb which has been given a positive charge by a power supply. Electrons are removed from the belt, leaving it positively charged. A similar comb at the top allows the net positive charge to spread to the dome.

Why do the hairs lift when VAN DE GRAAFF GENERATOR is touched?

9.2 ELECTRIC FIELD STRENGTH

We have learnt that a charge experiences an electrostatic force in the presence of other charges. Let us consider a positively charged object Q. If we place a small charge +q at point A, it will experience an electrostatic force F due to the charge Q. Thus, an electric field is said to exist at point A. Coulomb's law suggests that the field gets stronger as the point A gets closer to Q as represented in Fig. 9.4. The strength of the field at a position is known as its intensity at that point.

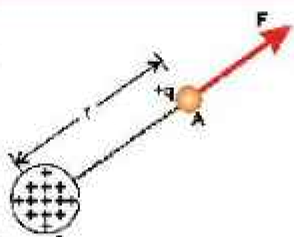


Fig. 9.4

The electric intensity of the field or simply electric field at any point is defined as the force experienced by a unit positive charge placed at that point.

Electric intensity is a force, so it is a vector quantity and is usually denoted by **E**. It can be obtained by the relation:

$$\mathbf{E} = \frac{\mathbf{F}}{q} \quad \text{.........................} \quad (9.8)$$

As
$$F = \frac{1}{4\pi\varepsilon_o}\frac{qQ}{r^2} \qquad \therefore \qquad E = \frac{1}{4\pi\varepsilon_o}\frac{Q}{r^2}$$

In vector form:
$$\mathbf{E} = \frac{1}{4\pi\varepsilon_o}\frac{Q}{r^2}\hat{\mathbf{r}}$$

From Eq. 9.8, the unit of electric intensity is newton per coulomb $N\,C^{-1}$. The direction of **E** is the same as that of **F**. The Eq. 9.5 can also be written as:

$$\mathbf{F} = q\mathbf{E} \quad \text{.........................} \quad (9.9)$$

Example 9.2 Two positive point charges $q_1 = 16.0\ \mu C$ and $q_2 = 4.0\ \mu C$ are separated by a distance of 3.0 m, as shown in Fig.9.5. Find the spot on the line joining the two charges where electric field is zero.

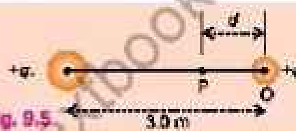


Fig. 9.5

Solution Between the two charges, the fields contributed by them have opposite directions, and electric field would be zero at a point P, where the magnitude of E_1 equals to E_2. In Fig. 9.5, let the distance of P from q_2 be d. At P, $E_1 = E_2$ which implies that:

$$\frac{1}{4\pi\varepsilon_o}\frac{q_1}{(3.0-d)^2} = \frac{1}{4\pi\varepsilon_o}\frac{q_2}{d^2}$$

or
$$\frac{16.0\times10^{-6}\,C}{9.0+d^2-6d} = \frac{4.0\times10^{-6}\,C}{d^2}$$
or $d^2 + 2d - 3 = 0$, which gives $d = +1$ m, -3 m

There are two values of d, the negative value corresponds to a location off to the right of both the charges where magnitudes of $\mathbf{E}_1$ and $\mathbf{E}_2$ are equal but directions are same. In this case $\mathbf{E}_1$ and $\mathbf{E}_2$ do not cancel at this spot. The positive value corresponds to the location shown in the figure and is the zero filed location, hence, $d = +1.0$ m.

Example 9.3 A proton experiences an electrostatic force equal to its weight at a particular point in an electric field. What is the field intensity at that point?

Mass of proton $= 1.67 \times 10^{-27}$ kg and charge, $e = 1.6 \times 10^{-19}$ C

Solution

Using
$$E = \frac{F}{q} = \frac{mg}{e}$$

Substituting the values,

$$E = \frac{1.67\times10^{-27}\ \text{kg}\times 9.8\ \text{m s}^{-2}}{1.6\times10^{-19}\ \text{C}} = 1.0\times10^{-7}\ \text{N C}^{-1}$$

Electric Field Lines

A visual representation of the electric field can be obtained in terms of electric field lines, an idea proposed by Michael Faraday. Electric field lines can be considered as a visual map used to represent the direction and strength of an electric field around a charged object. As electric field lines provide information about the electric force exerted on a charged object, these lines are commonly called "electric lines of force".

To introduce electric field lines, we place positive test charges $+q_o$ each of magnitude q_o at different places but at equal distances from a positive charge $+q$ as shown in the Fig. 9.6. Each test charge will experience a repulsive force, as indicated by arrows in Fig. 9.6(a). Therefore, the electric field created by the charge $+q$ is directed radially outward. Figure 9.6(b) shows corresponding field lines which show the field direction. Figure 9.7 shows the electric field lines in the vicinity of a negative charge $-q$. In this case the lines are directed radially "inward", because the force on a positive test charge is now of attraction, indicating the electric field points inward.

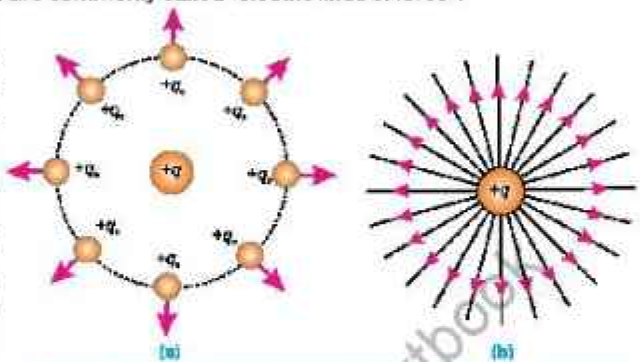


Fig 9.6
(a): A positive test charge $+q_o$ placed anywhere in the vicinity of a positive point charge $+q$ experiences a repulsive force directed radially outward.
(b): The electric field lines are directed radially outward from the positive point charge $+q$.

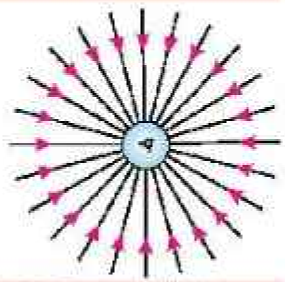


Fig 9.7: The electric field lines are directed radially inward towards a negative point charge $-q$.

Figures 9.6 and 9.7 represent two dimensional pictures of the field lines. However, electric field lines emerge from the charges in three dimensions, and an infinite number of lines could be drawn.

The electric field lines "map" also provides information about the strength of the electric field. As we notice in Figs. 9.6 and 9.7 that field lines are closer to each other near the charges where the field is strong while they continuously spread out indicating a continuous decrease in the field strength.

The number of lines per unit area passing perpendicularly through it is proportional to the magnitude of the electric field.

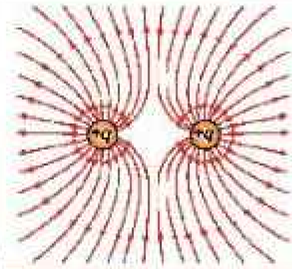


Fig.9.8: The electric field lines for two identical opposite point charges.

The electric field lines are curved in case of two identical separated charges. Figure 9.8 shows the pattern of lines associated with two identical positive point charges of equal magnitude. It reveals that the lines in the region between two like charges seem to repel each other. The behaviour of two identical negatively charges will be exactly the same. The middle region shows the presence of a zero field spot or neutral zone.

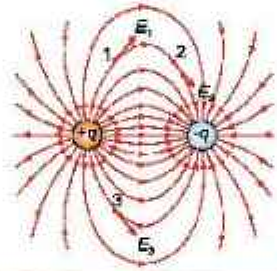


Fig 9.9: The electric field lines are directed radially inward towards a negative point charge -q.

Figure 9.9 shows the electric field pattern of two opposite charges of equal magnitudes. The field lines start from positive charge and end on a negative charge. The electric field at points such as 1, 2, 3 is the resultant of fields created by the two charges at these points. The directions of the resultant intensities is given by the tangents drawn to the field lines at these points.

In the regions where the field lines are parallel and equally spaced, the same number of lines pass per unit area and therefore, field is uniform on all points. Figure 9.10 shows the field lines between the plates of a parallel plate capacitor. The field is uniform in the middle region where field lines are equally spaced.

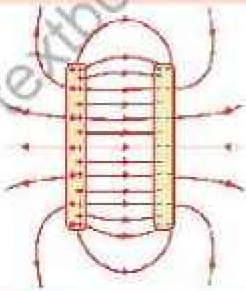


Fig 9.10: In the central region of a parallel plate capacitor, the electric field lines are parallel and evenly spaced, indicating that the electric field there has the same magnitude and direction at points.

We are now in a position to summarize the properties of electric field lines.

1. Electric field lines originate from positive charges and end on negative charges.
2. The tangent to a field line at any point gives the direction of the electric field at that point.
3. The lines are closer where the field is strong and the lines are farther apart where the field is weak.
4. No two lines cross each other. This is because E has only one direction at any given point. If the lines cross E could have more than one direction.

Do you know?

There is no electric field inside the conductor.

9.3 ELECTRIC FLUX

When we place an element of area in an electric field, some of the lines of force pass through it (Fig. 9.11). The number of the field lines passing through a certain element of area is known as electric flux through that area. It is usually denoted by Greek letter ϕ_e. For example, the electric flux ϕ_e through the area A is 4 while the flux through B is 2.

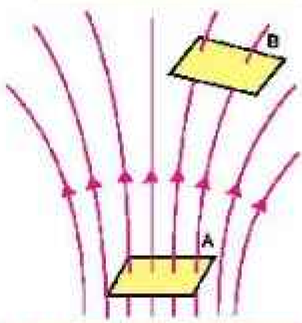


Fig.9.11: Electric flux through a surface normal to E.

In order to give a quantitative meaning to flux, the field lines are drawn such that the number of field lines passing through a unit area held perpendicular to field lines at a point represent the intensity **E** of the field at that point.

Usually, the element of area is represented by a vector area **A** whose magnitude is equal to the surface area *A* of the element and direction is along normal to the area.

In Fig.9.12 (a), area *A* is held perpendicular to the field lines, then *EA* lines pass through it. The flux ϕ_e, in this case is:

$$\phi_e = EA \quad \text{.......................(9.10)}$$

In Figure 9.12 (b), area *A* is held parallel to field lines and, as is obvious no lines cross this area, so that flux ϕ_e, in this case is:

$$\phi_e = EA = 0 \quad \text{......................(9.11)}$$

Figure 9.12(c) shows the case when **A** is neither perpendicular nor parallel to field lines but is inclined at an angle θ with the field **E**. In this case, we have to find the projection of the area which is perpendicular to the field lines. The area of this projection (Fig. 9.12-c) is $A\cos\theta$. The flux ϕ_e in this case is:

$$\phi_e = EA\cos\theta \quad \text{.................(9.12)}$$

The electric flux ϕ_e, through a patch of flat surface in terms of **E** and **A** is then given by

$$\phi_e = EA\cos\theta = \mathbf{E.A} \quad \text{.........(9.13)}$$

where θ is the angle between the field lines and the normal to the area. Electric flux being a scalar product, is a scalar quantity. Its SI unit is $N\,m^2\,C^{-1}$.

Electric Flux Through a Surface Enclosing a Charge

Let us calculate the electric flux through a closed surface, in shape of a sphere of radius *r* due to a point charge *q* placed at the centre of sphere as shown in Fig. 9.13. To apply the formula ϕ_e = **E.A** for the computation of electric flux, the surface area should be flat. For this reason the total surface area of the

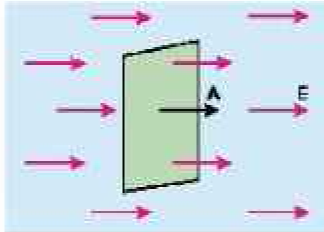


Fig.9.12(a): Maximum

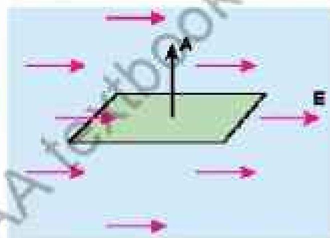


Fig.9.12(b): Minimum

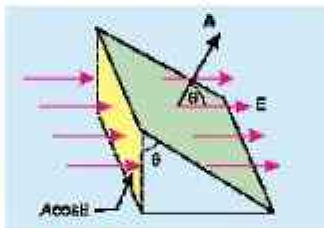


Fig.9.12 (c)

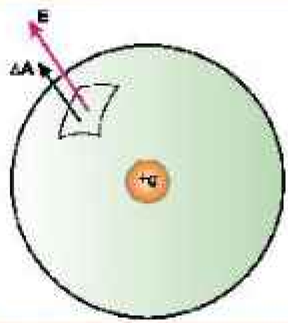


Fig.9.13: The total electric flux through the surface of the sphere due to a charge *q* at its centre is q/ε_o

sphere is divided into n small patches with areas of magnitudes ΔA_1, ΔA_2, ΔA_3, ………, ΔA_n, respectively as shown in Fig. 9.13. The direction of each vector area is along perpendicular drawn outward to the corresponding patch. The electric intensities at the centres of vector areas $\Delta \mathbf{A}_1$, $\Delta \mathbf{A}_2$, $\Delta \mathbf{A}_3$, ……, $\Delta \mathbf{A}_n$ are $\mathbf{E}_1$, $\mathbf{E}_2$, $\mathbf{E}_3$, ……, $\mathbf{E}_n$, respectively.

According to Eq. 9.13, the total flux passing through the closed surface is:

$$\Phi_e = \mathbf{E}_1.\Delta\mathbf{A}_1 + \mathbf{E}_2.\Delta\mathbf{A}_2 + \mathbf{E}_3.\Delta\mathbf{A}_3 + \ldots\ldots + \mathbf{E}_n.\Delta\mathbf{A}_n \quad \ldots\ldots (9.14)$$

The direction of electric intensity and vector area is the same at each patch. Moreover,

$$|\mathbf{E}_1| = |\mathbf{E}_2| = |\mathbf{E}_3| = \ldots\ldots = |\mathbf{E}_n| = E = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2} \quad \ldots\ldots (9.15)$$

Since **E** is parallel to vector area **A**, therefore, $\theta = 0°$ (Fig. 9.13)

so for each $\mathbf{E}.\Delta\mathbf{A} = EA\cos\theta$

$$= EA\cos 0° = EA \quad (\because \cos 0° = 1)$$

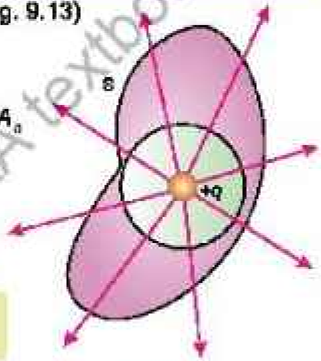


Fig: 9.14

$$\phi_e = E\Delta A_1 + E\Delta A_2 + E\Delta A_3 + \ldots\ldots + E\Delta A_n$$

$$= E\,(\Delta A_1 + \Delta A_2 + \Delta A_3 + \ldots\ldots + \Delta A_n)$$

$$= E\ \text{(Total spherical surface area)}$$

$$= \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2} \times 4\pi r^2$$

$$\phi_e = \frac{q}{\varepsilon_o} \quad \ldots\ldots (9.16)$$

Now imagine that a closed surface S is enclosing this sphere. It can be seen in Fig 9.14 that the flux through the closed surface S is the same as that through the sphere. So, we can conclude that total flux through a closed surface does not depend upon the shape or geometry of the closed surface. It depends upon the medium and the charge enclosed.

9.4 GAUSS'S LAW

Suppose point charges q_1, q_2, q_3, ………, q_n are arbitrarily distributed within an arbitrarily shaped closed surface, as shown in Fig. 9.15. Since $\phi_e = q/\varepsilon_o$, so the electric flux passing through the closed surface is:

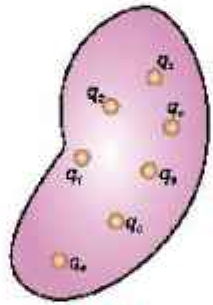


Fig: 9.15

$$\phi_e = \frac{q_1}{\varepsilon_o} + \frac{q_2}{\varepsilon_o} + \frac{q_3}{\varepsilon_o} + \ldots\ldots + \frac{q_n}{\varepsilon_o}$$

$$\phi_e = \frac{1}{\varepsilon_o} \times (q_1 + q_2 + q_3 + \ldots\ldots + q_n)$$

$$\phi_e = \frac{1}{\varepsilon_o} \times \text{(Total charge enclosed by closed surface)}$$

$$\phi_e = \frac{1}{\varepsilon_o} \times Q \quad \ldots\ldots (9.17)$$

where $Q = q_1 + q_2 + q_3 + \ldots\ldots\ldots\ldots + q_n$, is the total charge enclosed by closed surface. Equation 9.17 is mathematical expression of Gauss's law which can be stated as:

> The total electric flux through any closed surface is $1/\varepsilon_o$ times the total charge enclosed in it.

Applications of Gauss's Law

Gauss's law can be applied to calculate the electric intensity due to different charge configurations. In all such cases, an imaginary closed surface is considered which passes through the point at which the electric intensity is to be evaluated. This closed surface is known as Gaussian surface. Its choice is such that the flux through it can be easily evaluated.

As an example, let us find the electric field at any point outside a sphere on which a charge q is placed.

The Field of a Charged Conducting Sphere

Consider a conducting sphere of radius R containing a charge q. We know that all the charge is distributed uniformly over the surface of sphere as shown in Fig. 9.16. We can also conclude from the spherical symmetry that the electric field is radial everywhere and that its magnitude depends only on the distance r from the centre of the sphere. Thus, the magnitude E is uniform over a spherical surface with any radius r concentric with the spherical conductor. Therefore, we take our Gaussian surface as an imaginary sphere with radius r greater than the radius R of the conducting sphere.

The area of the Gaussian sphere is $4\pi r^2$, and because E is uniform over the sphere, the total flux through the whole surface will be:

Electric flux $\quad \phi_e = EA = E \times 4\pi r^2$

By Gauss's law, total flux is:

$$\phi_e = \frac{q}{\varepsilon_o}$$

Therefore $\quad E \times 4\pi r^2 = \dfrac{q}{\varepsilon_o}$

or $\quad E = \dfrac{q}{\varepsilon_o}\dfrac{1}{4\pi r^2}$

or $\quad E = \dfrac{1}{4\pi\varepsilon_o} \times \dfrac{q}{r^2}$

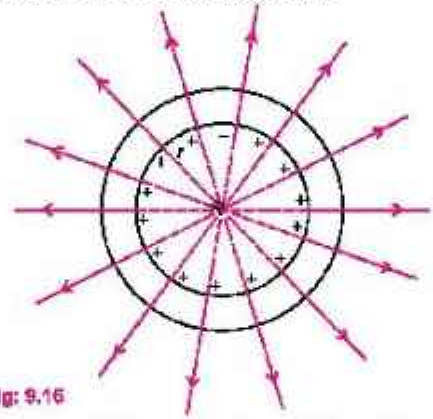


Fig: 9.16

This shows that the field at any point outside the sphere is the same as though the entire charge were concentrated at its centre. Just outside of the sphere, where $r = R$, i.e.,

$$E = \frac{1}{4\pi\varepsilon_o}\frac{q}{R^2} \quad \ldots\ldots\ldots\ldots\ldots(9.18)$$

In vector form; $\quad \mathbf{E} = \dfrac{1}{4\pi\varepsilon_o}\dfrac{q}{R^2}\hat{\mathbf{r}} \qquad$ where $\hat{\mathbf{r}}$ is the direction of $\mathbf{E}$.

9.5 ELECTRIC POTENTIAL

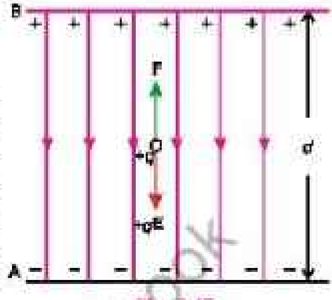


Fig: 9.17

Figure 9.17 shows two oppositely charged parallel plated which produce a uniform electric field.

Let us consider a positive charge q which is allowed to move in this uniform electric field. The positive charge will move from plate B to A and will gain *K.E.* If it is to be moved from A to B, an external force is needed to make the charge move against the electric field and will gain *P.E.* Let us impose a condition that as the charge is moved from A to B, it is moved keeping electrostatic equilibrium, i.e., it moves with uniform velocity. This condition could be achieved by applying a force **F** equal and opposite to qE at every point along its path. The work done by the external force against the electric field increases electrical potential energy of the charge that is moved.

Let W_{AB} be the work done by the force in carrying the positive charge q from A to B while keeping the charge in equilibrium. The change in its potential energy $\Delta U = W_{AB}$.

or $$U_B - U_A = W_{AB} \quad \text{.................................} \quad (9.19)$$

where U_A and U_B, are defined to be the potential energies at points A and B, respectively.

To describe electric field, we introduce the idea of electric potential difference. The **potential difference between two points A and B in an electric field is defined as the work done in carrying a unit positive charge from A to B while keeping the charge in equilibrium**, i.e.,

$$\Delta V = V_B - V_A = \frac{W_{AB}}{q} = \frac{\Delta U}{q} \quad \text{..............} \quad (9.20)$$

where V_A and V_B are defined as electric potentials at points A and B respectively. Electric potential energy difference and electric potential difference between the points A and B are related as:

$$\Delta U = q\Delta V \quad \text{.................................} \quad (9.21)$$

Thus, the potential difference between the two points can be defined as the difference of the potential energy per unit charge.

Do you know?

An ECG records the "voltage" between points on human skin generated by electrical process in the heart. This ECG is made in running position providing information about the heart's performance under stress.

As the unit of *P.E.* is joule, Eq. 9.20 shows that the unit of potential difference is joule per coulomb. It is called volt such that,

$$1\,\text{volt} = \frac{1\,\text{joule}}{1\,\text{coulomb}} \quad \text{.......................} \quad (9.22)$$

That is, a potential difference of 1 volt exists between two points if work done in moving a 1 coulomb positive charge from one point to the other, keeping electrostatic equilibrium, is one joule.

In order to give a concept of electric potential at a point in an electric field, we must have a reference to which we assign zero electric potential. This point is usually taken at infinity. Thus, in Eq. 9.20, if we take point A to be at infinity and choose $V_A=0$, the electric potential at B will be $V_B = \frac{W_{AB}}{q}$. Generally,

Point to ponder!

Why is it advised to wear rubber soled shoes while handling electric appliances?

$$V = \frac{W}{q} \quad \text{.................} \quad (9.23)$$

which states that the electric potential at any point in an electric field is equal to the work done in bringing a unit positive charge from infinity to that point keeping it in electrostatic equilibrium. So, the potential at a point is always relative to potential at infinity. Both potential and potential differences are scalar quantities because both W and q are scalars.

Electric Field as Potential Gradient

In this section, we will establish a relation between electric intensity and potential difference. Let us consider the situation shown in Fig. 9.17. The electric field between the two charged plates is uniform and its value is E. The potential difference between A and B is given by the equation:

$$V_B - V_A = \frac{W_{BA}}{q} \quad \text{...................}(9.24)$$

where $W_{AB} = Fd = -qEd$ (the negative sign is needed because F must be applied opposite to qE so as to keep it in equilibrium). With this, Eq. 9.24 becomes:

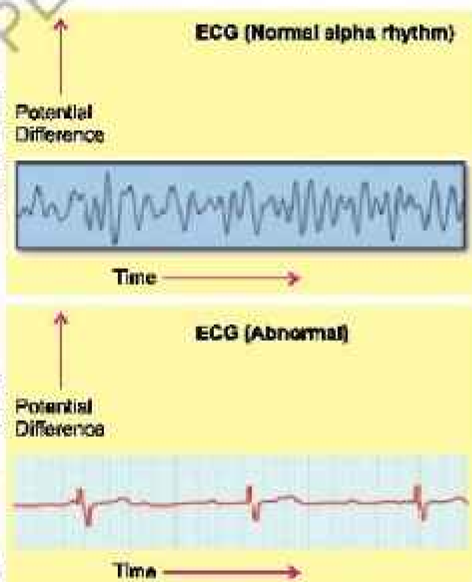
$$V_B - V_A = \frac{-qEd}{q} = -Ed$$

$$\text{or } E = -\frac{(V_B - V_A)}{d} = -\frac{\Delta V}{d} \quad \text{..........}(9.25)$$

If the plates A and B are separated by infinitesimally small distance Δd, the Eq. 9.25 is modified as:

$$E = -\frac{\Delta V}{\Delta d} \quad \text{...........................}(9.26)$$

The quantity $\Delta V / \Delta d$ gives the maximum value of the rate of change of potential with distance because the charge has been moved along a field line in which the



In electroencephalography the potential deference created by the electrical activity of the brain are used for diagnosing abnormal behavior.

distance Δd between the two plates is minimum. It is known as potential gradient. Thus, the electric field intensity is equal to the negative of the gradient of electric potential. The negative sign indicates that the direction of E is along the decreasing potential.

The unit of electric intensity from Eq.9.26 is volt/metre ($V\ m^{-1}$) which is equal to $N\ C^{-1}$ as given below:

$$1\frac{\text{volt}}{\text{metre}} = 1\frac{\text{joule/coulomb}}{\text{metre}} = 1\frac{\text{newton}\times\text{metre}}{\text{metre}\times\text{coulomb}} = 1\frac{\text{newton}}{\text{coulomb}} = 1\ N\ C^{-1}$$

Example 9.4 Two parallel metal plates are 1.0 cm apart. These are connected to a battery of 12 volts. Find the magnitude of electric field intensity between them.

Solution

Here, $\Delta V = 12\ V$, $\Delta d = 1.0\ cm = 1\times10^{-2}\ m$, $E = ?$

Using the equation $E = \frac{\Delta V}{\Delta d}$

Substituting the values,

$$E = \frac{12\ V}{1\times10^{-2}\ m} = 1200\ V\ m^{-1}$$

Example 9.5 Two horizontal parallel metal plates are connected to a 12 volt battery. An electron is released from the negative plate. Determine its velocity as it reaches the positive plate. Mass of electron = 9.1×10^{-31} kg and charge = $q = e = 1.6\times10^{-19}$ C.

Solution

The electron is repelled by the negative plate and attracted by the positive plate. It will be accelerated towards positive plate. Therefore, its *P.E.* will be lost that will be converted into its *K.E.*

Loss of *P.E* = Gain in *K.E*

$$\Delta V \times e = \frac{1}{2}mv^2$$

Submitting the values,

$$12\ V \times 1.6\times10^{-19}\ C = \frac{1}{2}\times 9.1\times10^{-31}\ kg \times v^2$$

$$v^2 = 4.2\times10^{12}\ m^2\ s^{-2}$$

or $$v = 2.1\times10^{6}\ m\ s^{-1}$$

9.6 ELECTRON VOLT

We know that when a particle of charge q moves from point A at potential V_A to a point B at potential V_B keeping electrostatic equilibrium, the change in potential energy ΔU of particle is:

$$\Delta U = q(V_B - V_A) = q\Delta V \qquad (9.27)$$

If no external force acts on the charge to maintain equilibrium, this change in *P.E.* appears in the form of change in *K.E.*

Suppose charge carried by the particle is $q = e = 1.6 \times 10^{-19}$C.

Thus, in this case, the energy acquired by the charge will be:

$$\Delta K.E. = q\Delta V = e\Delta V = (1.6 \times 10^{-19}\text{C})(\Delta V)$$

Moreover, assume that $\Delta V = 1$ volt, hence,

$$\Delta K.E. = q\Delta V = (1.6 \times 10^{-19}\text{C}) \times (1\text{volt})$$

$$\Delta K.E. = (1.6 \times 10^{-19}) \times (\text{C} \times \text{V}) = 1.6 \times 10^{-19}\text{J}$$

The amount of energy equal to 1.6×10^{-19}J is called one electron-volt and is denoted by 1 eV. It is defined as "the amount of energy acquired or lost by an electron as it traverses a potential difference of one volt". Thus,

$$1\text{eV} = 1.6 \times 10^{-19}\text{J} \quad \text{..............................(9.28)}$$

Example 9.6 A particle carrying a charge of $2e$ falls through a potential difference of 3.0 V. Calculate the energy required by it.

Solution $q = 2e$, $\Delta V = 3.0$ V

The energy acquired by the particle is:

$$\Delta U = q\Delta V = (2e)(3.0\text{ V}) = 6.0\text{ eV}$$

$$\Delta U = 6.0 \times 1.6 \times 10^{-19}\text{J} = 9.6 \times 10^{-19}\text{ J}$$

9.7 MOTION OF CHARGED PARTICLES IN A UNIFORM ELECTRIC FIELD

Two oppositely charged parallel metal plates produce uniform electric field between them. The direction of electric field is from positive to negative plate. A positive charge $+q$ placed in the field will move in the direction of electric field whereas a negative charge $-q$ will move opposite to the electric field. The magnitude of electric force acting on a charge q is represented in Fig 9.18 given by

$$F = qE \quad \text{....................(9.29)}$$

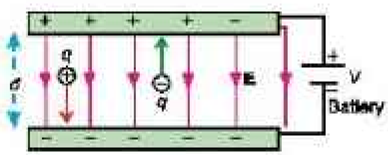


Fig. 9.18

where E is the electric intensity of the uniform electric field. If V is the potential difference between the plates and d is the separation of plates, then

$$E = \frac{V}{d} \quad \text{......................(9.30)}$$

To understand the effect of uniform electric field on the motion of charged particles, let us consider an electron placed between the two plates. The electron accelerates towards the positive plate due to a force F acting on it.

For example, let $V = 20\,V$, $d = 2.0\,cm = 2\times10^{-2}\,m$, the magnitude of E will be:

$$E = \frac{V}{d} = \frac{20\,V}{2\times10^{-2}\,m^{-1}} = 1000\,V\,C^{-1}$$

The acceleration for the electron will be given by

$$F = ma$$

or $$a = \frac{F}{m} = \frac{qE}{m}$$

The charge on an electron $q = e = 1.6\times10^{-19}\,C$ and mass of electron $m = 9.1\times10^{-31}\,kg$. So,

$$a = \frac{1.6\times10^{-19}\,C\times1000\,V\,C^{-1}}{9.1\times10^{-31}\,kg} = 1.76\times10^{14}\,m\,s^{-2}$$

If the electron is released from the negative plate, the velocity gained by it when it reaches positive plate can be found by the third equation of motion.

$$2aS = v_f^2 - v_i^2$$

Here $S = d = 2\times10^{-2}\,m$, $v_i = 0$, $v_f = v = ?$

Putting the values in the above equation

$$2\times1.76\times10^{14}\,m\,s^{-2}\times2\times10^{-2}\,m = v^2$$

or $$v^2 = 7.04\times10^{12}\,m^2\,s^{-2}$$

or $$v = 2.65\times10^{6}\,m\,s^{-1}$$

9.8 PATH OF A CHARGED PARTICLE

The path of a charged particle is determined by the electric field in the region. The path is typically straight if the field is uniform and the charged particle is moving along the field. However, if a charged particle enters perpendicularly to the uniform field between the oppositely charged parallel plates with a certain velocity as shown in Fig. 9.19, it will not go straight. Its path will be parabolic just like a projectile thrown horizontally in the gravitational field. The horizontal component of the velocity of the charged particle remains constant whereas vertical component is accelerated due to the electric force.

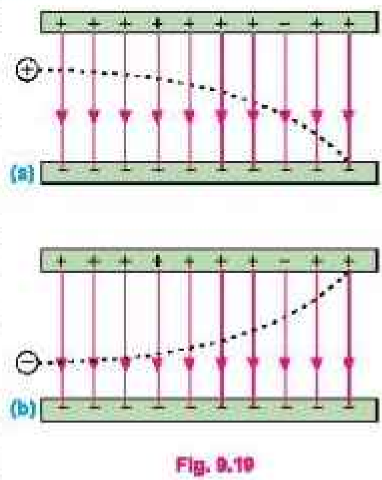


Fig. 9.19

Figure 9.19 shows that a positivity charged particle is attracted towards the negatively charged plate and thus undergoes deflection in that direction.

On the other hand, a negatively charged particle is attracted towards the positively charged plate and experiences deflection in that direction.

9.9 SHIELDING FROM EXTERNAL ELECTRIC FIELD

An English scientist Michael Faraday invented a structure in 1836, called Faraday cage or Faraday shield. Faraday cage is an enclosure that blocks the external electric fields in conductive materials. It acts like a hollow conductor where devices or objects can be put for protection from electrical external fields. Any electrical shock received by the cage runs through its outer surface without causing any harm inside. The electric field inside the hollow conductor remains zero.

To understand the working of Faraday cage, suppose that a piece of conductor (say copper) carries a number of free electrons. Each electron will experience a force of repulsion because of the electric field of its neighbouring electrons. As a consequence, all the electrons rush to the surface of the conductor. Once static equilibrium is established with all of the excess charges on the surface, no further movement of charge occurs. If some electrons shift from the conductor to another object due to friction etc., a net positive charge appears on the surface of the conductor. We can say,

> At equilibrium under electrostatic conditions, any excess charge resides on the surface of a conductor.

Now consider the interior of the hollow conductor. The excess charges arrange themselves on the conductor's surface precisely in the manner that the total field within the interior becomes zero. In other words,

> The conductor shields any charge within it from electric fields outside the conductor.

To eliminate the interference of external fields, circuits are often enclosed within metal boxes that provide shielding from such fields.

Figure 9.20 shows another aspect of how conductors alter the electric field lines created by external charges.

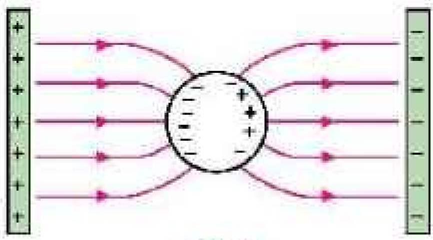


Fig. 9.20

The lines are altered because the electric field just outside the surface of a conductor is perpendicular to the surface at equilibrium under electrostatic conditions. If the field were not perpendicular, there would be a component of the field parallel to the surface. Since the free electrons on the surface of the conductor can move, they would do so under the force exerted by the parallel component. But in reality, no electron flow occurs at equilibrium. Therefore, there can be no parallel component, and the electric field is perpendicular to the surface.

The principle of Faraday cage demands a material that contains a lot of free electrons that can move freely to the surface of the material. Only the conductors have free electrons whereas insulators do not contain free electron, so the insulators can not be used to construct Faraday cage.

A good example of Faraday cage in our daily life is that of cars. The chassis and bodies of cars protect people inside due to its metal framed structure during the thunderstorms. The electrical charge travels over the metal surface of the vehicle into the ground and prevent the passengers inside.

A metal body of the microwave oven acts as a Faraday cage. Thus, they prevent the microwaves in an oven from expanding into the environment. Metal frame of an airplane also acts as a Faraday cage. When lightning strikes an airplane, electricity is distributed along its metal frame surface that keeps passengers and all devices inside the airplane safe.

9.10 ELECTRIC CURRENT

Usually, it is said that electric current is the flow of charge. Let us see what actually flows in a conductor. The charge carriers are the free electrons. When the ends of a conductor are connected to a battery or some other source of potential, an electric field is set up at every point within the conductor. The free electrons experience a force in the direction of –E and they start moving. As the free electrons are bumping among the atoms, so they are not accelerated in a straight line under this force. They keep on colliding with the atoms of the conductor. The overall effect of these collisions is to transfer the energy of accelerated eletrons to the lattice with the result that the electrons acquire an average velocity, called the drift velocity in the direction of –E. The drift velocity is of the order of 10^{-3} m s^{-1}. This drift velocity of electrons forms the electric current. The slow drift velocity does not mean that it takes long time for an electric current to set up. We know that as soon as we switch ON a bulb, it lit up immediately.

The reason is that on turning the switch ON, all the free electrons in the circuit start drifting. They repel the neighbouring ones and the disturbance propagates along the wire almost instantaneously. That is why, the electric current is set up very rapidly.

If a net charge Q passes through any cross-section of a conductor in time t, the current I flowing through it is:

$$I = \frac{Q}{t} \quad \text{.............. (9.31)}$$

The SI unit of current is ampere (A) and it is the current due to flow of one coulomb charge per second through any cross-section of a conductor. If the charges move around a circuit in the same direction at all times, the current is said to be direct current (D.C). For example, batteries produce direct current. If the charges move first one way and then the opposite way, changing direction in regular intervals, the current is said to be alternating current (A.C). Mostly the electric generators produce A.C. The electricity supplied to our homes, offices, factories etc., by power stations is an A.C.

Conventional Current

As we now understand, the electric current is due to flow of electrons through the metal wires, but early scientists believed that electric current was due to flow of positive charges. The scientists have kept the convention and take the direction of current flow to be the direction in which positive charges would move. We call it conventional current.

Conventional current is hypothetical flow of positive charges that would have the same effect in the circuit as the flow of negative charges that actually does occur.

In Fig. 9.21, negative electrons arrive at the positive terminal of the battery. The same effect would have been achieved if an equivalent amount of positive charge has left the positive terminal. Therefore, we can say that the conventional current flows from positive terminal towards the negative terminal of a battery. A conventional current is consistent with our earlier use of a positive test charge for defining electric fields and potential. The direction of conventional current is always from a point of higher potential towards a point of lower potential that is from the positive terminal towards the negative terminal. Now onward the current I always means the conventional current.

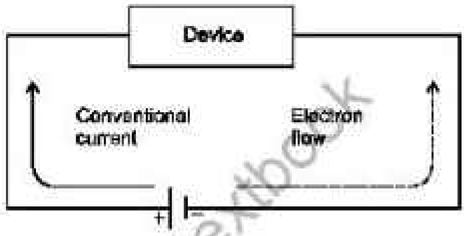


Fig. 9.21

9.11 CURRENT THROUGH A CONDUCTOR

Consider a segment of the current carrying conductor having length L and area of cross-section A. The volume of the segment is AL, as represented in Fig. 9.22. Let n be the number of charge carriers per unit volume, then total number of charge carriers in the segment at any time are nAL. If the charge on a charge carrier is q, the total change present inside the segment at any instant is:

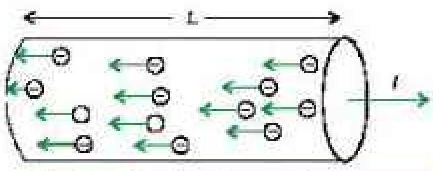


Fig. 9.22: Negative charge carriers

$$Q = nALq \quad \ldots\ldots\ldots\ldots\ldots\ldots \; (9.32)$$

Usually, the charge carriers in a conductor are free electrons which have negative charge.

Suppose that charge carriers move towards left face of the segment when a potential difference is applied across the conductor. Then electric current is set up in the conductor directed towards right face. Assuming that drift velocity of the charge carries to be v, the time taken t by all the charge carriers originally present in the

Do you know?

- Flow of current is directly proportional to the potential difference.
- Flow of heat is directly proportional to the temperature difference.
- Flow of fluid is directly proportional to the pressure difference.

For your information

Current is a flow of charge, pressured into motion by voltage and hampered by resistance.

segment to exit through the left face will be:

$$t = \frac{L}{v}$$

By definition of the current

$$I = \frac{Q}{t}$$

Putting the value of Q and t in the above equation, we have

$$I = \frac{nALq}{\frac{L}{v}}$$

or $$I = nAvq \quad \ldots\ldots\ldots\ldots\ldots\ldots (9.33)$$

Example 9.7 A copper wire has a cross-sectional area of $2 \times 10^{-6}\ m^2$ and carries a current of 3 A. If the number of elections per unit volume is $8.5 \times 10^{28}\ m^{-3}$, calculate the drift velocity of the electrons in the wire. Charge on an electron is 1.6×10^{-19} C.

Solution

$I = 3$ A, $A = 2 \times 10^{-6}\ m^2$, $n = 8.5 \times 10^{28}\ m^{-3}$, $q = 1.6 \times 10^{-19}$ C, $v = ?$

Using equation,

$$I = Anvq$$

$$3 = (2 \times 10^{-6}\ m^2)(8.5 \times 10^{28}\ m^{-3})v(1.6 \times 10^{-19}\ C)$$

$$v = 1.1 \times 10^{-4}\ m\,s^{-1}$$

9.12 OHM'S LAW

When a potential difference V is applied across the ends of a conductor, a current I starts flowing through it. The Ohm's law states that:

> The current flowing through a conductor is directly proportional to the potential difference applied across the conductor, provided there is no charge in the physical state of the conductor.

Mathematically,

$$I \propto V \quad \text{or} \quad V \propto I$$

or $$V = IR \quad \ldots\ldots\ldots\ldots\ldots\ldots (9.34)$$

where R is a constant of proportionality known as resistance of the conductor. The SI unit of resistance is ohm denoted by the Greek capital letter omega (Ω), and is defined as:

> The resistance of a conductor is 1 ohm if a current of 1 ampere flows through it when a potential difference of 1 volt is applied across its ends.

9.13 RESISTIVITY AND ITS DEPENDENCE UPON TEMPERATURE

It has been experimentally seen that the resistance R of a wire is directly proportional to its length L and inversely proportional to its cross sectional area A. Mathematically,

$$R \propto \frac{L}{A}$$

or $$R = \rho \frac{L}{A} \quad \text{..............} \quad (9.35)$$

where ρ (rho) is a constant of proportionality known as resistivity or specific resistance of the material of the wire. It may be noted that resistance is the characteristic of a particular wire whereas the resistivity is the property of the material of which the wire is made. From Eq. 9.35, we have

$$\rho = \frac{RA}{L} \quad \text{..............} \quad (9.36)$$

The above equation gives the definition of resistivity as **the resistance of a metre cube of a material.** The SI unit of resistivity is ohm-metre (Ω m)

Conductance is another quantity used to describe the electrical properties of materials. In fact, conductance is the reciprocal of resistance, i.e.,

$$\text{Conductance} = \frac{1}{\text{Resistance}}, \quad G = \frac{1}{R}, \text{ G is the conductance.}$$

Mathematically, conductivity, σ (sigma) is the reciprocal of resistivity (ρ), i.e.

$$\sigma = \frac{1}{\rho} \text{..........................} (9.37)$$

The SI unit of conductivity is $ohm^{-1}m^{-1}$ or mho m^{-1}. Resistivity of various materials is given in Table 9.2. It may be noted from Table 9.2 that silver and copper are two best conductors. That is the reason, most of electric wires are made of copper.

The resistivity of a substance depends upon the temperature also. It can be explained by recalling that the resistance offered by a conductor to the flow of electric current is due to collisions, which the free electrons encounter with atoms of the lattice. As the temperature of the conductor rises, the amplitude of vibration of the atoms in the lattice increases and hence, the probability of their collision with free electrons also increases. One may say that the atoms then offer a bigger target, that is the collision cross-section of the atoms increases with temperature. This makes the collisions between free electrons and the atoms in the lattice more frequent and hence, the resistance of the conductor increases.

Table 9.2

Substance	ρ(Ω m)	α(K^{-1})
Silver	1.52×10^{-8}	0.00380
Copper	1.69×10^{-8}	0.00390
Gold	2.27×10^{-8}	0.00340
Aluminum	2.63×10^{-8}	0.00390
Tungsten	5.00×10^{-8}	0.00460
Iron	11.00×10^{-8}	0.00520
Platinum	11.00×10^{-8}	0.00520
Constanton	49.00×10^{-8}	0.00001
Mercury	94.00×10^{-8}	0.00091
Nichrome	100.0×10^{-8}	0.00020
Carbon	3.5×10^{-5}	-0.00005
Germanium	0.5	-0.05
Silicon	20-2300	-0.07

Experimentally, the change in resistance of a metallic conductor with temperature is found to be nearly linear over a considerable range of temperature above and below 0°C (Fig.9.23). Over such a range the fractional change in resistance per kelvin is known as the temperature coefficient (α) of resistance, i.e.

$$\alpha = \frac{R_t - R_o}{R_o t} \quad \text{.......................... (9.38)}$$

where R_o and R_t are resistances at temperatures 0°C and t°C respectively. As resistivity ρ depends upon the temperature, Eq. 9.35 gives

$$R_t = \rho_t \frac{L}{A} \quad \text{and} \quad R_o = \rho_o \frac{L}{A} \quad \text{....... (9.39)}$$

Substituting the values of R_t and R_o in Eq. 9.39, we have

$$\alpha = \frac{\rho_t - \rho_o}{\rho_o t} \quad \text{.......................... (9.40)}$$

where ρ_o is the resistivity of a conductor at 0 °C and ρ_t is the resistivity at t°C. Values of temperature coefficients of resistance of some substances are also listed in Table 9.2. There are some substances like germanium, silicon, etc. whose resistance decreases with increase in temperature, these substances have negative temperature coefficients.

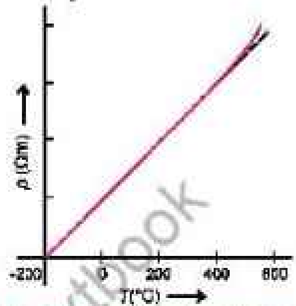


Fig. 9.23: Variation of resistivity of Cu with temperature

For your Information

Inspectors can easily check the reliability of a concrete bridge made with carbon fibers. The fibers conduct electricity. If sensors show that electrical resistance is increasing over time the fibers are separating because of cracks.

Example 9.8 0.75 A current flows through an iron wire when a battery of 1.5 V is connected across its ends. The length of the wire is 5.0 m and its cross-sectional area is $2.5 \times 10^{-7} m^2$. Compute the resistivity of iron.

Solution

The resistance R of the wire can be calculated by Eq. 9.34 i.e.,

$$R = \frac{V}{I} = \frac{1.5V}{0.75A} = 2.0\, V A^{-1} = 2.0\, \Omega$$

The resistivity ρ of iron of which the wire is made of is given by

$$\rho = R\frac{A}{L} = \frac{2.0\,\Omega \times 2.5 \times 10^{-7}\ m^2}{5.0\ m} = 1.0 \times 10^{-7}\, \Omega\, m$$

Example 9.9 A platinum wire has resistance of 10 Ω at 0°C and 20 Ω at 193°C. Find the value of temperature coefficient of resistance of platinum.

Solution $R_o = 10\,\Omega,\ R_t = 20\,\Omega,\ t = 466\,K - 273\,K = 193\,K$

Temperature coefficient of resistance can be found by

$$\alpha = \frac{R_t - R_o}{R_o t} = \frac{20\,\Omega - 10\,\Omega}{10\,\Omega \times 193\ K} = \frac{1}{193\ K} = 5.18 \times 10^{-3}\, K^{-1}$$

9.14 ELECTRICAL POWER

Consider a circuit consisting of a battery of emf ε connected in series with a resistance *R* (Fig. 9.24). A steady current *I* flows through the circuit and a steady potential difference *V* exists between the terminals A and B of the resistor *R*. Terminal A, connected to the positive pole of the battery, is at a higher potential than the terminal B. In this circuit the battery is continuously lifting charge uphill through the potential difference *V*. Using the meaning of potential difference, the work done in moving a charge *Q* up through the potential difference *V* is given by

Work done = $W = V \times Q$ ....(9.41)

This is the energy supplied by the battery.

The rate at which the battery is supplying electrical energy is the power output or electrical power of the battery. Using the definition of power, we have

$$\text{Electrical power} = \frac{\text{Energy supplied}}{\text{Time taken}} = V\frac{Q}{t}$$

Since $I = \frac{Q}{t}$

The above equation can also be written as:

Electrical power = VI ....(9.42)

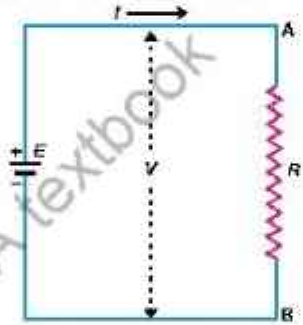


Fig. 9.24: The power of a battery appears as the power dissipated in the resistance *R*.

Equation 9.42 is a general relation for power delivered from a source of current *I* operating on a voltage *V*. In the circuit shown in Fig. 9.24 the power supplied by the battery is expended or dissipated in the resistor *R*. The principle of conservation of energy tells us that the power dissipated in the resistor is also given by Eq. 9.42.

Power dissipated $P = VI$

Alternative equation for calculating power can be found by substituting $V = IR$, $I = V/R$ in turn in Eq. 9.42.

$$P = VI = (IR)I = I^2R$$

or $$P = VI = V(V/R) = V^2/R$$

Thus, we have three equations for calculating the power dissipated in a resistor.

$$P = VI \quad , \quad P = I^2R$$

$$P = V^2/R \quad \text{.......................} \quad (9.43)$$

If *V* is expressed in volts and *I* in amperes, the power is expressed in watts.

9.15 ELECTROMOTIVE FORCE (EMF) AND POTENTIAL DIFFERENCE

We know that a source of electrical energy, say a cell or a battery, when connected across a resistance, maintains a steady current through it Fig. 9.25. The cell continuously supplies energy which is dissipated in the resistance of the circuit. Suppose when a steady current has been established in the circuit, a charge Q passes through any cross-section of the circuit in time t. During the course of motion, this charge enters the cell at its low potential end and leaves at its high potential end. The source must supply energy W to the positive charge to compel it to go to the point of high potential. The emf E of the source is defined as the energy supplied to unit charge by the cell.

Fig. 9.25: Electromotive force of a cell

i.e., $$E = \frac{W}{Q} \quad \text{.................(9.44)}$$

It may be noted that electromotive force is not a force and we do not measure it in newtons. The unit of emf is joule/coulomb which is volt (V). The energy supplied by the cell to the charge carriers is derived from the conversion of chemical energy into electrical energy inside the cell.

Fig. 9.26: An equivalent circuit of a cell of emf E and internal resistance r.

Like other components in a circuit, a cell also offers some resistance. This resistance is due to the electrolyte present between the two electrodes of the cell and is called the internal resistance of the cell. Thus, a cell of emf E having an internal resistance r is equivalent to a source of pure emf E with a resistance r in series as shown in Fig. 9.26.

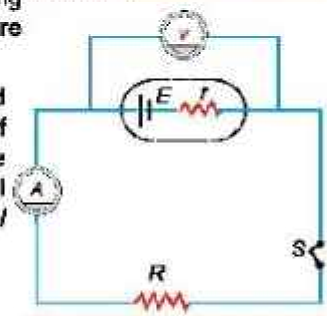


Fig. 9.27: The terminal potential difference V of a cell is $E-Ir$.

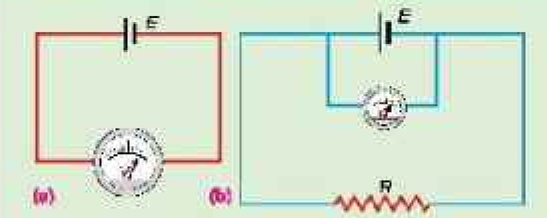
Let us consider the performance of a cell of emf E and internal resistance r as shown in Fig.9.27. A voltmeter of infinite resistance measures the potential difference across the external resistance R or the potential difference V across the terminals of the cell. The current I flowing through the circuit is given by

$$I = \frac{E}{R+r}$$

or $$E = IR + Ir \quad \text{...........(9.45)}$$

Here $IR = V$ is the terminal potential difference of the cell in the presence of current I. When the switch S is open, no current passes through the resistance. In this case, the voltmeter reads the emf E as terminal voltage. Thus, terminal voltage in the presence of the current (switch ON) would be less than the emf E by Ir.

Do you know?



A voltmeter connected across the terminals of a cell measures:
(a) the emf of the cell on open circuit,
(b) the terminal potential difference on a closed circuit.

Let us interpret the Eq. 9.45 on energy considerations. The left side of this equation is the emf *E* of the cell which is equal to energy gained by unit charge as it passes through the cell from its negative to positive terminal. The right side of the equation gives an account of the utilization of this energy as the current passes the circuit. It states that, as a unit charge passes through the circuit, a part of this energy equal to *Ir* is dissipated into the cell and the rest of the energy is dissipated into the external resistance *R*. It is given by potential drop *IR*. Thus, the emf gives the energy supplied to unit charge by the cell and the potential drop across the various elements account for the dissipation of this energy into other forms as the unit charge passes through these elements.

The emf is the "cause" and potential difference is its "effect". The emf is always present even when no current is drawn through the battery or the cell, but the potential difference across the conductor is zero when no current flows through it.

Example 9.10 The potential difference between the terminals of a battery in open circuit is 2.2 V. When it is connected across a resistance of 5.0 Ω, the potential falls to 1.8 V. Calculate the current and the internal resistance of the battery.

Solution

$$E = 2.2\,\text{V}, \qquad R = 5.0\,\Omega, \qquad V = 1.8\,\text{V}$$

We have to calculate *I* and *r*

Using

$$V = IR \quad \text{or} \quad I = \frac{V}{R} = \frac{1.8\,\text{V}}{5.0\,\Omega} = 0.36\,\text{A}$$

Internal resistance *r* can be calculated by using

$$E = V + Ir$$

$$2.2\,\text{V} = 1.8\,\text{V} + 0.36\,\text{A} \times r$$

or

$$r = 1.1\,\Omega$$

9.16 KIRCHHOFF'S RULES

Kirchhoff's rules are two fundamental principles in circuit analysis that help to determine the current and voltage in electrical circuits. They are particularly useful for analysing complex circuits that cannot be simplified by Ohm's law and series or parallel combinations.

Kirchhoff's First Rule

It states that the sum of all the currents meeting at a point in the circuit is zero.

i.e., $\Sigma I = 0$ .... (9.46)

It is a convention that a current flowing towards a point is taken as positive and that flowing away from a point is taken as negative.

For your information

A node is a point in an electric circuit which joins the two or more branches.

Consider a situation where four wires meet at a point A (Fig.9.28). The currents flowing into the point A are I_1 and I_2 and currents flowing away from the point are I_3 and I_4.

According to the convention, currents I_1 and I_2 are positive and currents I_3 and I_4 are negative. Applying Eq. 9.46, we have

$$I_1 + I_2 + (-I_3) + (-I_4) = 0$$

or $$I_1 + I_2 = I_3 + I_4 \quad \text{.....................(9.47)}$$

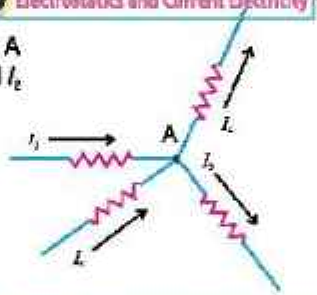


Fig. 9.28: According to Kirchhoff's 1st rule $I_1 + I_2 = I_3 + I_4$

Using Eq.9.47, Kirchhoff's first rule can be stated in other words as:

> The sum of all the currents flowing towards a point is equal to the sum of all the currents flowing away from the point.

Do you know?

The node at which potential is taken as zero is called datum node or reference node.

Kirchhoff's first rule which is also known as Kirchhoff's point rule is a manifestation of law of conservation of charge. If there is no sink or source of charge at a point, the total charge flowing towards the point must be equal to the total charge flowing away from it.

Kirchhoff's Second Rule

It states that the algebraic sum of voltage changes in a closed circuit or a loop must be equal to zero. Consider a closed circuit shown in Fig. 9.29. The direction of the current I flowing through the circuit depends on the cell having the greater emf. Suppose E_1 is greater than E_2, so the current flows in counter clockwise direction. We know that a steady current is equivalent to a continuous flow of positive charges through the circuit. We also know that a voltage change or potential difference is equal to the work done on a unit positive charge or energy gained or lost by it in moving from one point to the other. Thus, when a positive charge Q due to the current I in the closed circuit (Fig.9.29), passes through the cell E_1 from low (-ve) to high potential (+ve), it gains energy because work is done on it. Using Eq. 9.44 the energy gain is E_1Q. When the current passes through the cell E_2, it loses energy equal to $-E_2Q$ because here the charge passes from high to low potential. In going through the resistor R_1, the charge Q loses energy equal to $-IR_1Q$ where IR_1 is potential difference across R_1. The minus sign shows that the charge is passing from high to low potential. Similarly, the loss of energy while passing through the resistor R_2 is $-IR_2$ Q. Finally, the charge reaches the negative terminal of the cell E_1, from where we started. According to the law of conservation of energy, the total change in energy of our system is zero. Therefore, we can write:

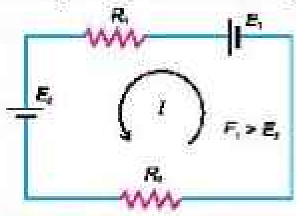


Fig. 9.29: According to Kirchhoff's 2nd rule $E_1 - IR_1 - E_2 - IR_2 = 0$

$$E_1Q - IR_1Q - E_2Q - IR_2Q = 0$$

or $$E_1 - IR_1 - E_2 - IR_2 = 0 \quad \text{.........(9.48)}$$

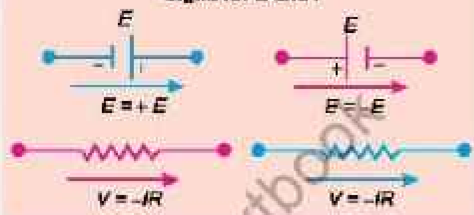
which is Kirchhoff's second rule and it states that:

The algebraic sum of potential changes in a closed circuit is zero.

We have seen that this rule is simply a particular way of stating the law of conservation of energy in electrical problems.

For your information

Signs for E and I

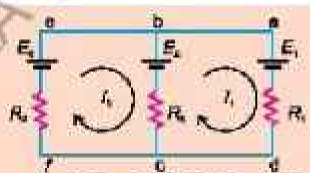


Before applying this rule for the analysis of complex network, it is worthwhile to thoroughly understand the rules for finding the potential changes.

(i) If a source of emf is traversed from positive to negative terminal, the potential change is positive. It is negative in the opposite direction.

(ii) If a resistor is traversed in the direction of current, the change in potential is positive. It is negative in the opposite direction.

Example 9.11 Calculate the currents in the three resistances of the circuit shown in Fig. 9.30.



$R_1 = 10\,\Omega$, $R_2 = 30\,\Omega$, $R_3 = 15\,\Omega$,
$E_1 = 40$ V, $E_2 = 60$ V, $R_3 = 50$V

Fig. 9.30

Solution

First we select two loops abcda and ebcfe. The choice of loops is quite arbitrary, but it should be such that each resistance is included at least once in the selected loops.

After selecting the loops, suppose a current I_1 is flowing in the first loop and I_2 in the second loop, all flowing in the same sense. These currents are called loop currents. The actual currents will be calculated with their help. It should be noted that the sense of the current flowing in all loops should essentially be the same. It may be clockwise or anticlockwise. Here we have assumed it to be clockwise.

We now apply Kirchhoff's second rule to obtain the equations required to calculate the currents through the resistances. We first consider the loop abcda. Starting at point a we follow the loop clockwise. The voltage change while crossing the battery E_1 is $-E_1$ because the current flows through it from positive to negative. The voltage change across R_1 is $-I_1R_1$. The resistance R_2 is common to both the loops I_1 and I_2, therefore, the currents I_1 and I_2 simultaneously flow through it. The directions of currents I_1 and I_2 as flowing through R_2 are opposite, so we have to decide that which of these currents is to be assigned a positive sign. The convention regarding the sign of the current is that if we are applying the Kirchhoff's second rule in the first loop, then the current of this loop i.e., I_1 will be assigned a positive sign and all currents flowing opposite to I_1 have a negative sign. Similarly, while applying Kirchhoff's second rule in the second loop, the current I_2 will be considered as positive and I_1 as negative. Using this convention the current flowing through R_2 is $(I_1 - I_2)$ and the voltage change across is $-(I_1 - I_2)R_2$. The voltage change across the battery E_2 is E_2. Thus, the Kirchhoff's second rule as applied to the loop abcda gives

$$-E_1 - I_1 R_1 - (I_1 - I_2)R_2 + E_2 = 0$$

Substituting the values, we have

$$-40\ \text{V} - I_1 \times 10\ \Omega - (I_1 - I_2) \times 30\ \Omega + 60\ \text{V} = 0$$

$$20\ \text{V} - 10\ \Omega \times [I_1 + 3\,(I_1 - I_2)] = 0$$

or

$$4\,I_1 - 3\,I_2 = 2\ \text{V}\,\Omega^{-1} = 2\ \text{A} \ \ldots\ldots\ldots\ldots\ldots \text{(i)}$$

Similarly, applying Kirchhoff's second rule to the loop ebcfe, we have

$$-E_2 - (I_2 - I_1)\,R_2 - I_2 R_3 + E_3 = 0$$

Substituting the values

$$-60\ \text{V} - (I_2 - I_1) \times 30\ \Omega - I_2 \times 15\ \Omega + 50\ \text{V} = 0$$

$$-10\ \text{V} - 15\ \Omega \times [I_2 + 2\,(I_2 - I_1)] = 0$$

$$6\,I_1 - 9\,I_2 = 2\ \text{V}\,\Omega^{-1} = 2\ \text{A} \ \ldots\ldots\ldots\ldots\ldots \text{(ii)}$$

Solving Eq.(i) and Eq. (ii) for I_1 and I_2, we have

$$I_1 = 0.66\ \text{A and } I_2 = 0.22\ \text{A}$$

Knowing the value of loop currents I_1 and I_2 the actual current flowing through each resistance of the circuit can be determined. Fig. 9.29 shows that I_1 and I_2 are the actual currents through the resistances R_1 and R_3. The actual current through R_2 is the difference of I_1 and I_2 and its direction is along the larger current. Thus,

The current through $R_1 = I_1 = 2/3\ \text{A} = 0.66\ \text{A}$ flowing in the direction of I_1 i.e., from a to d.

The current through $R_2 = I_1 - I_2 = 2/3\ \text{A} - 2/9\ \text{A} = 0.44\ \text{A}$ flowing in the direction of I_1 i.e., from c to b.

The current through $R_3 = I_2 = 2/9\ \text{A} = 0.22\ \text{A}$ flowing in the direction of I_2 i.e., from f to e.

Procedures of Solution of Circuit Problems

After solving the above problem, we are in a position to apply the same procedure to analyse other direct current complex networks. While using Kirchhoff's rules in other problems, it is worthwhile to follow the approach given below:

(i) Draw the circuit diagram.

(ii) The choice of loops should be such that each resistance is included at least once in the selected loops.

(iii) Assume a loop current in each loop. All the loop currents should be in the same sense. It may be either clockwise or anticlockwise.

(iv) Write the loop equations for all the selected loops. For writing each loop equation, the voltage change across any component is positive if traversed from low to high potential and it is negative if traversed from high to low potential.

(v) Solve these equations for the unknown quantities.

9.17 WHEATSTONE BRIDGE

It is an electric circuit. The Wheatstone bridge circuit shown in Fig. 9.31 consists of four resistances R_1, R_2, R_3 and R_4 connected in such a way so as to form a mesh ABCDA. A battery is connected between points A and C. A sensitive galvanometer of resistance R_g is connected between points B and D. If the switch S is closed, a current will flow through the galvanometer. We have to determine the condition under which no current flows through the galvanometer even after the switch is closed. For this purpose, we analyse this circuit using Kirchhoff's second rule. We consider two loops ABDA and BCDB and assume anticlockwise loop currents I_1 and I_2 through the loops respectively. The Kirchhoff's second rule as applied to loop ABDA gives:

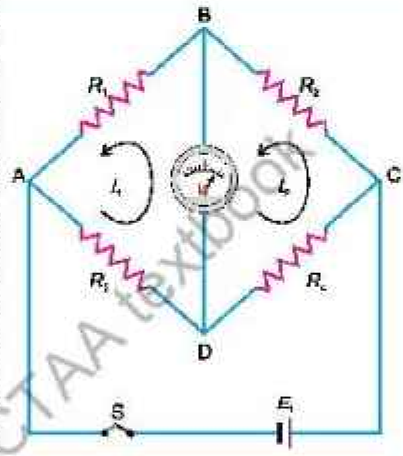


Fig. 9.31: Wheatstone bridge circuit

$$-I_1R_1-(I_1-I_2)R_g-I_1R_3=0 \quad \text{..............(9.49)}$$

Similarly, by applying the Kirchhoff's second rule to loop BCDB, we have

$$-I_2R_2-I_2R_4-(I_2-I_1)R_g=0 \quad \text{............(9.50)}$$

The current flowing through the galvanometer will be zero if, $I_1-I_2=0$ or $I_1=I_2$. With this condition Eq. 9.51 and Eq. 9.52 reduce to:

$$-I_1R_1=I_1R_3 \quad \text{........................(9.51)}$$

and $$-I_2R_2=I_2R_4 \quad \text{........................(9.52)}$$

Dividing Eq. 9.48 by Eq. 9.49, we have

$$\frac{-I_1R_1}{-I_2R_2}=\frac{I_1R_3}{I_2R_4} \quad \text{........................(9.53)}$$

Point to ponder!

Why is a three pin plug used in some electric appliances?

As $I_1=I_2$, therefore,

$$\frac{R_1}{R_2}=\frac{R_3}{R_4} \quad \text{.....................(9.54)}$$

Thus, whenever the condition of Eq. 9.54 is satisfied, no current flows through the galvanometer and it shows no deflection, or conversely when the galvanometer in the Wheatstone bridge circuit shows no deflection, Eq. 9.54 is satisfied. If we connect three resistances R_1, R_2 and R_3 of known adjustable values and a fourth resistance R_4 of unknown value and the resistances R_1, R_2 and R_3 are so adjusted that the galvanometer shows no deflection, then from the known resistances R_1, R_2 and R_3 , the unknown resistance R_4 can be determined by using Eq. 9.54.

9.18 POTENTIOMETER

A potentiometer is mainly used to compare potential differences and to find the value of an unknown resistance. It works on the principle of Wheatstone Bridge.

Working of Potentiometer

Potential difference is usually measured by an instrument called a voltmeter. The voltmeter is connected across the two points in a circuit between which potential difference is to be measured. It is necessary that the resistance of the voltmeter be large compared to the circuit resistance across which the voltmeter is connected. Otherwise, an appreciable current will flow through the voltmeter which will alter the circuit current and the potential difference to be measured. Thus, the voltmeter can read the correct potential difference only when it does not draw any current from the circuit across which it is connected. An ideal voltmeter would have an infinite resistance.

However, there are some potential measuring instruments such as digital voltmeter and cathode-ray oscilloscope which practically do not draw any current from the circuit because of their large resistance and are thus very accurate potential measuring instruments. But these instruments are very expensive. A very simple instrument which can measure and compare potential differences accurately is a potentiometer.

A potentiometer consists of a resistor R in the form of a wire on which a terminal C can slide (Fig. 9.32-a). The resistance between A and C can be varied from 0 to R as the sliding contact C is moved from A to B. If a battery of emf E is connected across R (Fig. 9.32-b) the current flowing through it is $I = E/R$. If we represent the resistance between A and C by r, the potential drop between these points will be $r\,I = r\,E/R$. Thus, as C is moved from A to B, r varies from 0 to R and the potential drop between A and C changes from 0 to E.

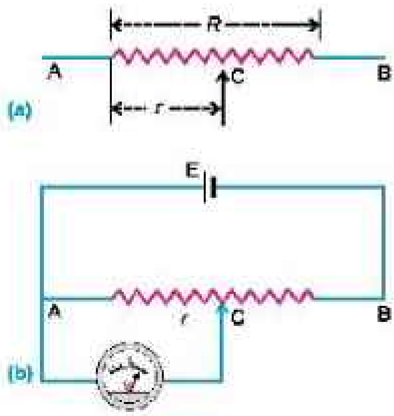


Fig. 9.32

Such an arrangement also known as potential divider can be used to measure the unknown emf of a source by using the circuit shown in Fig. 9.33. Here R is in the form of a straight wire of uniform area of cross-section. A source of potential, say a cell whose emf E_x is to be measured, is connected between A and the sliding contact C through a galvanometer G. It should be noted that the positive terminal of E_x and that of the potential divider are connected to the same point A. If, in the loop AGCA, the point C and the negative terminal of E_x are at the same potential, then the two terminals of the galvanometer will be at the same potential and no current will flow through the galvanometer. Therefore, to measure the potential E_x, the position of C is so adjusted that the galvanometer shows no deflection. Under this condition, the emf E_x of the cell is equal to the potential difference between A and C whose value Er/R is known. In case of a wire of uniform cross-section, the resistance is proportional to the length of the wire. Therefore, the unknown emf is also given by

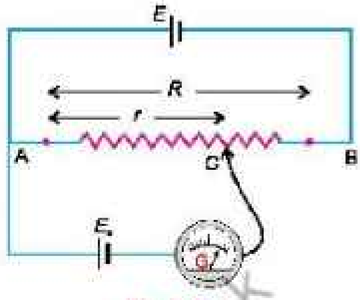


Fig. 9.32

$$E_x = E\frac{r}{R} = E\frac{\ell}{L} \quad \text{.....................(9.55)}$$

where L is the total length of the wire AB and ℓ is its length from A to C, after C has been adjusted for no deflection. As the maximum potential that can be obtained between A and C is E, so the unknown emf E_x should not exceed this value, otherwise the null condition will not be obtained. It can be seen that the unknown emf E_x is determined when no current is drawn from it and therefore, potentiometer is one of the most accurate methods for measuring potential.

The method for measuring the emf of a cell as described above can be used to compare the emfs E_1 and E_2 of two cells. The balancing lengths ℓ_1 and ℓ_2 are found separately for the two cells. Then,

$$E_1 = E\frac{\ell_1}{L} \text{ and } E_2 = E\frac{\ell_2}{L}$$

Dividing these two equations, we have

$$\frac{E_1}{E_2} = \frac{\ell_1}{\ell_2} \quad \text{.....................................(9.56)}$$

So, the ratio of the emfs is equal to ratio of the balancing lengths.

9.19 USE OF A GALVANOMETER

A galvanometer is an instrument for detecting a current. We are not going to discuss its internal structure and how does it work. We focus only on its use. It is often used in null methods to achieve precise measurements in electrical circuits. The null method involves adjusting the circuit until the galvanometer shows no deflection i.e., a zero reading. This indicates that certain required conditions are met in the circuit. In this state, the electric potentials at both ends of the galvanometer are the same. Although a galvanometer has its own resistance, but at the null reading, its resistance does not come into play. The reason is that, in this condition no current is passing through it. The null method is widely used in bridge circuits such as Wheatstone and potentiometer setups.

As we have studied in the previous section, the null method is used to measure an unknown resistance in the Wheatstone bridge circuits. The galvanometer is connected between the mid-points of opposite sides. The variable resistance is adjusted until the galvanometer shows no deflection. At this point, the bridge is balanced and the unknown resistance can be calculated using the ratio of the known resistances.

In a potentiometer, null method is used to measure an unknown voltage by comparison with a known reference voltage applied across the resistance wire of the potentiometer. A galvanometer and a jockey are used to make contact along the wire. At null point, the potential difference between the jockey and the end of the wire equals the unknown voltage. The position of the jockey gives the measure of the unknown voltage.

There are some advantages of using a galvanometer in null method.

1. Null method, eliminates the effect of the galvanometer's internal resistance on the measurement resulting in more accurate readings.
2. Galvanometers are highly sensitive and can detect very small currents of the order of 10^{-9} ampere.
3. "No deflection" indicates a direct and clear condition of balance making it easier to identify the null point.

9.20 THERMISTORS

A thermistor is a heat sensitive resistor. Most thermistors have negative temperature coefficient of resistance, i.e., the resistance of such thermistors decreases when their temperature is increased. Thermistors with positive temperature coefficient are also available.

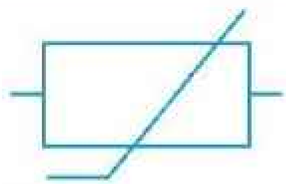


Fig. 9.34: Thermistors symbols

In the thermistors, resistance decreases as temperature increases. This is because increasing temperature provides more energy to the charge carriers (electrons or holes), enabling them to move more freely and thus reducing resistance.

Fig. 9.35: Types of thermistors

Thermistors are made by heating under high pressure semiconductor ceramic made from mixtures of metallic oxides of manganese, nickel, cobalt, copper, iron, etc. These are pressed into desired shapes and then baked at high temperature. Different types of thermistors are shown in Fig. 9.34. They may be in the form of beads, rods or washers.

Applications of Thermistors

Temperature Measurement

Thermistors are used in thermometers, and electric devices such as air conditioners, refrigerators, heaters, microwave ovens, incubators, etc.to monitor temperature.

Thermistors with high negative temperature coefficient are very accurate for measuring low temperatures especially near 10 K. The higher resistance at low temperature enables more accurate measurement possible.

Thermistors have wide applications as temperature sensors, i.e. they convert changes of temperature into electrical voltage which is duly processed. For example, these are used in coolant temperature sensors in automobile engines to prevent the engine overheating and in digital thermometers.

Temperature Compensation

Thermistors are used in circuits where temperature changes could affect performance. Such as in oscillators, battery charging circuits and power systems.

Inrush Current Limiting

Thermistors are used to limit the initial flow of current when a device is first turned on.

Voltage Divider

Thermistors are widely used as voltage divider. As shown in Fig. 9.36 when temperature of a thermistor increases, its resistance decreases. This decreases the voltage drop across the thermistor. As a result, the potential at point B increases that can be used to trigger a circuit connected to it. In case of a fire alarm, the use of a thermistor turns the NOT gate low when it gets heated. The output of NOT gate goes high and turns the siren ON.

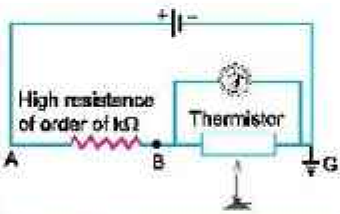


Fig. 9.36

9.21 LIGHT DEPENDENT RESISTOR

Light dependent resistor (LDR) is a resistor whose resistance decreases with increasing light intensity. Due to this property, it is also known as photo resistor. The LDRs are typically made from semiconductor material like cadmium sulphide. The material is deposited in a special pattern on an insulating plate.

Working Principle

The principle used in an LDR is the increase in the conductivity of the material on exposing it to light. In darkness, the semiconductor material has a few free electrons (charge carriers) resulting in high resistance. When light photons hit the material, they transfer energy to electrons in the outer orbits, thus, making them free to conduct electricity. This decreases the resistance of LDR. The amount of light hitting the LDR's surface determines the number of free elections. Conversely, less light results in lowering the free elections, thus, making higher resistance. This change in resistance can be measured and used in circuits to sense light levels.

Applications of LDRs

Light Sensors

LDRs are commonly used in light sensing circuits such as automatic lighting systems in homes and street lights. An LDR works just like a switch that turns ON at dusk and OFF at dawn.

Camera Exposure Control

LDRs help in adjusting the exposure time in cameras based on the amount of available light.

Voltage Divider

In a typical circuit, an LDR can be a part of voltage divider, that converts the resistance change into measurable voltage change. This voltage can then be reed by a micro-controller or other control circuitry to perform actions based on light levels. A circuit is shown in Fig. 9.37 in which an LDR is used as voltage divider. In the dark, the LDR has a very high resistance as compared to the standard resistance (100 kΩ) in the circuit. Therefore the voltage drop across the LDR is very large as registered by a voltmeter. When the LDR is exposed to light, the resistance of LDR decreases to very low. Now, the voltmeter registers a lower reading. Hence, the change in light intensity gives rise to change in voltage. Therefore, by connecting mid-point B to a NOT gate. LDR can be used as a switch.

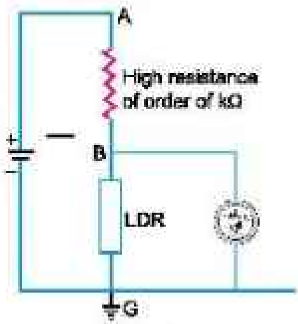


Fig. 9.37

Reliability of A Concrete Bridge

Inspectors can easily check the reliability of concrete bridge with the help of carbon fibers embedded in its slab. This is possible because of the conducting property of the carbon fibers. Let us know step by step how does it work?

1. First step is to know the electrical properties of carbon fibers. Carbon fibers are known to be good conductors of electricity due to their high carbon content.
2. Secondly, we can embed the carbon fibers within the slab of the concrete bridge during its construction.


Fig. 9.38: Carbon fibre sheets

Then we can connect them to form a conductors network.

3. Inspectors can check the reliability of the concrete bridge by applying small electric current to the carbon fiber network. They can determine the integrity of the concrete structure by measuring the resistance of the network.
4. The sensor installed into the network can show whether the electric resistance is changing or not. If the resistance remains the same over time, it indicates that the concrete bridge is maintaining its structural integrity. However, if the resistance increases, it means that the concrete is deteriorating or that the carbon fibers are being damaged.

Some other methods are also used to check the strength of the concrete bridge. For example, a type of sensors continuously monitor strain, vibration and temperature. Internal flaws, such as cracks or voids are detected by using ultrasound waves.

QUESTIONS

Multiple Choice Questions

Tick (✓) the correct answer.

9.1 Two point charges A and B are separated by 10 m. If the distance between them is reduced to 5m, the force exerted on each:

(a) decreases to half its original value

(b) increases to twice the original value

(c) decreases to one quarter of its original value

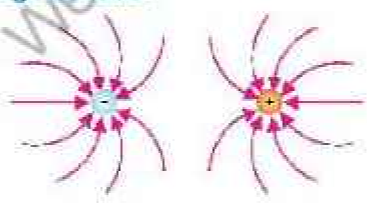
(d) increases four times to its original value

9.2 Which electric charge is possible on a particle?

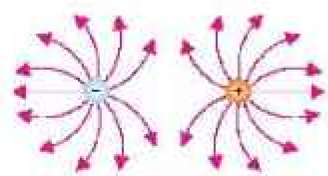
(a) 2.5×10^{-19} C (b) 3.2×10^{-19} C (c) 1.6×10^{-19} C (d) 6.02×10^{23} C

9.3 Which diagram best represents the electric field lines around two oppositely charged particles?

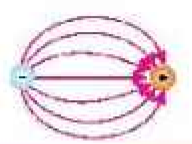
(a)



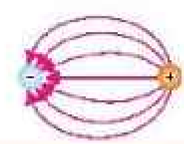
(b)



(c)



(d)



9.4 What is the work done on an election by potential difference of 100 volts?

(a) 1.6×10^{-19} eV (b) 1.6×10^{-17} eV (c) 6.25×10^{-17} eV (d) 100 eV

9.5 The potential at a point situated at a distance of 50 cm from a charge of 50 μC is:

(a) 9×10^{-4} volts (b) 18×10^{-4} volts (c) 9×10^{5} volts (d) 18×10^{4} volts

9.6 A ball of weight 0.1 N having a charge of 100 μC remained suspended between two oppositely charged horizontal metal plates. The electric intensity between the plates is:

(a) 10 $N\,C^{-1}$ (b) 100 $N\,C^{-1}$ (c) 1000 $N\,C^{-1}$ (d) 10000 $N\,C^{-1}$

9.7 A piece of wire has resistance of 4 Ω. It is doubled on itself so that its length becomes half but area of cross-section is doubled. Its resistance now will be:

(a) 8 Ω (b) 4 Ω (c) 2 Ω (d) 1 Ω

9.8 The current through a conductor is 3.0 A when it is attached across a potential difference of 6.0 V. How much power is used?

(a) 0.5 W (b) 2.0 W (c) 9.0 W (d) 18 W

9.9 The algebraic sum of potential changes for a complete circuit is zero. It is the statement of:

(a) Ohm's law (b) Gauss's law

(c) Kirchhoff's first law (d) Kirchhoff's second law

9.10 The radius of curvature of the path of a charged particle in a uniform magnetic field is directly proportional to:

(a) the particle's charge (b) the particle's momentum

(c) the particle's energy (d) the flux density of the field

Short Answer Questions

9.1 How does a moving conductor like an aeroplane acquire charge as it flies through the air? Describe briefly.

9.2 Define electric intensity and electric potential.

9.3 A battery is rated at 100 A h (ampere-hour). How much charge can this battery supply?

9.4 Is electron-volt a unit of potential difference or energy? Explain.

9.5 A copper wire of length L has resistance R. It is stretched to double of its length. What will be the resistance of the new length of wire?

9.6 Why does the resistance of a conductor rise with increase in temperature?

9.7 Is the filament resistance lower or higher in a 500W-220V light bulb than in a 100 W-220V bulb?

9.8 Why does resistance of a thermistor changes as temperature increases?

9.9 Which materials can be used to construct Faraday's cage and why?

Constructed Response Questions

9.1 Electric lines of force never cross each other. Why?

9.2 Is **E** necessarily zero inside a charged rubber balloon if the balloon is spherical? Assume that charge is distributed uniformly over the surface.

9.3 Electrostatic force is 10^{39} times stronger then gravitational force. Argue that our galaxy should be almost electrically neutral.

9.4 An uncharged conducting hollow sphere is placed in the field of a positive charge *q*. What will be the net flux through the shell?

9.5 A potential difference is applied across the ends of a copper wire. What is the effect on the drift velocity of free elections by

(i) increasing the potential difference?

(ii) decreasing the length and the temperature of the wire?

9.6 Why the terminal potential difference of a battery decreases when the current drawn from it is increased?

Comprehensive Questions

9.1 Explain the electric potential and prove that electric field intensity is equal to the negative of potential gradient.

9.2 State and explain Kirchhoff's rules.

9.3 What is a Wheatstone bridge? Explain its working with the help of a diagram.

9.4 What is a light dependent resistor (LDR)? How can this be used as ON-OFF switch for lighting?

9.5 What is a potentiometer? Describe its working.

Numerical Problems

9.1 Two unequal point charges repel each other with a force of 0.4 N when they are 5.0 cm apart. Find the force which each charge exerts on the other when they are (a) 2.5 cm apart (b) 15.0 cm apart. [Ans: (a) 1.6 N (b) 0.04 N]

9.2 A particle of charge +20 μC is placed between two parallel plates, 10 cm apart and having a potential difference of 0.5 kV between them. Calculate the electric field between the plates, and the electric force exerted on the charged particle.

(Ans: 5 kN C^{-1}, 100 mN)

9.3 The electron and proton in a hydrogen atom are separated (on the average) by a distance of approximately 5.3×10^{-11} m. Find the ratio of the electric force and the gravitational force between the electron and proton in this state.

(Ans: $\approx 2.3 \times 10^{39}$)

9.4 After a pleasant showering, a water droplet of mass 1.2×10^{-11} kg is located in the air near the ground. An atmospheric electric field of magnitude 6.0×10^{3} N C^{-1} points vertically downward in the vicinity of the water droplet. The droplet remains suspended at rest in the air. Find the electric charge on the droplet.

(Ans: -1.96×10^{-14} C)

9.5 An electron enters the region of a uniform electric filed, with $v_i = 2.99 \times 10^{6}$ m s^{-1} and $E = 300$ N C^{-1}. The horizontal length of the plates is 10.0 cm. Find the acceleration of the electron while it is in the electric field. How long will it take to pass through the field? (Ans: 5.27×10^{13} m s^{-2}, 3.34×10^{-8} s)

9.6 A disc of 10 cm^2 area is placed in a vertical electric field $E = 5 \times 10^{5}$ N C^{-1}. If the plane of the disc makes an angle of 30° with the horizontal, determine the electric flux through the disc. (Ans: $250\sqrt{3}$ N m^2 C^{-1})

9.7 A circular copper rod is 50 cm long and has 1 cm diameter. Find the resistance across its ends. What should be the side of square cross-section of a 50 cm long tungsten rod if its resistance is the same? [Resistivity of copper is 1.69×10^{-8} Ω m and that of tungsten is 5.0×10^{-8} Ω m.] (Ans: 1.08×10^{-4} Ω, 1.52 cm)

9.8 The copper winding of an electric fan has a resistance of 50 Ω at 30 °C. After running for some time, the resistance becomes 52 Ω. How much is the increase in temperature of the winding? [For copper $\alpha = 0.0039$ K^{-1}] (Ans: 10.3 °C)

9.9 During an experiment, a copper wire of 50 m long and 150 μm thick is hung vertically. Then a current of 1 A is passed across its ends for 50 s. Find the resistance of the wire and the heat dissipated during this process. [Resistivity of copper is 1.69×10^{-8} Ω m.] (Ans: 47.8 Ω, 2390 J)

9.10 The emf of a battery is 12 V. It is connected to a 3.6 Ω resistor. If the internal resistance of the battery is 0.2 Ω, what will be the terminal voltage across the battery? (Ans: 11.4 V)

Chapter 10

Electromagnetism

Learning Objectives

After studying this chapter, the students will be able to:

- State that a force might act on a current-carrying conductor placed in a magnetic field
- Use the equation $F = BIL\sin\theta$ [with directions as interpreted by Fleming's left-hand rule to solve problems]
- Define magnetic flux [as the product of the magnetic flux density and the cross-sectional area perpendicular to the direction of the magnetic flux density]
- Use $\phi = BA$ to solve problems
- Use the concept of magnetic flux linkage
- Define magnetic flux density [as the force acting per unit current per unit length on a wire placed at right angles to the magnetic field]
- Use $F = BqV\sin\theta$ to solve problems
- Describe the motion of a charged particle moving in a uniform magnetic field perpendicular to the direction of motion of the particle
- Explain how electric and magnetic fields can be used in velocity selection
- Explain experiments that demonstrate Faraday's and Lenz's laws [(a) that a changing magnetic flux can induce an emf in a circuit, (b) that the induced emf is in such a direction as to oppose the change producing it, (c) the factors affecting the magnitude of the induced emf.]
- Use Faraday's and Lenz's laws of electromagnetic induction to solve problems
- Describe how ferrofluids work [they make use of temporary soft magnetic materials suspended in liquids to develop fluids that react to the poles of a magnet and have many applications in fields such as electronics]
- Explain how seismometers make use of electromagnetic induction to the earthquake detection [specifically in terms of:
  (i) any movement or vibration of the rock on which the seismometer rests (buried in a protective case) results in relative motion between the magnet and the coil (Suspended by a spring from the frame.)
  (ii) The emf induced in the coil is directly proportional to the displacement associated

We have already studied that a magnetic field is produced around a current-carrying conductor. Also, a changing magnetic field gives rise to a current in a conductor placed in it. Electromagnetism is a key area of physics that studies how electric charges and magnetic fields interact.

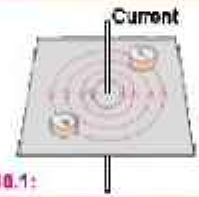


Fig. 10.1: The direction of magnetic field produced by a current

In 1820, Hans Christian Oersted found that electricity and magnetism are correlated.

For your Information

Magnetism started with lodestone, a natural mineral discovered in ancient Türkiye. Lodestone, or magnetite (Fe_3O_4), can attract metals like iron and steel and aligns with the Earth's magnetic poles, leading to the invention of the compass.

Electromagnetism is crucial for modern technology, including phones, computers, and medical devices. In this chapter, we will explore basic concepts like electric fields, magnetic forces, and electromagnetic induction, and see how these principles affect both natural phenomena and technology. Understanding these concepts help us appreciate how electromagnetism influences our world and drives innovation.

10.1 FORCE ON A CURRENT-CARRYING CONDUCTOR IN A UNIFORM MAGNETIC FIELD

It has been observed experimentally that a current-carrying conductor placed in a magnetic field experiences a force. Consider a straight conductor carrying a steady current placed perpendicular to uniform magnetic field. Assume the direction of the current is out of the paper as shown by ⊙ in Fig. 10.2. The direction of magnetic field produced by the current is also shown.

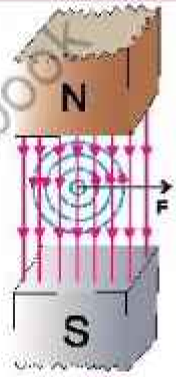


Fig. 10.2

Just as two magnets exert forces on each other through their magnetic fields, a current-carrying conductor experiences a force due to the interaction between its own magnetic field and the external magnetic field. To determine the direction of this force, consider the interaction between the two fields.

The magnetic field produced by the current and the external uniform magnetic field reinforce each other on the left side of the conductor and oppose each other on the right side. Consequently, the conductor moves towards the side where the field is weaker. That is, the force on the conductor is directed to the right as viewed from the front. Thus, the force *F* is perpendicular to both the conductor and the magnetic field. Fleming's left-hand rule is used to predict the direction of the force experienced by a current-carrying conductor in a magnetic field. To apply the rule:

> Position your left hand such that the first finger points in the direction of magnetic field, the second finger points in the direction of current, the thumb will then point in the direction of force.

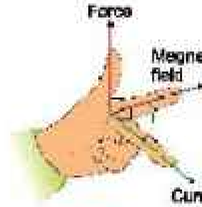


Fig. 10.3

The Fleming's left hand rule is illustrated in Fig. 10.3.

However, direction of force can also be found by using right hand rule that can be stated as:

> Curl fingers of your right hand from current to magnetic field through smaller angle, the stretched thumb will indicate the direction of force.

Let us now determine the magnitude of the force on a current-carrying conductor placed inside a magnetic field. Experimentally, it has been observed that the magnitude of the force acting on the conductor is directly proportional to the current I in the conductor, the length L of the conductor, and the strength of the external magnetic field B. The strength of the magnetic field is also known as the magnetic induction B, which has the same direction as the field. Thus, the force F on a conductor of length L, carrying a current I and placed perpendicular to a magnetic field of strength B, is given by

$$F \propto BIL$$

$$F = kBIL$$

In SI units, the value of k = 1. Therefore,

$$F = BIL \quad \text{..................... (10.1)}$$

From Eq. (10.1) we can see that $B = \frac{F}{IL}$, so we can define B as:

The magnetic strength is numerically equal to the force exerted on a conductor of length one metre carrying one ampere current, placed perpendicular to the magnetic field.

Equation $B = \frac{F}{IL}$ also gives us the unit of B. The SI unit of B is tesla (T).

$$1\,T = 1\,N\,A^{-1}\,m^{-1}$$

It may be noted that magnetic induction is a vector quantity. Its direction is the same as that of magnetic field.

We can also consider a vector **L** which has a magnitude equal to the length of the conductor and its direction is along the flow of current.

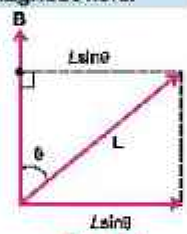


Fig. 10.4

Now consider a conductor **L** placed at an angle 'θ' w.r.t the magnetic field, then we will use the component of L perpendicular to **B** i.e., (L sinθ), as shown in Fig. 10.4. Then the Eq. (10.1) will become,

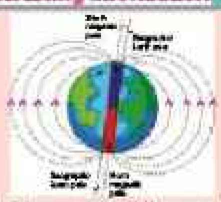
$$F = BIL\sin\theta \quad \text{......................... (10.2)}$$

In the vector form the Eq. 10.2 can be written as

$$\mathbf{F} = I\mathbf{L} \times \mathbf{B} \quad \text{......................... (10.3)}$$

Equation (10.2) shows that the force will be maximum (BIL) when the conductor is perpendicular to the field, i.e., θ = 90°, and it will be zero when the conductor is along the field i.e., θ = 0.

Interesting Information



The Earth's magnetic field is approximately that of a dipole, like that of the fictitious bar magnet, where the south magnetic pole is towards the geographic north pole and the north magnetic pole is towards the geographic south pole.

Example 10.1 A 20.0 cm wire carrying a current of 10.0 A is placed in a uniform magnetic field of 0.30 T. If the wire makes an angle of 40° with the direction of magnetic field, find the magnitude of the force acting on the wire.

Solution Length of the wire $= L = 20.0\,cm = 0.20\,m$

Current $= I = 10.0\,A$

Strength of magnetic field $= B = 0.30\,T$

Angle $= \theta = 40°$

Substituting the values in Eq. (10.2): $F = 10.0\,A \times 0.30\,T \times 0.20\,m \times \sin 40° = 0.39\,N$

10.2 MAGNETIC FLUX AND FLUX DENSITY

We can represent the strength of a magnetic field B by the lines of force in the same way as for electric field. Then, the population of these lines in the field per unit area passing through a surface perpendicular to the field will represent the magnetic flux. Thus,

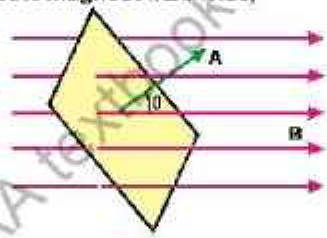


Fig. 10.5

The magnetic flux through a patch of area A is the number of magnetic lines passing through this area.

If B represents the number of lines passing through unit area placed perpendicular to the field, then the total flux through area A perpendicular to the field will be:

$$B = \frac{\phi_B}{A} \quad \text{..........................(10.4)}$$

The surface A may not be perpendicular to the field, that is the normal to the surface makes an angle θ with **B** as shown in Fig. 10.5. Then, we will have to use component of **B** ($B\cos\theta$) along the vector area **A** (Fig. 10.6). Therefore, the flux passing through the surface will be:

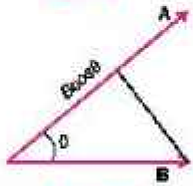


Fig. 10.6

$$\phi_B = BA\cos\theta \quad \text{..................(10.5)}$$

As **B** and **A** both are vectors, so we can write

$$\phi_B = \mathbf{B}.\mathbf{A} \quad \text{.....................(10.6)}$$

Equation (10.6) shows that ϕ_B is a scaler quantity. Therefore, we can define magnetic flux as:

The magnetic flux ϕ_B, through a plane element of area **A** in a uniform magnetic field **B** is given by dot product of **B** and **A**.

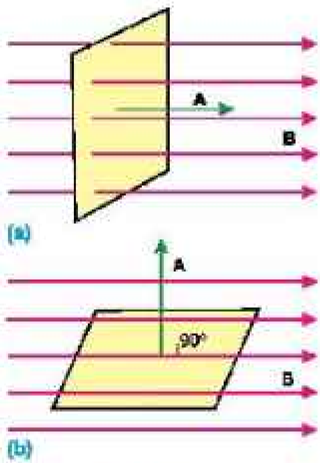


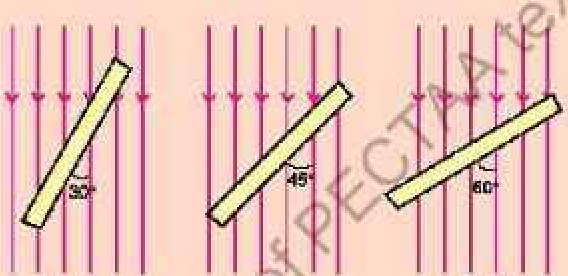
Fig. 10.7

Note that **A** is a vector whose magnitude is the area of the element and whose direction is along the normal to the surface of the element and θ is the angle between the directions of the vectors **B** and **A** (Fig. 10.5). In Fig. 10.7(a), the field is directed along the normal to the area, so θ is zero ($\cos 0° = 1$) and the flux is maximum, equal to BA. When the field is parallel to the plane of the area (Fig. 10.7-b), the angle between the field and normal to area is 90° i.e., $\theta = 90°$ ($\cos 90° = 0$), so the flux through the area in this orientation is zero.

In case of a curved surface placed in a uniform magnetic field, the curved surface is divided into a number of small surface elements, each element being assumed plane and the flux through the whole curved surface is calculated by the sum of the contributions from all the elements of the surface using Eq.(10.5).

From the definition of tesla, the unit of magnetic flux is $N\,m\,A^{-1}$ which is called weber (Wb). According to Eq. 10.5, the magnetic induction **B** is the flux per unit area of a surface perpendicular to **B**, hence, it is also called as magnetic flux density. Its unit is $Wb\,m^{-2}$. Therefore, magnetic induction, i.e. the magnetic field strength is measured in terms of $Wb\,m^{-2}$ or $N\,A^{-1}\,m^{-1}$ (tesla).

Example 10.2 A rectangular loop of wire is placed in a uniform magnetic field of magnitude 1.2 T. If the loop is 25 cm long and 20 cm wide, determine the magnetic flux through the loop for the three orientations as shown in Fig. 10.8.



(a) (b) (c)

Fig. 10.8

Solution For orientation (a), angle between **B** and area vector **A** is $\theta = 60°$

Using $\phi_B = BA\cos\theta$

$$= 1.2\,T \times 20\,cm \times 25\,cm \times \cos 60°$$

$$= 1.2\,T \times 5\times10^{-2}\,m^2 \times 0.5$$

$$= 3\times10^{-2}\,Wb$$

For orientation (b), angle $\theta = 45°$

$$\phi_B = 1.2\,T \times 5\times10^{-2}\,m^2 \times 0.707$$

$$= 4.2\times10^{-2}\,Wb$$

For orientation (c), angle $\theta = 30°$

$$\phi_B = 1.2\,T \times 5\times10^{-2}\,m^2 \times 0.866$$

$$= 5.2\times10^{-2}\,Wb$$

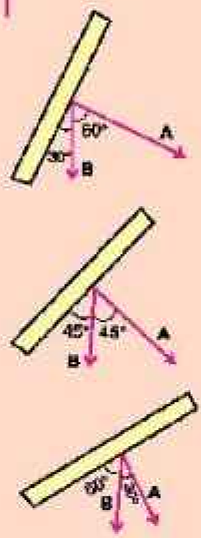


Fig. 10.9

10.3 MAGNETIC FLUX LINKAGE

Magnetic flux linkage is a key concept in electromagnetism, particularly in the study of inductance and electromagnetic induction. Magnetic flux linkage refers to the product of the magnetic flux through a coil and the number of turns in the coil. It essentially measures how much magnetic flux is linked with the coil due to its multiple turns, and is an important factor in understanding how coils and inductors operate in electrical circuits.

Magnetic flux linkage = $\phi = N\phi_s$ .... (10.7)

where ϕ_s is the magnetic flux through a single loop of area A and N is the total number of turns of the coil. Magnetic flux linkage plays a crucial role in the design and operation of transformers, electric motors, generators, and inductors. This concept is particularly important in Faraday's law of electromagnetic induction, which we will explore later in this chapter.

For your information

A magnetic strip on the ATM card contains millions of tiny magnetic domains held together by a resin binder. The machine reads the information encoded on the magnetic strip and it makes your access to your account.

10.4 MOTION OF A CHARGED PARTICLE IN A MAGNETIC FIELD

We have observed that a current-carrying conductor placed perpendicularly in a uniform magnetic field experiences a force **F**. Since a current is the flow of electric charges, it raises the question: do individual charges moving through a magnetic field also experience a force? The answer is yes. Experiments show that a charged particle does experience a force when it moves across a magnetic field. We can calculate this force by examining the behaviour of a current-carrying conductor in a magnetic field.

Consider a conductor of length L through which N charged particles, each with charge q, are passing in time t. The motion of these charged particles produces a current I in the conductor, which is given by

Do you know?

Like electric field lines, magnetic field lines also never cross each other. However, they can attract or repel each other.

$$I = \frac{Q}{t} = \frac{Nq}{t}$$

where Q is the total charge flowing in time t. If **v** is the velocity of charged particles, then the velocity of the particles along the conductor is

$$\mathbf{v} = v\hat{\mathbf{L}}$$

where $\hat{\mathbf{L}}$ is the unit vector in the direction of the current. The sign of the force depends on whether the charge q is positive or negative. However, the unit vector $\hat{\mathbf{L}}$ is directed along the direction of the current, which is the direction of motion of positive charges.

Since the particles take time t to move across the conductor of length L, therefore,

$$t = \frac{L}{v}$$

Then $I = \frac{Nq}{t} = \frac{Nqv}{L}$ ....(10.8)

Point to ponder!

Why does a picture become distorted when a magnetic bar is brought near to the screen of TV, computer monitor or oscilloscope?

If this conductor is placed in a uniform magnetic field **B**, it will experience a force **F**, as given by Eq.10.3.

$\mathbf{F} = I\mathbf{L} \times \mathbf{B}$ ....(10.9)

As $\mathbf{L} = L\hat{\mathbf{L}}$ ....(10.10)

Substituting the values of I and L in Eq. (10.3), we have

$$\mathbf{F} = \frac{Nqv}{L}(L\hat{\mathbf{L}} \times \mathbf{B})$$

or $\mathbf{F} = Nqv\hat{\mathbf{L}} \times \mathbf{B}$..........................(10.11)

As velocity **v** is directed along the conductor **L**, so we can write: $v\hat{\mathbf{L}} = v\hat{\mathbf{v}} = \mathbf{v}$ (as $\hat{\mathbf{L}} = \hat{\mathbf{v}}$)

$\mathbf{F} = Nq(\mathbf{v} \times \mathbf{B})$ ....(10.12)

Therefore the force on a single particle will be

$\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$ ....(10.13)

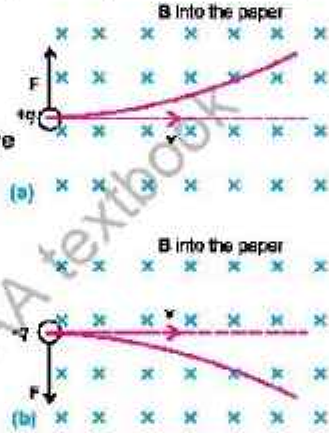


Fig. 10.10

If θ is the angle between **B** and **v**, then magnitude of force F is given by

$F = qvB \sin\theta$ ....(10.14)

Therefore, the force is maximum when **B** is perpendicular to **v**, i.e., $\theta = 90°$, and force is zero when **B** is in the direction of **v**, i.e., $\theta = 0$. The direction of force can be known by applying Fleming's left hand rule or right hand rule for vector product.

(a) The positively charged particle enters into the magnetic field along the dotted line on plane of paper. It experiences a force in the upward direction due to which it is deflected along a curved path (Fig. 10.10-a).

(b) The negatively charged particle is deflected downward by the force acting on it downwards (Fig. 10.10-b).

Example 10.3 An electron enters into a uniform magnetic field perpendicularly with a speed of 10^4 m s^{-1}. What path the electron will move along inside the field?

($B = 2.5$ Wb m^{-2}, $m = 9.11 \times 10^{-31}$ kg, $e = 1.6 \times 10^{-19}$ C)

Solution The Force F acting on the electron will be:

$$F = qvB \sin\theta$$

As the velocity **v** is perpendicular to **B**, i.e., $\theta = 90°$, the charge $q = e$, so

$$F = evB \times 1 = evB$$

Since F acts perpendicular to **v**, so this force provides centripetal force to the electron to keep it in a circle of radius *r*. Then

$$evB = \frac{mv^2}{r}$$

$$r = \frac{mv}{eB}$$

Putting the values in above equation, we have

$$r = \frac{9.11 \times 10^{-31}\ \text{kg} \times 10^{4}\ \text{m s}^{-1}}{1.6 \times 10^{-19}\ \text{C} \times 2.5\ \text{Wb m}^{-2}}$$

$$r = 2.3 \times 10^{-8}\ \text{m}$$

The path of electron will be a circle of radius 2.3×10^{-8} m.

10.5 VELOCITY SELECTOR

A velocity selector is a device used to determine the velocity of a charged particle. In this device, electric and magnetic forces are applied to the moving particle in such a way that they balance each other only for one value of velocity, allowing the particle to continue moving with a constant velocity.

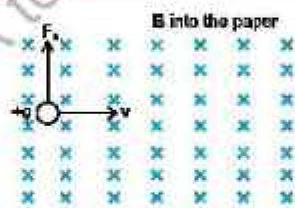


Fig. 10.11

Consider a particle with a positive charge +*q* that enters a uniform magnetic field **B** at a right angles to it, with a velocity **v**. The magnetic force acts on the particle in the upward direction, as shown in Fig.10.11. To balance this magnetic force, an electric force must act downward on the particle.

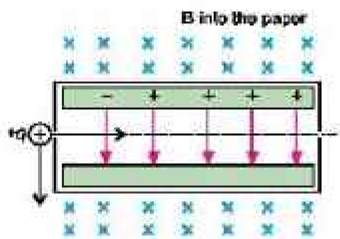


Fig. 10.12

A velocity selector consists of a cylindrical tube located within a magnetic field **B**. Inside the tube is a parallel plate capacitor that creates a uniform electric field **E**. The electric field **E** is oriented perpendicular to the magnetic field **B**, as shown in Fig. 10.12.

When the charged particle enters the left end of the tube, the magnetic force acts upward, while the electric force acts downward in the direction of the electric field **E** on the positively charged particle. If the strengths of the electric and magnetic fields are adjusted appropriately, these forces will cancel each other out. With no net force acting on the particle, its velocity **v** remains constant in accordance with Newton's first law. As a result, the particle moves in a straight line at a constant velocity and exits the right end of the tube.

The particles with velocities different from **v** will be deflected and will not exit at the right end of the tube.

The magnitude of the velocity selected can be determined as below.

As the velocity **v** is perpendicular to both **B** and **E**, therefore,

Magnetic force (upward) $= Bqv$

Electric force (downward) $= qE$

For no deflection of particle, $Bqv = qE$

or $$v = \frac{E}{B} \quad \text{.............................. (10.15)}$$

Example 10.4 Alpha particles ranging in speed from 1000 m s^{-1} to 2000 m s^{-1} enter into a velocity selector where the electric intensity is 300 V m^{-1} and the magnetic induction 0.20 T. Which particle will move undeviated through the field?

Solution $E = 300\ V\ m^{-1} = 300\ N\ C^{-1}$, $B = 0.20\ T$

Only those particles will be able to pass through the plate for which the electric force qE acting on the particles balances the magnetic force Bqv on the particles as shown in the Fig. 10.12.

Therefore $qE = Bqv$

Thus, the selected speed is:

$$v = \frac{E}{B} = \frac{300\ N\ C^{-1}}{0.20\ N\ A^{-1}\ m^{-1}} = 1500\ m\ s^{-1}$$

Point to ponder!

A force is exerted on a moving charged particle in a magnetic field. In what direction it should move that the force is not exerted on it?

The alpha particles having a speed of 1500 m s^{-1} will move undeviated through the field.

Example 10.5 A charged particle moves through a velocity selector at a constant velocity in a straight line. The electric field of the velocity selector is 4.8×10^3 N C^{-1}, while the magnetic field is 0.2 T. When the electric filed is turned OFF, the charged particle travels on a circular path of radius 3.0 cm. Find the charge to mass ratio of the particle.

Solution Since the particle is moving in a direction perpendicular to both **E** and **B**, so the magnitude of velocity **v** will be given by

$$qE = Bqv \quad \text{or} \quad v = \frac{E}{B}$$

When the electric filed is turned off, the particle will move along a circular path of radius r. Then the magnetic force provides necessary centripetal force. Then

$$Bqv = \frac{mv^2}{r}$$

So, charge to mass ratio becomes:

$$\frac{q}{m} = \frac{v}{Br}$$

Putting the value of $v = \frac{E}{B}$, we have

$$\frac{q}{m} = \frac{E}{B^2 r}$$

Putting the values of E, B and r, we have

$$\frac{q}{m} = \frac{4.8\times10^{3}\,\text{N C}^{-1}}{(0.20\,\text{T})^{2}(3\times10^{-2}\,\text{m})}$$

$$= \frac{4.8\times10^{3}\,\text{N C}^{-1}}{1.2\times10^{-3}\,\text{m}} = 4\times10^{6}\,\text{C kg}^{-1}$$

10.6 INDUCED EMF AND FARADAY'S LAW

It has been observed experimentally that when a conductor moves across a magnetic field, an electromotive force (emf) is induced between its ends. The induced emf in the moving conductor is similar to that of a battery. That is, if the ends of the conductor are connected by a wire to form a closed circuit, a current will flow through it.

The emf induced by the motion of a conductor across a magnetic field is called motional emf.

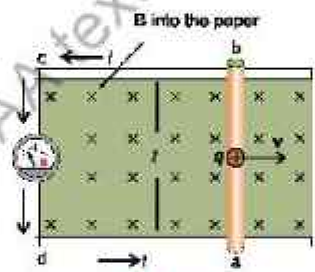


Fig. 10.13

Consider an experiment as shown in Fig. 10.13. A conducting rod of length L is placed on two parallel metal rails separated by a distance L. A galvanometer is connected between the ends c and d of the rails. This forms a complete conducting loop abcda. A uniform magnetic field **B** is applied directed into the page. Initially, when the rod is stationary, galvanometer indicates no current in the loop. If the rod is pulled to the right with constant velocity **v**, the galvanometer indicates a current flowing through the loop. Obviously, the current is induced due to the motion of the conducting rod across the magnetic field. The moving rod is acting as a source of emf $\varepsilon = V_b - V_a = \Delta V$.

When the rod moves, a charge q within the rod also moves with the same velocity **v** in the magnetic field **B** and experiences a force given by, Eq. 10.13.

$$\mathbf{F} = q\,(\mathbf{v}\times\mathbf{B})$$

The magnitude of the force is:

$$F = q\,v\,B\sin\theta$$

Since angle θ between v and B is 90°, so

$$F = qvB \quad \ldots\ldots\ldots\ldots (10.16)$$

Applying right hand rule, we see that the force **F** acting on the charge q is directed from point a to point b along the rod. As a result, charges migrate to the top end of the conductor. As more charges move, a concentration of charge builds up at the top end b, while there is a deficiency of charges at the bottom end a. This redistribution of charge creates an electrostatic field E directed from b to a. The electrostatic force on the charge is $F_e = qE$ directed from b to a. The system quickly reaches an equilibrium state where these two forces on the charge are balanced. If E is the electric field intensity in this state, then

$$qE = qvB$$

or $$E = vB \quad \text{.................... (10.17)}$$

The motional emf ε will be equal to the potential difference $\Delta V = V_+ - V_-$ between the two ends of the moving conductor in this equilibrium state. The gradient of potential is given by $\Delta V/L$. As the electric intensity is given by the negative of the gradient, therefore,

$$E = -\frac{\Delta V}{L} \quad \text{..............(10.18)}$$

or $$\Delta V = -LE = -(LvB)$$

The motional emf is:

$$\varepsilon = \Delta V = LvB \quad \text{................(10.19)}$$

This is the magnitude of motional emf. However, if the angle between **v** and **B** is θ, then

$$\varepsilon = -vBL\sin\theta \quad \text{................. (10.20)}$$

Due to the induced emf, positive charges flow along the path abcda; therefore, the induced current is anticlockwise in the diagram. As the current flows, the quantity of charge at the top decreases, which reduces the electric field intensity, while the magnetic force remains unchanged. This imbalance disturbs the equilibrium in favour of the magnetic force. Consequently, as the charges reach the end 'a' of the conductor due to the current flow, they are carried back to the top end 'b' by the unbalanced magnetic field, and the current continues to flow.

Do you know?

Wireless charging works under the principle of electromagnetic induction.

Point to ponder!

This heater operates on the principle of electromagnetic induction. The water in the metal pot is boiling whereas that in the glass pot is not. Even the glass top of the heater is cool to touch. The coil just beneath the top carries AC that produces changing magnetic flux. Flux linking with pots induces emf in them. Current is generated in the metal pot that heats up the water, but no current flows through the glass pan, why?

Faraday's Law

The motional emf induced in a rod moving perpendicular to a magnetic field is $\varepsilon = -vBL$. The motional emf as well as other induced emfs can be described in terms of magnetic flux. Consider the experiment shown in Fig. 10.14 again. Let the conducting rod be moving from position 1 to position 2 in a

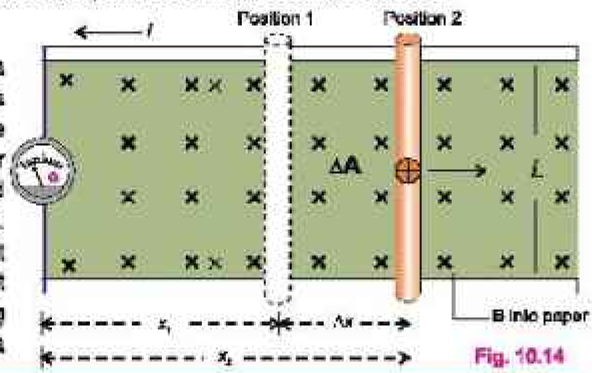


Fig. 10.14

small interval of time Δt. The distance travelled by the rod in time Δt is $x_2 - x_1 = \Delta x$.

Since the rod is moving with constant velocity **v**, therefore,

$$v = \frac{\Delta x}{\Delta t} \quad \text{.....................(10.21)}$$

Putting this value of v in Eq. 10.19, we have

$$\varepsilon = -vBL = -\frac{\Delta x}{\Delta t}BL \quad \text{.....................(10.22)}$$

As the rod moves through the distance Δx, the increase in the area of loop is given by $\Delta A = \Delta x L$. This increases the flux through the loop by $\Delta\phi_B = (\Delta A)B$. Putting $(\Delta x L)B = \Delta\phi_B$ in Eq. 10.22, we have

$$\varepsilon = \frac{\Delta\phi_B}{\Delta t} \quad \text{.....................(10.23)}$$

Eqation(10.20) shows that if the magnetic flux is changing through the single loop of a conducting coil, then the negative of the rate of change of magnetic flux is equal to the emf induced in the loop.

If there is a coil of N loops instead of a single loop, then the induced emf will become N times.

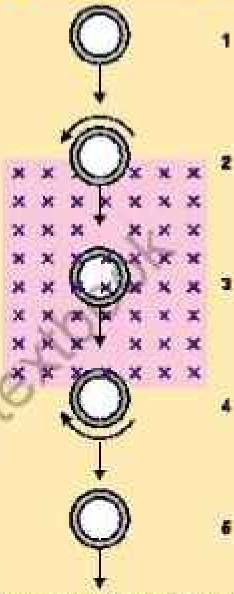
i.e., $$\varepsilon = -N\frac{\Delta\phi_B}{\Delta t} \quad \text{.....................(10.24)}$$

Although the above expression is derived on the basis of motional emf, but it is true in general. This conclusion was first arrived at by Faraday, so this is known as Faraday's law of electromagnetic induction which states that:

> The average emf induced in a conducting coil of N loops is equal to the negative of the rate at which the magnetic flux through the coil is changing with time.

The minus sign indicates that the direction of the induced emf is such that it opposes the change in flux.

Point to ponder!



A copper ring passes through a rectangle region where a constant magnetic field is directed into the page. What do you guess about the current in the ring at the positions 2, 3 and 4 ?

For your information

Faraday's homopolar generator with which he was able to produce a continuous induced current.

10.7 LENZ'S LAW AND DIRECTION OF INDUCED EMF

In the previous section, a mathematical expression for Faraday's law of electromagnetic induction was derived as:

$$\varepsilon = -N\frac{\Delta\Phi}{\Delta t}$$

The minus sign in the expression is very important; it relates to the direction of the induced emf. To determine the direction, we use a principle based on the discovery made by the Russian physicist Heinrich Lenz in 1834. He found that the polarity of an induced emf always produces an induced current that opposes the change in the magnetic field that causes the emf. This principle is known as Lenz's Law, which states that;

> The direction of the induced current is always such that it opposes the change that causes the current.

Lenz's Law specifically applies to induced currents and not directly to induced emf. This means we can apply Lenz's Law to closed conducting loops or coils. If the loop is not closed, we can imagine it as if it were closed to determine the direction of the induced current, and from this, we can infer the direction of the induced emf.

Let us apply Lenz's law to a coil in which a current is induced by the movement of a bar magnet. A current-carrying coil generates a magnetic field similar to that of a bar magnet, with one face of the coil acting as the north pole and the other as the south pole. To oppose the motion of the bar magnet, the face of the coil towards the magnet must become a north pole (Fig. 10.15). This arrangement causes the two north poles to repel each other. According to the right-hand rule, the induced current in the coil must flow anticlockwise when viewed from the side of the bar magnet.

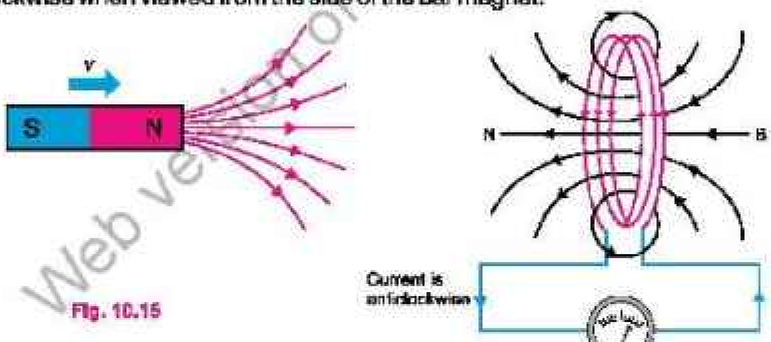


Fig. 10.15

According to Lenz's law, the "push" of the magnet is the "change" that produces the induced current, and the current acts to oppose the push. On the other hand, if we pull the magnet away from the coil, the induced current will oppose the "pull" by creating a south pole on the face of coil towards the bar magnet.

Lenz's law is also a manifestation of the law of conservation of energy and can be conveniently applied to circuits involving induced currents. To understand this, let us revisit the experiment depicted in Fig. 10.16. When the rod moves to the right, an emf is induced in it, causing an induced current to flow through the loop in anticlockwise direction. Because the current-carrying rod is moving within the magnetic field, it experiences a magnetic force $\mathbf{F}_m$ with the magnitude of $F_m = ILB\sin 90°$.

According to the right-hand rule, the direction of the magnetic force $\mathbf{F}_m$ is opposite to that of the velocity **v**, so it tends to stop the rod (Fig. 10.16-a). To keep the rod moving with a constant velocity, an external force equal in magnitude to $\mathbf{F}_m$ but opposite in direction must be applied. This external force provides the energy necessary for the induced current to flow. Thus, electromagnetic induction adheres to the law of conservation of energy.

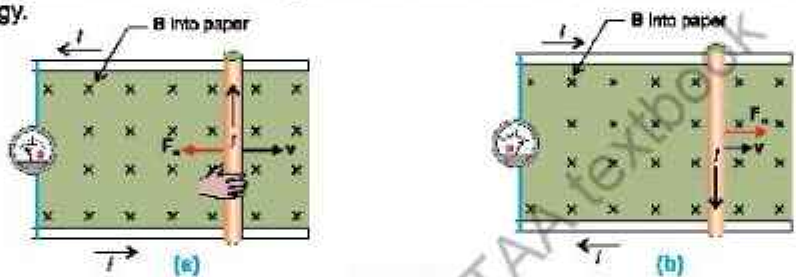


Fig.10.16

The Lenz's law forbids the induced current directed clockwise in this case, because the force $\mathbf{F}_m$ would be, then, in the direction of **v** that would accelerate the rod towards right (Fig.10.16-b). This in turn would induce a stronger current, the magnetic field due to it also increases and the magnetic force increases further. Thus, the motion of the wire is accelerated more and more. Starting with a minute quantity of energy, we obtain an ever-increasing kinetic energy of motion apparently from nowhere. Consequently, the process becomes self-perpetuating which is against the law of conservation of energy.

Example 10.6 A metal rod of length 25 cm is moving at a speed of $0.5\ \mathrm{m\ s^{-1}}$ in a direction perpendicular to a 0.25T magnetic field. Find the emf produced in the rod.

Solution

Speed of rod $= v = 0.5\ \mathrm{m\ s^{-1}}$

Length of rod $= L = 25\ \mathrm{cm} = 0.25\ \mathrm{m}$

Magnetic flux density $= B = 0.25\ \mathrm{T} = 0.25\ \mathrm{N\ A^{-1}\ m^{-1}}$

Induced emf $= \varepsilon = ?$

Using the relation,

$$\varepsilon = vBL$$

$$\varepsilon = 0.5\ \mathrm{m\ s^{-1}} \times 0.25\ \mathrm{NA^{-1}\ m^{-1}} \times 0.25\ \mathrm{m}$$

$$\varepsilon = 3.13 \times 10^{-2}\ \mathrm{J\ C^{-1}} = 3.13 \times 10^{-2}\ \mathrm{V}$$

Point to ponder!

By neglecting the resistance, can a constant current in a coil set up a potential difference across the coil?

Example 10.7 A loop of wire is placed in a uniform magnetic field that is perpendicular to the plane of the loop. The strength of the magnetic field is 0.6 T. The area of the loop begins to shrink at a constant rate of $\frac{\Delta A}{\Delta t} = 0.8\ m^2\ s^{-1}$. What is the magnitude of emf induced in the loop while it is shrinking?

Solution Rate of change of area = $\frac{\Delta A}{\Delta t} = 0.8\ m^2 s^{-1}$

Magnetic flux density $= B = 0.6\ T = 0.6\ N\ A^{-1} m^{-1}$

Number of turns $= N = 1$

Induced emf $= \varepsilon = ?$

Rate of change of flux $= \frac{\Delta\Phi}{\Delta t} = B\frac{\Delta A}{\Delta t}\cos 0° = B\frac{\Delta A}{\Delta t}$

Applying Faraday's law, magnitude of induced emf is:

$$\varepsilon = N\frac{\Delta\Phi}{\Delta t} = NB\frac{\Delta A}{\Delta t}$$

$$= 1 \times 0.6\ N\ A\ m^{-1} \times 0.8\ m^2\ s^{-1}$$

$$\varepsilon = 0.48\ J\ C^{-1} = 0.48\ V$$

10.8 FACTORS AFFECTING EMF

1. Rate of Change of Magnetic Flux

Faraday's law suggests that faster changes in magnetic flux result in greater induced emf.

2. Number of Turns of the Coil

According to Faraday's law, induced emf is also proportional to the number of turns of the coil. More turns result in a greater induced emf.

3. Relative Speed

The speed of the coil (or conductor) through a magnetic field also affects the magnitude of the induced emf. Faster speed increases the rate of change of magnetic flux that results into an increase in the induced emf.

10.9 FERROFLUIDS

Ferrofluid is a unique material that exhibits both liquid and magnetic properties. It operates through a combination of magnetic and fluid dynamics principles.

Essentially, the ferrofluid is a colloidal suspension of magnetic particles in a carrier fluid (such as oil or water). Typically, the magnetic particles are iron oxide, ground to the nano-scale and approximately 10 nanometres in size. These particles are coated with a surfactant, a substance that reduces surface tension. This coating prevents the particles from clumping together, ensuring they remain evenly dispersed in the fluid. The viscosity of the fluid, the nanometre size of the particles, and their constant movement prevent the particles from settling down.

When there is no magnet around, a ferrofluid acts like a liquid, but when there is a magnet nearby, the particles are temporarily magnetized and the fluid becomes a magnet. They form structures within the fluid causing the ferrofluid to act more like a solid. When the magnet is removed, the particles are demagnetized and the ferrofluid acts like a liquid again.

For your information

The first ferrofluid developed by NASA in 1960, was ground from natural magnetite. Ferrofluid was invented to move liquids through space.

This phenomenon is due to the competition between magnetic forces, surface tension and gravity. In the presence of strong magnetic field, the formation of chain-like structures is a result of magnetic forces pulling the fluid upwards while gravity and surface tension work to pull it back down. These chains align along the magnetic field lines and increase the viscosity, making it behave like a solid bulging in certain directions. These are commonly known as spikes. However, the spikes are formed where the magnetic forces overcome the other forces (Fig.10.17).

Fig.10.17

The following experiment will exhibit this phenomenon.

Experiment

You need some laser printer toner, some cooking oil, a test tube, a glass bottle, a small stick and a magnet.

Procedure

Pour some toner in the test tube. Remember that laser printer toner contains 40 % iron oxide in nanometre particle size. Add some cooking oil in it and mix it well with the stick to form ferrofluid. Put this fluid in the bottle. The fluid will act like a liquid on shaking the bottle. Now bring the magnet near to fluid outside of the bottle.

You will observe that the fluid jumps towards the magnet, because it has itself become a magnet. If we hold the magnet on the side of the bottle, you will see a structure with spikes formed by the fluid as shown in Fig. 10.18.

Fig.10.18

Applications of Ferrofluids

There are many applications of ferrofluids in the fields of electronics, medicine, engineering, and active research in Physics and Material science. In **electronics,** ferrofluids are used in rotary seals for computer hard drives and other rotating shaft motors. In loudspeakers, ferrofluids are used to cool the voice coil which can heat up during operation. The magnetic field holds the fluid in place around the coil, allowing it to absorb and dissipate heat more effectively. Ferrofluids are also used in speakers to dampen vibrations and improve sound quality.

In **medical** applications, ferrofluids can be directed to specific areas in the body using external magnets, allowing for targeted drug delivery. The magnetic particles can carry drugs directly to a tumor or other targeted site, reducing side effects and improving treatment efficiency. Ferrofluids can also be used as contrast agents in magnetic resonance imaging (MRI).

Other applications of ferrofluids include damping or precisely controlling the flow of liquids by manipulating the magnetic field.

10.10 A SEISMOMETER

A seismometer is an instrument that responds to any movement of the rocks under the ground or vibration caused by earthquakes, volcano eruption and explosion. A seismometer detects earthquakes by using electromagnetic induction to convert ground motion into electrical signals. Typically, a seismometer includes a weight suspended by a spring. When an earthquake occurs, the ground moves but the weight tends to stay stationary due to inertia. This results in relative motion between the weight and the frame of the seismometer which is attached to the ground. The weight is often attached to a magnet which moves inside a coil of wire (Fig.10.19). This setup works according to Faraday's law of electromagnetic induction, that is, the changing magnetic flux through the coil induces an emf in the coil. This gives rise to an induced electric current.

Fig. 10.19 Seismometer

The induced current is proportional to the velocity of the ground motion. The electrical signals generated are then amplified and recorded. Thus, data is provided on the amplitude, frequency and the duration of the earthquake waves.

For your information

Most earthquakes are caused by plate tectonics (displacement) and occur at a depth of 60 km. These earthquakes are categorized as shallow. Intermediate and deep intermediate can be as deep, as 280 km beneath the crust, while deep earthquakes can reach depths past 280 km.

The data is analyzed to determine various characteristics of the earthquake, such as its location, magnitude and depth.

For your Information

There are two main types of seismic waves that generate earthquakes i.e., P-waves are primary waves which are longitudinal and S-waves are secondary waves which are transverse in nature.

Usually, a seismometer is buried under the ground at a depth of 50-1000 metres. It is placed in a protective case called a vault. This is a cylindrical steel tank that is approximately 1 metre wide and 2 metres deep with a concrete pad at the bottom (Fig.10.20).

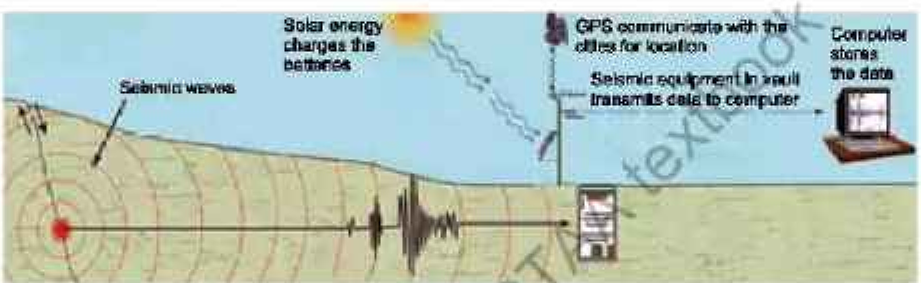


Fig. 10.20

QUESTIONS

Multiple Choice Questions

Tick (✓) the correct answer.

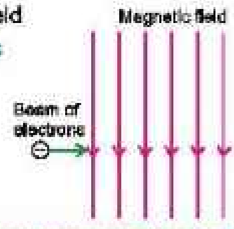
10.1 A current is flowing towards north along a power line. The direction of the magnetic field over the wire is directed towards:

(a) north (b) south (c) east (d) west

10.2 The radius of curvature of the path of a charged particle in a uniform magnetic field is directly proportional to:

(a) the particle's charge (b) the particle's momentum
(c) the particle's energy (d) the flux density of the field

10.3 The diagram shows a beam of electrons entering a magnetic field. What is the effect of magnetic field on the electrons?



(a) They are deflected into the plane of the diagram.
(b) They are deflected out of the plane of the diagram.
(c) They are deflected towards the bottom of the diagram.
(d) They are deflected towards the top of the diagram.

10.4 The force exerted on a wire of 1 metre length carrying 1 ampere current placed at right angle to the magnetic field is called:

(a) magnetic field intensity (b) magnetic flux
(c) magnetic induction (d) none of these

10.5 The unit of flux density is:

(a) $NA^{-1}m^{-1}$ (b) NAm^{-1} (c) NmA^{-2} (d) NmA

10.6 A moving charged particle is surrounded by:

(a) electric field only (b) magnetic field only

(c) both electric and magnetic field (d) no field

10.7 Magnetic force on the charge q moving parallel to magnetic field with velocity v is:

(a) $qvB\sin\theta$ (b) qvB (c) zero (d) ILB

10.8 The unit $NA^{-1}m^{-1}$ is called:

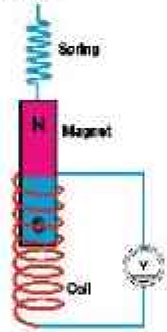
(a) weber (b) tesla (c) coulomb (d) none of these

10.9 Electrons while moving perpendicularly through a uniform magnetic field are:

(a) deflected towards north pole (b) deflected towards south pole

(c) deflected along circular path (d) not deflected at all

10.10 A magnet is suspended from a spring. The magnet oscillates and moves in and out of the coil connected to a galvanometer. When the magnet oscillates, the galvanometer shows:



(a) deflection to the left and to the right with constant amplitude

(b) deflected on one side

(c) no deflection

(d) deflection to the left and right, but the amplitude steadily decreases

Short Answer Questions

10.1 It is said that Lenz's law specifically applies to induced currents and not directly to induce emf. Explain briefly.

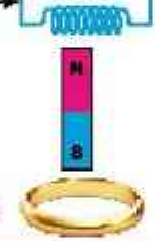
10.2 A square loop of wire is moving through a uniform magnetic field. The normal to the loop is oriented parallel to the magnetic field. Is an emf induced in the loop? Give a reason for your answer.

10.3 Does the induced emf always act to decrease the magnetic flux through a circuit?

10.4 When a magnet is pushed into the solenoid, as shown in the figure (a), the galvanometer indicates a small current. Why is the current produced? What will be the magnetic pole produced at the left end of the solenoid?

(a)

10.5 A bar magnet falls through a fixed metal ring (Fig-b). Will the magnet fall with an acceleration of a freely falling body? Give reason.



(b)

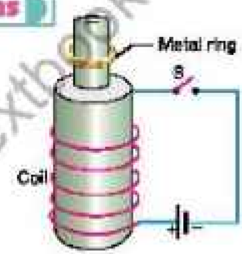
10.6 Which of the two charged particles of the same mass will be deflected most in the magnetic field (a) fast moving (b) slow moving?

10.7 An electron and a proton are projected into a magnetic field at right angles to it with a certain velocity. Which of the particles will suffer greater deflection? Why?

10.8 Can a single moving proton produce magnetic field?

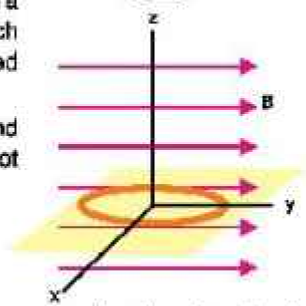
10.9 A magnetic field is necessary if there is to be a magnetic flux passing through a coil of wire. Yet, just because there is a magnetic field does not mean that a magnetic flux will pass through a coil. Account for this situation.

Constructed Response Questions



10.1 A charge is lying stationary between the opposite poles of two magnets. Is a magnetic force exerted on it? Why?

10.2 When the switch in the circuit is closed, a current is established in the coil and the metal ring jumps upward. Why? Describe what would happen to the ring if the battery polarity were reversed?



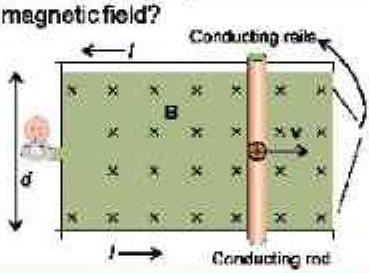
10.3 The figure shows a coil of wire in the x – y plane with a magnetic field directed along the y-axis. Around which of the three-coordinate axis should the coil be rotated in order to generate an emf and a current in the coil?

10.4 Is it possible to change both the area of the loop and the magnetic field passing through the loop and still not have an induced emf in the loop? Give reason.

10.5 Does the application of uniform magnetic field to a moving charged particle result in a change in kinetic energy of the particle? Explain.

10.6 A uniform electric field and a magnetic field act in the same direction. A proton is projected, into the space, with a uniform velocity in opposite direction. What will happen to the proton?

10.7 A conductor moves in a magnetic field when a current is passed through the conductor. Would you expect the reverse effect to occur? That is, would a current be produced if a conductor is moved across the magnetic field?



10.8 Consider a conducting rod of length L moving with velocity v to the right as shown in the figure. Left ends of the conducting rails are connected to a bulb. Due to motion of the rod through the magnetic field, an emf is produced across the ends of the rod. This emf gives rise to a current I. As a result, the bulb lights up. Explain where does the electrical energy consumed by the bulb come from?

10.9 What will you do if you want to save a sensitive instrument from stray magnetic fields?

Comprehensive Questions

10.1 Distinguish between magnetic flux and flux density. How are they related?

10.2 Find an expression for the force exerted on a current-carrying conductor placed in a uniform magnetic field.

10.3 State and explain Faraday's law and Lenz's law. Also describe factors affecting the induced emf.

10.4 Determine the force acting on a charged particle moving through a uniform magnetic field.

10.5 What is a velocity selector? Explain its working.

10.6 Explain how ferrofluids work?

Numerical Problems

10.1 A positively charged particle is projected perpendicularly into a magnetic field at a speed of 1500 m s^{-1}. It experiences a force of magnitude *F*. At what angle θ with the field, the particle should be projected at a speed of 2000 m s^{-1}, so that it experiences the same magnitude of force? (Ans: $\theta = 49°$)

10.2 Electrons are accelerated from rest through a potential difference of 15 kV in an oscilloscope. The electrons then pass through a 0.35 T magnetic field that deflects them to the desired position on the screen. Find the magnitude of the maximum force that an electron can experience. (Ans: 4.1×10^{-12} N)

10.3 A square coil of side 15 cm each consists of 60 turns. Initially, it is located in a uniform magnetic field of magnitude 0.8 T such that plane of the coil is perpendicular to the field. The coil is then turned through an angle of $\theta = 30°$ in a time of 2 s. Determine the average induced emf. (Ans: 0.07 V)

10.4 A metallic rod is moving through a uniform magnetic field of 0.2 T. The emf induced across its ends is found to be 0.8 V. It is required to induce an emf of 2.4 V across its ends. How much field strength is needed for this? (0.6 T)

10.5 A copper ring has a radius of 4.0 cm and resistance of 1.0 mΩ. A magnetic field is applied over the ring, perpendicular to its plane. If the magnetic field increases from 0.2 T to 0.4 T in a time interval of 5×10^{-3} s, what is the current in the ring during this interval? (Ans: 201 A)

10.6 A coil of 10 turns and 35 cm^2 area is in a perpendicular magnetic field of 0.5 T. The coil is pulled out of the field in 1.0 s. Find the induced emf in the coil as it is pulled out of the field. (Ans: 1.75×10^{-2} V)

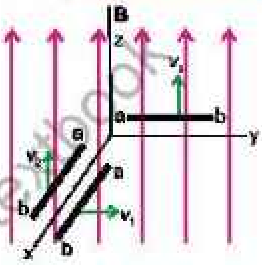
10.7 A proton is accelerated by a potential difference of 6×10^5 volts. It then enters perpendicularly in a uniform magnetic field $B = 1.0$ weber m^{-2}. Find the radius of curvature of the path of the proton. $m = 1.67\times10^{-27}$ kg, $e = 1.6\times10^{-19}$ C.

(Ans: 11.2 cm)

10.8 A proton enters a uniform magnetic field $B = 0.3$ weber m^{-2} in a direction making an angle 45° with the magnetic field. What will be the radius of the circular path if the velocity of proton is 10^4 m s^{-1}. (Ans: 2.46×10^{-4} m)

10.9 Three identical conducting rods L_1, L_2 and L_3 are moving in different planes with the same speeds $v_1 = v_2 = v_3 = 2.5$ m s^{-1} as shown in the figure. The length of each rod is 60 cm. A constant magnetic field of magnitude $B = 0.5$ T is directed along z-axis. Find the magnitude of emf induced in each rod and indicate which end of the rod is positive.

[Ans: (Rod L_1) emf = 0.75 V, and end a is positive, (Rod L_2) emf = 0 (Rod L_3) emf = 0]



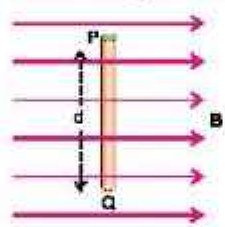
10.10 An emf of 0.5 V is induced across the ends of a metal rod moving through a magnetic field of 0.4 T. If an emf of 1.5 V has to be induced, what field strength would be needed for that? Assume that all other factors remain the same.

(Ans: 1.2 T)

10.11 A charged particle moves through a velocity selector at a constant velocity of 4.96×10^4 m s^{-1} in a direction perpendicular to both **E** and **B**. If the magnetic field strength is 0.114 T, what should be the magnitude of electric field intensity so that the particle moves undeflected? (Ans: 5.65×10^3 N C^{-1})

10.12 A current-carrying conductor PQ of length 2 m is placed perpendicularly to a magnetic field of flux density 0.5 T as shown in the figure. The resulting force on the conductor is 1 N acting into the plane of the paper. What is the magnitude and direction of the current?

(Ans: 1 A, Q to P)



Chapter 11 Special Theory of Relativity

Learning Objectives

After studying this chapter, the students will be able to:

- Distinguish between inertial and non-inertial frames of reference.
- Describe the significance of Einstein's assumption of the constancy of the speed of light.
- Describe that if c is constant then space and time become relative.
- State the postulates of Special theory of relativity
- Explain qualitatively and quantitatively the consequences of special relativity Specifically in the case of:
  - a. The relativity of simultaneity.
  - b. The equivalence between mass and energy
  - c. Length contraction.
  - d. Time dilation.
  - e. Mass increase
- State that spacetime is a mathematical model in relativity that treats time as a fourth dimension of the traditional three dimensions of space
  (It can be thought of as a metaphorical sheet of paper that can bend, and when it bends it can cause effects such as stretching and compression seen when gravitational waves pass through objects.)

At the beginning of the 20th century, new experiments and theoretical calculations revealed that classical physics, based on Newton's laws, could not explain phenomena involving extremely small particles or very high velocities. This led to the development of relativistic mechanics, which offered a more comprehensive framework than classical mechanics and fundamentally changed our view of the universe. Albert Einstein's Special Theory of Relativity, introduced in 1905, addressed these issues by proposing that the laws of Physics are the same for all observers and that the speed of light is constant, regardless of the observer's motion. This theory not only resolved the conflicts between classical mechanics and electromagnetic theory but also revolutionized our understanding of time, space, and motion, forming the basis of what is now known as modern physics. This chapter will explore how Einstein's theory reshaped our view of the universe and continues to influence our understanding of the physical world.

11.1 RELATIVE MOTION

Consider throwing a ball to your right. For someone facing you, this direction appears to his left. This illustrates that direction is a relative concept. Similarly, the state of rest or motion of an object depends on the observer. For example, the walls of a moving train seem stationary to passengers inside the train but appear to be moving to someone standing on the ground. Thus, we cannot definitively say whether an object is absolutely at rest or in motion; all motions are relative to the observer or to the reference frame being used. This becomes evident with the following example: An observer in a closed train compartment uses the compartment as his frame of reference. To determine the train's motion, the observer drops a ball and measures the horizontal distance travelled by the ball, keeping the vertical distance the same in each case. It is assumed that the vertical distance is covered in "t" seconds in all scenarios.

Case (a): Suppose the train is stationary. In that case, the horizontal velocity of the ball will be zero, and the horizontal distance travelled will also be zero. In this scenario, the observations made by the observer inside the train and by someone outside the train will be identical. The ball will have fallen to a point on the floor directly below the point from where it was dropped (Fig. 11.1-a).

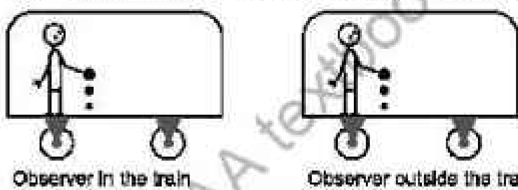


Fig. 11.1(a)

Case (b): The train is moving with a uniform velocity v_x. When the ball is released, it has an initial horizontal velocity v_x and behaves like a projectile. It travels a horizontal distance v_xt in time t it takes for the ball to reach the floor. Since both the train and the observer inside it are moving with the same velocity v_x, they both travel the same horizontal distance v_xt in the same time t. Therefore, the observer inside the train sees that the ball falls to a point on the floor directly below where it was dropped. In contrast, an observer outside the train will see the ball following a projectile path, as shown in Fig. 11.1(b). Thus, observers in different frames of reference, moving with uniform velocity relative to each other, will perceive motion differently.

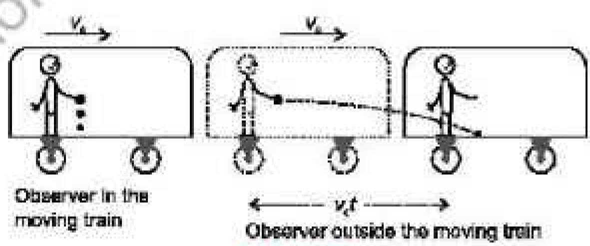


Fig. 11.1(b)

11.2 FRAMES OF REFERENCE

We have discussed the most commonly used Cartesian Coordinate System. In effect, a frame of reference is any coordinate system relative to which measurements are taken. For example, the position of a table in a room can be described relative to the walls of the room, making the room the frame of reference. Similarly, the laboratory is the reference frame for measurements taken there. If the same experiment is performed in a moving train, the train becomes the frame of reference. The position of a spaceship can be described relative to the positions of distant stars. A coordinate system based on these stars is then the frame of reference.

Inertial and Non-inertial Frame of Reference

An inertial frame of reference is defined as a coordinate system in which the law of inertia is valid. This means a body at rest remains at rest unless acted upon by an unbalanced force that produces acceleration. Other laws of nature also apply in such a system. For instance, a body placed on the Earth remains at rest unless an unbalanced force acts upon it, indicating that the Earth can be considered an inertial frame of reference. A body in a car moving with uniform velocity relative to the Earth also remains at rest, so the car is also an inertial frame of reference. Thus, any frame of reference moving with uniform velocity relative to an inertial frame is also an inertial frame.

However, if the moving car is suddenly stopped or accelerated, the body inside no longer remains at rest. In such cases, the car is not an inertial frame of reference. Therefore, an accelerated frame of reference is a non-inertial frame. While Earth is rotating and revolving, making it strictly speaking a non-inertial frame, it is often treated as an inertial frame due to its relatively small acceleration.

For your information

Relativity is the study of the way in which observers from moving frame of reference affect your perception of the world.

11.3 SPECIAL THEORY OF RELATIVITY

The theory of relativity deals with how observers in different states of relative motion describe physical phenomena. The special theory of relativity addresses problems involving inertial (non-accelerating) frames of reference. There is another theory, called the general theory of relativity, that deals with problems involving frames of reference that are accelerating relative to one another. The special theory of relativity is based on two postulates, which can be stated as follows:

Do you know?

The speed of light c emitted by the flashlight is measured same by two observers, one moving in the car with speed v and other standing on the road.

1. The laws of physics are the same in all inertial frames (Principle of Relativity).
2. The speed of light in free space has the same value for all observers, regardless of the state of motion of the source or the observer (Principle of Constancy of Light).

The first postulate generalizes the fact that all physical laws are the same in frames of reference moving with uniform velocity relative to one another. If the laws of Physics differed for observers in relative motion, those observers could determine which was stationary and which was moving. However, such a distinction does not exist, implying that there is no way to detect absolute uniform motion.

The second postulate states the experimental fact that the speed of light in free space is a universal constant, denoted as c ($c = 3 \times 10^8$ m s^{-1}). Since c is constant, space and time become relative. For example, if you are sitting in a train moving at the speed of light and you hold up a mirror in front of you at arm's length, you will still see your reflection in the mirror. This is because, according to the principle of relativity, no experiment can detect the constant motion of the train relative to the person inside it.

These simple postulates have far-reaching consequences. They include phenomena such as the slowing down of clocks and the contraction of lengths in moving reference frames as observed by a stationary observer. Some interesting results of the special theory of relativity can be summarized as follows, without going into their mathematical details.

The Relativity of Simultaneity

If two events in different locations are observed by one observer to be simultaneous, they will generally not be observed as simultaneous by another observer in a different frame of reference moving relative to the first observer. In other words, whether two events are seen as simultaneous depends on the observer's frame of reference.

Consider a train equipped with light-operated doors. The light switch is located in the centre of the roof and is operated by a traveler standing in the middle of the compartment. When the train is travelling at half the speed of light, the traveler turns on the light. The light travels forward and backward at equal speed and reaches both doors at the same time. Consequently, the traveler sees both doors opening simultaneously. However, an observer outside the train will see the back door open before the front door. This is because the back door is moving towards the light waves, while the front door is moving away from the light waves.

For your information

If you are in a frame of reference moving at constant velocity from which you cannot see any other frame of reference, there is no way to know if you are moving or at rest.

Time Dilation

According to the special theory of relativity, time is not an absolute quantity; it depends on the motion of the frame of reference.

Suppose an observer is stationary in an inertial frame and measures the time interval between two events in this frame. Let this time interval be t_o. This is known as proper time. If the observer is moving with respect to the frame of events with relativistic velocity v, or if the frame of events is moving with respect to the observer with a uniform relativistic velocity v, the time measured by the observer will not be t_o, but rather t, given by

$$t = \frac{t_o}{\sqrt{1-\frac{v^2}{c^2}}} \qquad \text{.................................} \quad (11.1)$$

As the quantity $\sqrt{1-\frac{v^2}{c^2}}$ is always less than one, so t is greater than t_o i.e., time has dilated or stretched due to the relative motion of the observer and the frame of reference of the events. This astonishing result applies to all timing processes—physical, chemical, and biological. Even the aging process of the human body is slowed by motion at very high speeds or relativistic speeds.

For example, if a traveler on a plane moving at 0.8 c picks up and opens a book, the event takes one second as measured by the traveler. However, to a person standing outside the plane, the same event takes 1.7 seconds.

Length Contraction

The distance from Earth to a star measured by an observer in a moving spaceship would appear smaller than the distance measured by an observer on the Earth. In other words, if you are in motion relative to two points that are a fixed distance apart, the distance between the two points appears shorter than if you were at rest relative to them. This effect is known as length contraction. Length contraction occurs only along the direction of motion; no such contraction is observed perpendicular to the direction of motion. The length of an object or the distance between two points measured by an observer who is at rest relative to them is called the proper length ℓ_o. If an object and an observer are in relative motion with speed v, then the contracted length 'ℓ' is given by

$$\ell = \ell_o\sqrt{1-\frac{v^2}{c^2}} \quad \text{..................} \quad (11.2)$$

Let a train that is measured to be 100 metres long when at rest travel at 80% of the speed of light (0.8 c). A person inside the train will measure its length as 100 metres. However, a person standing by the side of the track will observe the train to be only 60 metres long. This effect of relativity, which is the shortening of length in the direction of motion, is due to length contraction.

Mass Variation

According to the special theory of relativity, the mass of an object is a variable quantity that depends on the object's speed. An object whose mass is measured at rest is called its rest mass m_o, will have an increased mass m when observed to be moving at speed v. They are related by

$$m = \frac{m_o}{\sqrt{1-\frac{v^2}{c^2}}} \quad \text{..................} \quad (11.3)$$

The increase in mass indicates the increase in inertia that an object has at high speeds. As v approaches c, it requires a greater force to change the object's speed.

As $\quad v \to c,\ \frac{v}{c} \to 1, \quad$ therefore, $\quad \sqrt{1-\frac{v^2}{c^2}} \to 0$

Thus $\quad m \to \infty$

An infinite mass would require an infinite force to accelerate it. Since infinite forces are not available, an object cannot be accelerated to the speed of light c in free space.

In our everyday life, we deal with speeds that are extremely small compared to the speed of light. Even Earth's orbital speed is only 30 km s^{-1}, while the speed of light in free space is 300,000 km s^{-1}. This is why Newton's laws are valid in everyday situations. However, when dealing with subatomic particles moving at velocities approaching the speed of light, relativistic effects become very prominent, and experimental results cannot be explained without considering Einstein's equations.

11.4 THE EQUILANCE BETWEEN MASS AND ENERGY

According to the special theory of relativity, mass and energy are distinct entities but are interconvertible. The total energy E and mass m of an object are related by the expression:

$$E = mc^2 \qquad \text{.......................................} \quad (11.4)$$

where m depends on the speed of the object. At rest, the energy equivalent of an object's mass m_o is called its rest mass energy E_o. Thus,

$$E_o = m_o c^2$$

As mc^2 is greater than $m_o c^2$, the difference of energy $(mc^2 - m_o c^2)$ is due to the motion, and it represents the kinetic energy of the mass. Hence,

$$K.E. = (m - m_o)c^2$$

From Eq. 11.4, the change in mass m due to change in energy ΔE is given by

$$\Delta m = \frac{\Delta E}{c^2}$$

Because c^2 is a very large quantity, this implies that small changes in mass require very large changes in energy. In our everyday world, energy changes are too small to provide measurable mass changes. However, energy and mass changes in nuclear reactions are found to be exactly in accordance with the aforementioned equations.

11.5 SPACE-TIME IN RELATIVITY

Space is said to be a three-dimensional extent in which all objects and events occur. It provides a framework to define the position and motion of various objects under the influence of some force.

Time measures the sequence and duration of events. In the theory of relativity, time is not absolute; it is considered the fourth dimension. For example, oscillatory motion, such as that of a swinging pendulum, relies on time to determine the frequency of oscillations. Another example is time dilation, a phenomenon discussed earlier in this chapter, where time passes more slowly for an observer moving at extremely high speeds compared to one at rest. The special theory of relativity explains that space and time are related to each other. It describes how space and time are influenced by gravity and speed, such as the bending of light around massive objects like stars.

Space-time is, in fact, a mathematical model that unifies space-time into a single continuum. It is a concept used to describe all points of space and time and their relation to each other. According to Einstein's theory, space-time is curved especially near massive bodies and for speeds approaching the speed of light. We can hypothetically visualize this as a fabric sheet. If a heavy ball is placed over this sheet, it curves as shown in Fig. 11.2.

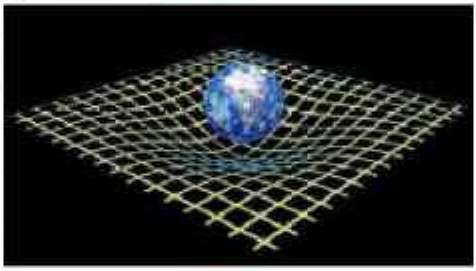


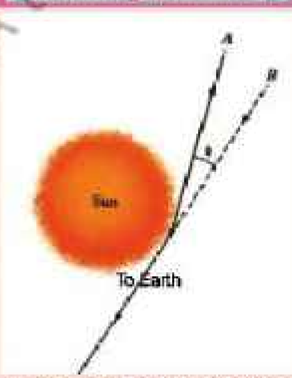
Fig. 11.2

Objects such as stars and planets cause space-time to curve around themselves, much like an elastic fabric deforms when holding a ball. The more massive the object, the deeper the curve. Consequently, we do not speak of a force of gravity acting on bodies; instead, we say that bodies and light rays move along geodesics (analogous to straight lines in plane geometry) in curved space-time. Thus, a body at rest or moving slowly near a massive object would follow a geodesic toward that object.

Einstein's theory provides a physical picture of how gravity works. Newton discovered the inverse square law of gravity but explicitly stated that he offered no explanation for why gravity should follow this law. Einstein's theory also describes gravity as following an inverse square law (except in strong gravitational fields), but it explains why this is so. This is why Einstein's theory is considered an advancement over Newton's, even though it encompasses Newton's theory and yields the same results as Newton's theory in all but very strong gravitational fields.

The bending of starlight caused by the Sun's gravity was measured during a solar eclipse in 1919. The results matched Einstein's theory rather than Newton's, leading to Einstein's theory being hailed as a scientific triumph. Another success of Einstein's theory was the detection of gravitational waves, produced by some celestial events causing disturbances (squeezes and stretches) in the curvature of space-time. These waves were detected in 2015 and announced in 2016.

Interesting information



Bending of starlight by the Sun. Light from the star A is deflected as it passes close to the Sun on its way to Earth. We see the star in the apparent direction B, shifted by the angle ϕ. Einstein predicted that ϕ = 1.745 seconds of angle which was found to be the same during the solar eclipse of 1919.

Example 11.1 The period of a pendulum is measured to be 3.0 s in the inertial reference frame of the pendulum. What is its period measured by an observer moving at a speed of 0.95 c with respect to the pendulum?

Solution

$t_o = 3.0\,s$, $\quad v = 0.95\,c$, $\quad t = ?$

Using $$t = \frac{t_o}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$t = \frac{3.0\,s}{\sqrt{1-\frac{(0.95\,c)^2}{c^2}}} = \frac{3.0\,s}{\sqrt{1-(0.95)^2}} = 9.6\,s$$

Example 11.2 A bar 1.0 m in length and located along x-axis moves with a speed of 0.75 *c* with respect to a stationary observer. What is the length of the bar as measured by the stationary observer?

Solution

$\ell_o = 1.0$ m , $v = 0.75\,c$, $\ell = ?$

Using $$\ell = \ell_o\sqrt{1-\frac{v^2}{c^2}}$$

$$\ell = 1.0\,m \times \sqrt{1-\frac{(0.75\,c)^2}{c^2}} = 1.0\,m \times \sqrt{1-(0.75)^2} = 0.66\,m$$

For your information

The faster you are moving or close to a strong source of gravity, the slower the time goes for you.

Example 11.3 Find the mass *m* of a moving object with speed 0.8 *c*.

Solution

Using $$m = \frac{m_o}{\sqrt{1-\frac{v^2}{c^2}}}$$

or $$m = \frac{m_o}{\sqrt{1-\frac{(0.8\,c)^2}{c^2}}} = \frac{m_o}{\sqrt{1-(0.8)^2}} = 1.67\,m_o$$

or $$m = 1.67\,m_o$$

Interesting information

If you are on some spaceship moving extremely fast through space near a black hole like in movie, "Interstellar" then you could miss 7 years on the Earth in everyhour.

Hypothetical Example of Space Time

Let a spaceship be travelling to a star with half of the speed of light. Let it takes eight years to reach to the star, from the point of view of the observer on the Earth. From the Earth's point of view, the clocks on the spaceship are moving slowly, so that less time passes on the spaceship compared to the Earth.

For the spaceship occupants, the length of the journey has contracted which they cover in less time. The occupants of the spaceship record 7 years to reach their destination, rather than 8 years.

QUESTIONS

Multiple Choice Questions

Tick (✓) the correct answer.

11.1 Relativistic mechanic yields results different from classical mechanics for objects moving with:

(a) low velocity (b) velocity equal to that of sound waves
(c) velocity greater than sound waves (d) velocity approaching that of light

11.2 If an observer is moving in the same direction as a sound wave, the velocity of the wave seems to be:

(a) more (b) less
(c) constant (d) sum of the two velocities

11.3 If the rest mass of a particle m_o increases to m due to its high speed, then its kinetic energy is:

(a) $1/2\ mc^2$ (b) $1/2\ mv^2$ (c) $(m - m_o)\ c^2$ (d) $1/2\ (m - m_o)\ c^2$

11.4 The speed of beam light of a car while moving with high speed as compared to its rest position is:

(a) greater (b) less (c) same (d) zero

11.5 A photon is a particle of light. What is its mass when it moves with 0.9 c?

(a) 9.1×10^{-31} kg (b) 1.67×10^{-19} kg (c) 1.67×10^{-27} kg (d) Zero

Short Answer Questions

11.1 What is meant by inertial frame of reference and a non-inertial frame of reference?

11.2 What are the two postulates of special theory of relativity?

11.3 Describe why it is impossible for a material particle to move with speed of light.

11.4 Does theory of relativity contradicts Newton's laws of motion? Explain briefly.

11.5 What is meant by proper time, and proper length?

11.6 What is meant by relativistic mass, length and time?

11.7 Why mass of a moving object increases?

11.8 All motion are relatives. Does space-time is absolute? Explain briefly.

11.9 Explain that speed of light is an ultimate limit for any object.

11.10 Give examples where the results of special theory of relativity have been verified.

Constructed Response Questions

11.1 Speed of sound is affected by relative motion between the observer and the source. Does this apply to speed of light as well? Describe briefly.

11.2 Is it ever possible to see a star moving away from us at a uniform velocity equal to the velocity of light?

11.3 If the speed of light is just 50 m s^{-1}, how would every day events appear to?

11.4 If the speed of light were infinite, what would the equations of special theory of relativity reduce to?

11.5 According to Einstein's equation; $E = mc^2$, is it possible to create a single electron from energy? Explain.

Comprehensive Questions

11.1 What is meant by the "frame of reference"? Distinguish between inertial frame of reference and non inertial frame of reference by giving examples.

11.2 Describe the Einstein's mass-energy equation; why cannot we observe its effects in everyday life? What are its significant consequences? Give examples.

11.3 State the Einstien's concept about the space-time. Describe the view of gravity according to this concept.

Numerical Problems

11.1 An electron is accelerated to a speed of 0.995 *c* which passes down an evacuated tube 500 m long. How long will the tube appear to the electron? (Ans: 50 m)

11.2 A neutron, being not a stable particle, disintigerates in 20 minutes on the average. How long will it seem to exist if shoots out from a nucleus with a speed of 0.8 *c*?

(Ans: 33.3 min)

11.3 A spaceship is measured 100 m long while it is at rest with respect to an observer, if this spaceship now flies by the observer with a speed of 0.99 c, what length will the observer find for the spaceship? (Ans: 14 m)

11.4 The rest mass of an electron is 9.11×10^{-31} kg. Calculate the corresponding rest-mass energy. (Ans: 8.2×10^{-14} J or 0.51 MeV)

11.5 An electron is accelerated to a speed $v = 0.85$ c. Calculate its total energy and kinetic energy in electron volt. (Ans: 0.97 Mev, 0.459 MeV)

11.6 At what speed, would the mass of a proton in a particle accelerator be tripled?

(Ans: 0.943 *c*)

11.7 The period of pendulum is measured to be 3 s in an inertial frame of reference. What will be the period measured by an observer in a spaceship with a constant speed of 0.95 *c* with respect to the pendulum? (Ans: 9.6 s)

11.8 Hypothetically, If a ball of mass 0.5 kg is projected with a velocity of 0.9 *c*, what will be its mass in flight? (Ans: 1.15 kg)

Chapter 12 Nuclear and Particle Physics

Learning Objectives

After studying this chapter, the students will be able to:

- state that nucleon number and charge are conserved in nuclear processes
- describe the composition, mass and charge of α-, β- and γ-radiations [both β− (electrons) and β+ (positrons) are included]
- Explain that an anti-particle has the same mass but opposite charge to the corresponding particle [give the example that a positron is the anti-particle of an electron]
- state that (electron) anti-neutrinos are produced during β-decay and (electron) neutrinos are produced during β+ decay
- Explain that α-particles have discrete energies but that β-particles have a continuous range of energies because (anti)neutrinos are emitted in β-decay
- Describe quarks and anti- quarks (as a fundamental [including that there are six flavors (types) of quark: up, down, strange, charm, top and bottom]
- describe protons and neutrons in terms of their quark composition
- state that a hadron may be either a baryon (consisting of three quarks) or a meson (consisting of one quark and an antiquark)
- describe the changes to quark composition that take place during β- and β+ decay
- state that electrons and neutrinos are fundamental particles called leptons
- State, W, Z, gluon, and photons as fundamental particles called exchange particles or force carriers
- State the Higgs Boson as a fundamental particle which is responsible for the particle's mass.
- Explain that every subatomic particle has a corresponding anti- particle [that has the same mass as a given particle but opposite electric or magnetic properties according to the Standard Model of Particle Physics)]
- Explain that there are various contending theories about what 'mass' and 'force' are generated from [e.g. that these are generated from quantum fields when they are energized, or from multi dimensional 'strings' that vibrate in higher dimensions to give rise to particles (no further technical knowledge beyond these simple descriptions is expected at this level)]
- Illustrate that anti -particles usually have the same weight, but opposite charge, compared to their matter Counterparts
- State that most of the matter in the observable universe is matter
- Describe the asymmetry of matter and anti -matter in the universe as an unsolved mystery
- Describe annihilation reactions [a particle meets its corresponding anti -particle, they undergo annihilation reactions in which either all the mass is converted to heat and light energy, or some mass is left over in the form of new sub-atomic particles.]

We believe that all atoms are made up of neutrons, protons, and electrons. The antiparticles of these three particles are also known. The positron (a positive electron), the neutrino, and the photon are also known. By the end of 1960s, many new types of particles similar to the neutron and the proton were discovered. These were called *mesons* whose masses were mostly less than nucleon masses but more than the electron mass. Afterwards, other mesons were also found that have masses greater than nucleons. Physicists started to look for more fundamental particles which must have even smaller constituents which was later confirmed by experiments. These were named as *quarks*. We will discuss in this chapter, the basic building blocks of matter.

12.1 STRUCTURE AND PROPERTIES OF THE NUCLEUS

The atomic nucleus comprises two types of particles: protons and neutrons. A **proton** is the nucleus of the simplest atom, hydrogen, called a **protium.** The proton has a positive charge of the same magnitude as that of the electron (1.6×10^{-19} C) and its mass is 1.67×10^{-27} kg. The neutron, whose existence was pointed out in 1932 by James Chadwick, is electrically neutral as its name suggested and its mass is nearly the same as that of proton. Now we can say that the nucleus has two types of particles (neutrons and protons) called nucleons.

Besides hydrogen, the nuclei of all other elements consist of both neutrons and protons. The different nuclei are called **nuclides**. The number of protons in a nucleus is called the **atomic number** represented by the symbol Z. The total number of nucleons, the sum of neutrons and protons, is represented by the symbol A and is called the **atomic mass number**, or simply **mass number**. It is written as

$$A = N + Z \quad \text{.............. (12.1)}$$

where N represents the neutron number.

In order to specify the given nuclei, the symbol X is commonly used as $^{A}_{Z}X$, where X is the chemical symbol for the element. To indicate the mass of atomic particles, instead of kilogram, **unified mass scale** (u) is generally used. By definition 1u is exactly one twelveth the mass of carbon-12 atom ($1u = 1.6606 \times 10^{-27}$ kg = 931 MeV). Using this value of u, the mass of a proton is 1.007276 u and that of a neutron is 1.008665 u while that of an electron is 0.00055 u.

For a particular atom (e.g., carbon), nuclei are found to contain different numbers of neutrons, although they all have the same number of protons. For example, carbon nuclei always have 6 protons, but they may have different number of neutrons. Nuclei that contain the same number of protons but different numbers of neutrons are called **isotopes**. Isotopes of carbon are $^{12}_{6}C$, $^{13}_{6}C$, and $^{14}_{6}C$, amongst them $^{12}_{6}C$ and $^{13}_{6}C$ are stable but $^{14}_{6}C$ is unstable and decays into nitrogen along with the emission of β and neutrino

12.2 FUNDAMENTAL FORCES OF NATURE

To understand the structure of the nucleus, it is important to know the nature of the forces that bind the nucleon together. But before that, we should know the basic forces in nature. Despite the apparent complexity within the universe, all interactions in the universe are governed by the four basic forces, known as fundamental forces. These forces control how objects move, interact and behave at different scales from nucleons in the atom to massive galaxies. The four fundamental forces are gravity, electromagnetism, weak nuclear force, and strong nuclear force.

Gravitational force or gravity is one of the four fundamental forces of nature. It is the weakest of the four but it is a long-range force. It is an attractive force and arises due to the gravitational interaction between the bodies. The gravitational force between two bodies is proportional to the product of their masses and inversely proportional to the square of the distance between them. When considered for massive objects, such as the Sun, or giant planets, gravitational force is considered to be significant as the masses of these objects are large. However, on an atomic level, this force is considered to be negligibly weak.

The **electromagnetic force** is responsible for electric field and magnetic field interactions. Like the gravitational force, the electromagnetic force follows an inverse square law but is much stronger than gravity. It governs a vast range of phenomena, from atomic structure, chemical bonding, electricity, magnetism, and light propagation. **James Clerk Maxwell** (1861) formulated a set of four fundamental equations named as "Maxwell's equations" that unified electricity and magnetism into **electromagnetism**. These equations describe how electric and magnetic fields interact and how electromagnetic waves propagate. These equations showed that electric and magnetic fields are not separate forces but are two aspects of a single **electromagnetic force**.

Out of the four fundamental forces, **nuclear forces** are the strongest attractive forces. Electromagnetism holds the matter together, but there was no explanation on how the nucleus is held together in the atom. If we only consider the forces of electromagnetism and gravity, the nucleus should fly off in different directions. The stability of the nucleus implies that another force should exist within the nucleus which is stronger than the gravitational force and electromagnetic force. This is where nuclear forces come into play. **Strong nuclear forces** are responsible for holding the nuclei of atoms together. They only exist inside the nucleus. So, we call them as short-range forces. The strong nuclear force acts as an attractive force between all nucleons. Thus, protons attract each other via the strong nuclear forces at

Table 12.1
Relative strength and range of four fundamental forces

Force	Approximate Relative Strength (compared to strong force)	Range
Gravity	10^{-38}	∞
Weak nuclear forces	10^{-13}	$<10^{-18}$ m
Electromagnetic	10^{-2}	∞
Strong nuclear forces	1	$<10^{-15}$ m

the same time they repel each other via the electric force. A neutron, being electrically neutral, can attract other neutrons or protons via the strong nuclear force. **Weak nuclear forces** are responsible for the radioactive decay, particularly the beta decay and interactions involving neutrino. Unlike the other fundamental forces, the weak force can **change the identity of particles**, making it essential for processes like nuclear fusion in stars and the decay of unstable atomic nuclei. The relative strength and range of the above four forces are given in Table 12.1.

12.3 MATTER AND ANTI-MATTER

It was predicted by Paul Dirac in 1928 that the fundamental particles have their anti-particles. The rest masses of the anti-particles are the same as that of their corresponding particles but with opposite charges and magnetic moments. For example, positron is the anti-particle of an electron. It is represented by e^+. The rest mass of the positron is the same as that of an electron but it carries positive charge with magnitude the same as that of an electron. It is noted that the positron was the first discovered anti-particle by Anderson in 1932 in a cloud chamber experiment. This was first experimental discovery of an anti-particle. After that a lot of anti-particles were discovered. Usually, the anti-particles are represented by a letter with a bar over it, e.g., anti-proton is represented by $\bar{p}$ anti-neutrino by $\bar{\nu}$ and so on.

The quarks and leptons have been recognized as the fundamental particles also known as elementary particles among the too many discovered particles. These elementary particles have also anti-particles.

For your information

1. A particle accelerator is a huge machine that accelerates charged particles, such as electrons, protons, or ions, to extremely high energies and speeds, approaching the speed of light.
2. Linear Accelerators, Cyclotrons and Betatrons are important particle accelerators.

Interesting information

For their work on this discovery, Dirac and Anderson received the Nobel Prize in Physics-Dirac in 1933, and Anderson in 1936. In 1955, Segre and Chamberlain discovered anti-proton using a particle accelerator and were awarded the Nobel Prize in physics in 1959 for their discovery of anti-proton.

Do you know?

(i) The cosmic rays are high-energy particles coming from the outer-space with unknown sources. Their source may be the Sun or the other stars. These particles consist mostly of protons, neutrons and heavier nuclei, which are continually bombarding the Earth. When these particles interact with the atoms of the gases of the Earth's atmosphere, they produce showers of secondary particles which rain down on us all the time.

(ii) When nuclei of unstable radioactive element say ^{235}U undergo fission reactions in the nuclear reactors, they emit a variety of particles, such as; neutrons, neutrinos, α-particles, photons, electrons and positrons.

(iii) When the charged particles, such as electrons and protons are accelerated by an accelerator and then bombarded on the target material, which is hydrogen, these accelerated charged particles may also collide head-on with each other. As a result, the debris from these reactions contain particles like pions, kaons, muons and even anti-protons.

Pair Production

Pair production occurs when a γ-ray (high energy photon) passes nearby an atomic nucleus. As a result, an electron-positron pair is emitted as shown in Fig. 12.1. The presence of a third particle, such as a nucleus, is necessary to conserve linear momentum. According to the law of mass-energy equivalence, the minimum energy of a photon for pair production must be equal to the sum of the rest mass energies of the created particles. The rest mass energy of the electron-positron pair is $2m_oc^2$ = 1.02 MeV which has been verified experimentally. A gamma ray photon with energy less than 1.02 MeV cannot produce an electron-positron pair whereas a photon with energy greater than 1.02 MeV creates an electron-positron pair and the excess energy goes into the kinetic energies of the particles.

The process of pair production satisfies the laws of conservation of charge, momentum and energy. It can occur for any particle and anti-particle.

Do you know?

The pair production cannot take place in vacuum or space. The pair production can happen only in the presence of an external object like an atomic nucleus which can experience some recoil during the collision process to conserve the energy and the momentum at the same time.

Annihilation of Matter

It is the opposite process of pair production. For example, when an electron and a positron interact to each other, they annihilate into two gamma ray photons as shown in Fig. 12.2. The reaction can be written as

$$e^- + e^+ \rightarrow \gamma + \gamma$$

The energy of each gamma ray photon is 0.51 MeV which is equal to the rest mass energy of an electron or a positron, i.e. $E=m_oc^2$. In an annihilation reaction, energy and momentum are conserved. Besides the electron and positron annihilation, the annihilation reactions of other particles and their anti-particles can also be carried out e.g., proton and anti-proton, lepton and anti-lepton, quark and anti-quark, etc.

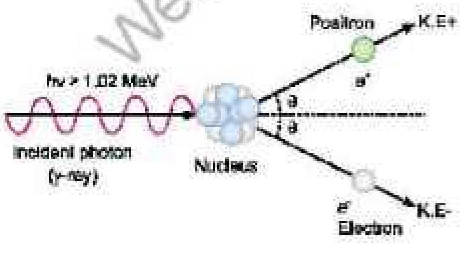


Fig. 12.1: A high energetic photon (γ-ray) interacting with a nucleus and results into an electron and a positron pair.

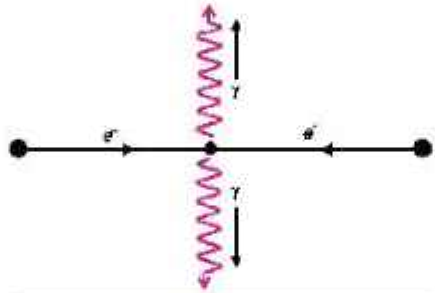


Fig. 12.2: Annihilation of matter

A particle accelerator, named as **Large Hadron Collider (LHC)** at **CERN** have revealed that some mass of colliding particles is changed to electromagnetic radiation according to Einstein's equation and left over mass appears in the form of new sub-atomic particles (Fig. 12.3).

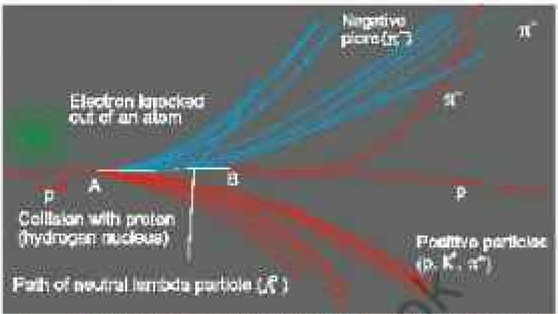


Fig. 12.3: A high energetic $p\bar{p}$ collision producing 18 new particles.

12.4 RADIOACTIVITY

It has been observed that the nuclei whose atomic numbers are greater than 82 are found naturally unstable, and these nuclei spontaneously emit radiations. Such nuclei are called radioactive and the emission of radiation is known as natural radioactivity. These radiations are of three types, α, β and γ-radiations. Unstable isotopes can also be produced artificially in the laboratory by nuclear reactions. This occurs when a stable element is bombarded with high-energy particles, such as neutrons, protons, alpha particles, or gamma rays, causing it to become unstable and emit radiation. This is called artificial radioactivity and radioactive isotopes are named **radioisotopes** or **radionuclides**.

The α-particles, β-particles and γ-radiations are traversed differently when passed through the electric field as shown in Fig.12.4. It is seen that α-particles deflect towards the negative terminal of the electric field, showing that they have a positive charge. The α-particles are emitted at high speeds, typically a few percent of the speed of light. However, α-particles can travel only several centimetres in the air due to their large mass. The β-particles deflect towards the positive terminal of the electric field, showing

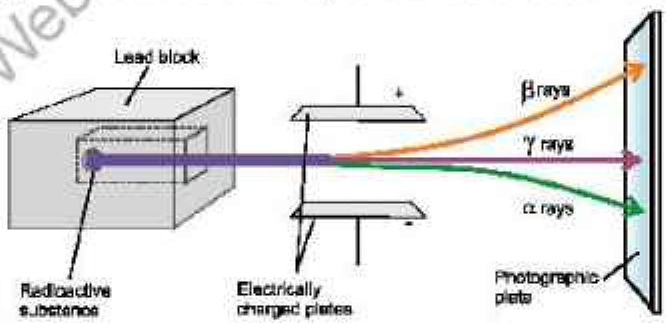


Fig. 12.4: The three radioactive radiations, namely alpha, beta and gamma rays

they have a negative charge. The deflection of β-particles is more than the α-particles, proving that they are lighter particles than α-particles. The β-particles are fast-moving electrons and move with speeds up to 0.9995 of the speed of light. The γ-radiations passed through the electric field without deflection, showing that they have no charge. The γ-radiations are electromagnetic radiations which consist of photons. They move with the speed of light with the highest penetrating power but the lowest ionization power.

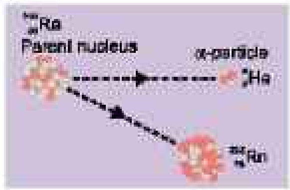
The process of emitting α-particles, β-particles and γ-radiations from the nucleus is called α-decay, β-decay and γ-decay, respectively, and are discussed below.

α-Decay

If the nucleus has more protons than the number of neutrons, the electrostatic force of repulsion becomes greater than the strong nuclear force of attraction. In this case, the nucleus becomes unstable and emits alpha particles in radioactive decay. An α-particle is equivalent to a helium (He) nucleus which consists of two protons and two neutrons. This means the nucleus loses two protons and neutrons in the α-decay. Hence, the atomic number Z decreases by 2 while its mass number A decreases by 4. Alpha decay can be written in general as:

$$^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}X + {}^{4}_{2}He \quad \text{.................} \quad (12.2)$$

Here $^{A}_{Z}X$ is the parent nucleus which decays into the daughter nucleus $^{A-4}_{Z-2}X$ and $^{4}_{2}He$ is the alpha particle. In the α-decay, it is experimentally observed that the number of nucleons (A) and electric charge are conserved. An example of α-decay is given below:



A radium-226 isotope ($^{226}_{88}Ra$) emits an alpha particle and decays into a daughter nucleus radon-222 ($^{222}_{86}Rn$).

$$^{226}_{88}Ra \rightarrow {}^{222}_{86}Rn + {}^{4}_{2}He \quad \text{.................} \quad (12.3)$$

In the above nuclear reaction, the **daughter** nucleus ($^{222}_{86}Rn$) is different from the **parent** nucleus ($^{226}_{88}Ra$). This transition of one element into another is called the **transmutation** of the elements. It is experimentally found that the mass of the parent nucleus is greater than the total mass of the daughter nucleus and the mass of the α-particle. Thus, the total mass-energy ($E=mc^2$) of the decay products is less than the mass-energy of the original nuclide. This difference in mass-energy is called the **disintegration energy** *Q*, or the **Q-value** of the decay.

β-Decay

There are two types of β-decay; β^--decay and β^+-decay.

(i) β^--Decay

Some nuclides have neutron-to-proton ratio (N/P) too large and are the source of β^--decay. The β-particles are not the orbital electrons but they are created within the nucleus at the moment of emission. In this process, a neutron in the nucleus decays into a proton and an electron, plus another particle called anti-neutrino which is the anti-particle of neutrino. The neutrino is denoted by a Greek symbol ν (nu) and anti-neutrino is denoted by a bar over the $\bar{\nu}$. The decay process is given by the following relation:

$$^{1}_{0}n \rightarrow ^{1}_{1}p + ^{0}_{-1}e + \bar{\nu}$$

One of the neutrons changes to a proton and in order to conserve charge it emits an electron. These electrons are called beta particles. However, they are indistinguishable from orbital electrons. Both the neutrino and the anti-neutrino have zero charge and very small mass, that is why they are very difficult to observe when passing through the matter. No nucleons are lost when a β-particle is emitted, and the total number of nucleons A remains the same but the mass number Z changes. Beta decay process can be written as:

$$^{A}_{Z}X \rightarrow ^{A}_{Z+1}X + ^{0}_{-1}e + \bar{\nu}$$

From the above equation, it is clear that the parent element of atomic number Z is transmuted to another element of atomic number (Z+1). An example is the isotope of thorium, which is unstable and decays into protactinium by beta emission. The reaction is represented as:

$$^{A}_{Z}Th \rightarrow ^{239}_{91}Pa + ^{0}_{-1}e + \bar{\nu}$$

For your information

The neutrino was first proposed by Wolfgang Pauli in 1930 to obey the energy conservation in the beta-minus and beta-plus decays. Later in 1953, neutrino was detected by F. Reines and C. L. Cowan in a high-power nuclear reactor. On this discovery, F. Reines received the Nobel prize in 1995.

(ii) β⁺-Decay

There are also nuclides that have neutron-to-proton ratio (N/P) too small for stability and decay by emitting a positron instead of an electron. The positron (e+) has the same mass as the electron but it has a positive charge. The positron is the anti-particle of the electron. In this process, a proton in the nucleus decays into a neutron and a positron, plus a neutrino. The generalized decay is given below:

$$^{A}_{Z}X \rightarrow ^{A}_{Z-1}X + ^{0}_{+1}e + \nu$$

An example of a decay of Neon into Flourine by emitting positron and neutrino is:

$$^{19}_{10}Ne \rightarrow ^{19}_{9}F + ^{0}_{+1}e + \nu$$

Energy of Alpha and Beta Particles in Radioactive Decay

In both α-decay and β-decay, for a particular radionuclide, the same amount of energy is released. In α-decay of a particular radionuclide, every emitted α-particle has the same sharply defined kinetic energy. When the number of α-particles is plotted against kinetic energy, there are distinct spikes that appear on the graph as shown in Fig. 12.5. This demonstrates that **α-particles have discrete energies.**

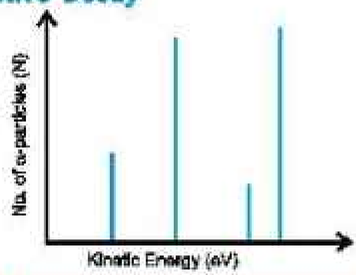


Fig. 12.5: Discrete energy values of α-particles

However, in the case of β-particle emission, energy is shared between β-particle and anti-neutrino in varying proportions. The sum of electron (or positron) energy and the anti-neutrino's (or neutrino's) energy, however, in every case remains the same. Thus, in β-decay, the energy of an electron or a positron may range from zero to a maximum value. When the number of β-particles is plotted against kinetic energy, the graph shows a curve as shown in Fig. 12.6. This demonstrates that beta particles (electrons or positrons) have a continuous range of energies. The principle of conservation of momentum and energy applies in both alpha and beta emission.

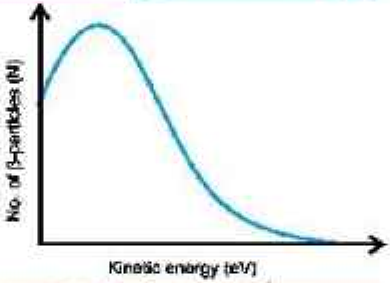


Fig. 12.6: Continuos spectrum of β-particles

(iii) γ-Radiation

The emission of γ-radiation from a nucleus is generally represented by this equation:

$$^{A}_{Z}X^{*} \rightarrow {}^{A}_{Z}X + \gamma$$

where $^{A}_{Z}X^{*}$ represents an excited nucleus while $^{A}_{Z}X$ shows ground state of the nucleus.

Table 12.2: The summary of nature of alpha, beta and gamma radiations

Characteristics	α-particles	β-particles	γ-rays
1. Nature	Helium nuclei of charge 2e	Electrons or positrons from the nucleus of charge ±e	E. M. waves from excited nuclei with no charge
2. Typical sources	Radon-222	Strontium-94	Cobalt-60
3. Ionization (ion pairs mm^{-1} in air)	About 10^4	About 10^2	About 1
4. Range in air	Several centimetres	Several metres	Obeys inverse square law
5. Absorbed by	A paper	1-5 mm of Al sheet	1-10 cm of lead sheet
6. Energy spectrum	Emitted with the same energy	Variable energy	Variable energy
7. Speed	$\sim 10^7$ m s^{-1}	$\sim 1 \times 10^8$ m s^{-1}	$\sim 3 \times 10^8$ m s^{-1}

12.5 FUNDAMENTAL PARTICLES

By the term **fundamental particle**, we mean a particle that has no internal structure, which means that it is indivisible. Presently, the fundamental constituents of matter are considered to be **quarks** (protons, neutrons and mesons are made up of quarks) and **leptons** (including electrons, positrons, and neutrinos). They are considered the basic building blocks of matter.

When a nucleus is smashed in an ultra high energy particle accelerator, or two high energy particles are collided, entirely new types of particles are created which apparently do not exist within the atoms of the ordinary matter. They are the outcome of the violent collisions needed to probe the basic structure of matter. More than a hundred new particles have been identified to be classified into families with similar properties. Many of these were accounted well with the scheme of theoretical physicist while the rest were named "strange particle". They are always created in pairs, e.g., when a pion (π^-) collides with a proton, two strange particles k^0 (Kaon), and Λ^0 (Lamda) are created. The nuclear reaction is:

$$\pi^- + p \rightarrow k^0 + \Lambda^0$$

All particles spin on their axes and the spin of charged particles makes them tiny magnets. The characteristic spin of electrons, protons, neutrons is 1/2 and the spin of photon is 1 and of pions is taken as zero. Half spin particles obey the Pauli's exclusion principle which says that only one particle of a kind occupies a given quantum state. These particles are called **"fermions"**. The particles with zero or whole number spin do not obey this principle. They are called **"bosons"** as they obey Bose-Einstein statistics. Further major classifications are:

1. The nucleons and the heavier particles such as Λ^0 and k^0 which decay to nucleons are called **"baryons"** (heavy).
2. The particles that do not interact strongly with nucleons, and are called **leptons** (small) along with the electrons, tau and neutrino.

Both of the β-decay (beta-minus and beta-plus) processes provide evidence that the protons and neutrons are not the fundamental particles. By the 1960s many new types of particles similar to the neutron and proton were discovered, as well as many "midsized" particles called *mesons* whose masses were mostly less than nucleon masses but more than the electron mass (other mesons, found later, have masses greater than nucleons). The strongly attractive particles are called π meson or pions while the weakly interacting particles were named μ mesons or muons.

This discovery led to the conclusion that these particles could not be fundamental particles, and must be made up of even smaller constituents , which were given the name *quarks*.

Hadrons and Leptons

Particles can also be classified based on the four fundamental forces that act on them. While the gravitational force affects all particles. Its impact at the subatomic level is so minimal that it is generally disregarded at the sub-atomic level. The electromagnetic force, which acts on all electrically charged particles, is well understood and can be considered when necessary; however, in this chapter, we will largely ignore its effects.

Particles can be broadly classified based on whether they interact via the strong force or do not. Those that experience the strong force are known as **hadrons**, while those that do not, are called **leptons**. Examples of hadrons include protons, neutrons, and pions, whereas electrons and neutrinos are classified as leptons.

Hadrons are composite subatomic particles that can be further divided into two broad categories: some are bosons, referred to as **mesons** such as pion, while others are fermions, known as **baryons**, with protons and neutrons being the key examples. Baryons are made of an odd number of quarks (usually three quarks), and mesons are made up of an even number of quarks (usually two quarks: one quark and one anti-quark).

The leptons interact only through weak or electromagnetic interactions. No experiments have yet been able to reveal any internal structure for the leptons; **they appear to be truly fundamental particles** that cannot be split into smaller particles. All known leptons have spin ½, so they all are fermions. The **six known leptons** are grouped as three pairs of particles as shown in Table 12.3. Each pair includes a charged particle (e^-, μ^-, τ^-), its associated neutrino and corresponding anti-particles. Charged leptons can combine with other particles to form various composite particles such as atoms and positronium, while neutrinos rarely interact with anything, and are consequently rarely observed. The best-known of all leptons is the electron.

Table 12.3: The lepton family

Family	Particle	Symbol	Mass (MeV/c^2)	Charge *q*	Antiparticle
Electron	Electron Electron neutrino	e^{-1} ν_e	0.511 » 1×10^{-7}	-1 0	e^{+1} $\bar{\nu}_e$
Muon	Muon Muon neutrino	μ^{-1} ν_m	105.7 » 1×10^{-7}	-1 0	μ^{+1} $\bar{\nu}_m$
Tau	Tau Tau Neutrino	τ^{-1} ν_τ	1777 » 1×10^{-7}	-1 0	τ^{+1} $\bar{\nu}_\tau$

12.6 QUARKS

In 1964, M. Gell-Mann and George Zweig proposed that none of the hadrons, not even the proton and neutron, are truly fundamental, but instead are made up of combinations of three more fundamental entities called **quarks** or **quark flavours**. Quarks are considered to be truly fundamental particles, like leptons. Three quarks originally proposed were named **up**, **down**, and **strange**, with abbreviations **u, d and s,** respectively. Presently, we are aware of six quarks, just as there are six leptons-based on a presumed *symmetry* in nature. The other three quarks are called **charm**, **bottom**, and **top (c, b, t)**. These new quarks can be distinguished from the 3 original quarks (Table 12.4). All quarks have a spin and an electric charge (a fraction of the previously thought smallest charge e on an electron). Quarks are invisible. They never appear on their own.

All hadrons are considered to be made up of combinations of quarks (plus the gluons that hold them together), and their properties are described by looking at their quark content. Mesons consist of a quark–antiquark pair. For example, a π^+ meson is a ud combination. A π^0 can be made of u$\bar{u}$).

Each baryon, on the other hand, consists of three quarks, such as:

The proton has a quark composition of uud and so its charge quantum number is:

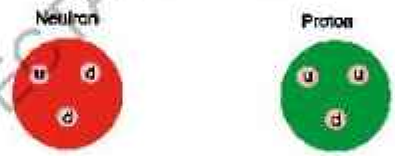
$$q(\text{uud}) = \frac{2}{3} + \frac{2}{3} + \left(-\frac{1}{3}\right) = +1$$

Neutron has a quark composition of udd and its charge quantum number is:

$$q(\text{udd}) = \frac{2}{3} + \left(\frac{1}{3}\right) + \left(-\frac{1}{3}\right) = 0$$

Table 12.4: Quark flavours

Particles	Symbol	Charge (q)	Mass (MeV/c²)	Anti - Particle	Charge (q)
Up	u	$+\frac{2}{3}$	5	$\overline{m}$	$-\frac{2}{3}$
Down	d	$-\frac{1}{3}$	10	$\overline{d}$	$+\frac{1}{3}$
Charm	c	$+\frac{2}{3}$	1500	$\overline{c}$	$-\frac{2}{3}$
Strange	s	$-\frac{1}{3}$	200	$\overline{s}$	$+\frac{1}{3}$
Top	t	$+\frac{2}{3}$	175000	$\overline{t}$	$-\frac{2}{3}$
Bottom	b	$-\frac{1}{3}$	4300	$\overline{b}$	$+\frac{1}{3}$



Mesons are quark–antiquark pairs. Consider the meson π^+, which consists of an up-quark u and an antidown quark $\bar{d}$. We see that the charge quantum number of the up quark is +2/3 and that of the antidown quark is +1/3. This adds nicely to a charge quantum number of +1 for the π^+ meson; that is, q(u) = 2/3 + 1/3 = +1.

Not long after the quark theory was proposed, it was suggested that quarks have another property (or quality) called **colour**, or "**colour charge**" (analogous to electric charge). According to this theory, each flavour of quark can have one of three colours, usually designated red, green, and blue. Note that the names "colour" and "flavour" have nothing to do with our senses, but are purely whimsical—as are other names, such as charm, in this new field. The anti-quarks are coloured antired, antigreen, and antiblue. Baryons are made up of three quarks, one of each colour. Mesons consist of a quark–anti-quark pair

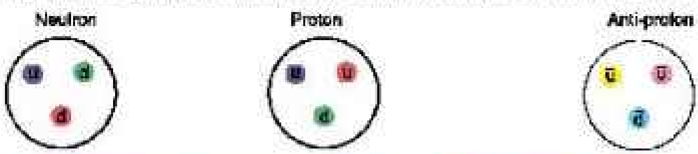


Fig. 12.7 (a): Colourless baryons: blue + red + green = white

Fig. 12.7 (b): A colourless anti-baryons: anti-blue + anti-red + anti-green = white of a particular colour and its anti-colour. Both baryons and mesons are thus colourless or white. Each quark is assumed to carry a *colour charge*, analogous to an electric charge, and the strong force between quarks is referred to as the **colour force**. This theory of the strong force is called **quantum chromodynamics** or **QCD**, to indicate that the force acts between colour charges (and not between, say, electric charges). The strong force between two hadrons is considered to be a force between the quarks that make them up.

Beta Decay in Terms of Quarks

When a neutron in the nucleus decays into a proton and an electron (beta particle), an anti-neutrino is also produced in the reaction and the decay process is given as:

$$ {}_{0}^{1}n \rightarrow {}_{1}^{1}p + {}_{-1}^{0}e + \bar{\nu} $$

Now, we can identify that a neutron with composition udd can convert into a proton with composition uud by changing a down quark into an up quark. The fundamental decay process can now be expressed as:

$$ d \rightarrow u + {}_{-1}^{0}e + \bar{\nu} $$

Do you know?

A neutron is stable only inside a nucleus. Free neutrons decay with a half-life of 900 s.

Thus, as our understanding of the fundamental nature of matter deepens, we can analyze familiar processes at increasingly intricate levels. The quark model not only enhances our comprehension of particle structures but also provides insight into their interactions.

12.7 HIGGS BOSON

Fundamental particles are considered to be the six quarks, the six leptons and the gauge bosons (Higgs bosons), which are the carriers of the fundamental forces. Leptons and quarks interact with each other by sending and receiving bosons. For example, electromagnetic interactions occur when two positively charged particles send and receive (exchange) photons. The photons are said to "carry" the force between charged particles. Likewise, attraction between two quarks in an atomic nucleus occurs when two quarks send and receive gluons. Similarly, the W^+, W^- and Z are the bosons that are the carriers of weak nuclear force and gravitons are the carriers of gravitational force. The Higgs boson is a special particle discovered in 2012 at large hadron collider at CERN. It is associated with the Higg's field that permeates all of the space. Its crucial role is that it provides explanation for how the other particles get mass by interacting with it.

The particles that interact strongly with Higg's field get more mass such as carriers of electroweak interaction i.e., W^+, W^- and Z bosons whereas the particles like photons do not interact with Higg's field and hence, their rest mass is considered zero. Higgs boson has a mass of around 125 giga-electron-volts (GeV/c^2) and decays rapidly into other particles. Here are some key facts:

1. The Higgs boson gets its mass from its interactions with its associated Higg's field.
2. It can be a unique portal to find the conditions of universe shortly after the Big Bang and signs of dark matter due to its distinctive characteristics and properties.

12.8 CONSERVATION LAWS

All nuclear processess such as nuclear reactions and nuclear decays obey conservation laws such as:

1. conservation of energy, momentum and charge. It includes the nucleon number N and charge number Z.
2. baryon number
3. lepton number

For your information

Hadrons

Mesons, e.g. pions, kaons

baryons, e.g. protons, neutrons, omega, sigma and lambda particles

Non-hadrons

Leptons, including electrons, muon, neutrinos

photons, gravitons

12.9 THE ASYMMETRY OF MATTER AND ANTI-MATTER IN THE UNIVERSE

Many observations show that there is the asymmetry between matter and anti-matter. This is the most remarkable features of our universe. A number of hypothesis are existed which show that the universe consists almost entirely of matter rather than anti-matter. e.g., the universe is assumed to be composed up 5% of ordinary matter (electrons, protons and neutrons), 71% of hydrogen atoms and 24% of helium atoms, there are no any contribution of anti-hydrogen or anti-helium atoms in the composition of the universe.

The experimental results explain that the matter and anti-matter asymmetry is indeed due to the violation of conservation of baryon number. i.e., there is imbalance number of baryons and its anti-baryons. Now if the particle-antiparticle symmetry is also violated, then there will be a mechanism for making more quarks than anti-quarks, more leptons than anti-leptons and eventually more matter than anti-matter. Hence, it is concluded that the problem asymmetry of matter and anti-matter is still mystery.

12.10 MOST OF THE MATTER IN THE OBSERVABLE UNIVERSE IS PLASMA

Our universe is more vast than our thinking, because we still know about its 5% part but we still do not know about its remaining 95% part. For example, our universe consists of about 27% of unknown matter called dark matter and about 68% a mysterious antigravity material known as dark energy. By adding 27% and 68% we have 95%. It means 95% universe is out of our thinking, i.e., we know nothing about 95% of the universe.

On the other hand, the 5% of the universe that we know about it, is an ordinary matter, where hydrogen and helium are almost in the plasma state. Therefore, these figures indicate that 95% of the 5% of the observable universe is the plasma state, while the remaining 5% is in the form of matter. Hence, it is concluded that most of the matter in the observable universe is plasma.

12.11 THE THEORIES ABOUT THE FORCES BETWEEN THE MASSES OF PARTICLES

To explain the interactions between the masses of the particles through different mediators, we have the following two theories, the quantum field theory and the string theory.

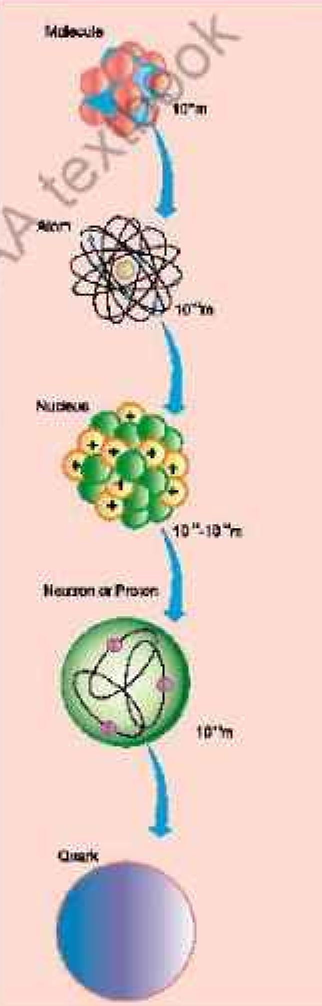
1. The Quantum Field Theory

According to this theory, each particle is represented by a field called quantum field and it is responsible to transmit a force from one particle to another by a mediator. For example, let a positively charged particle produces an electric field in the space around it. This charged particle exerts an attractive force on a nearby negatively charged particle through its field. Moreover, the field can also carry energy and momentum from one particle to another. Where the energy and momentum of all fields are quantized. The quanta that exchange momentum and energy from one type of particle to another in their field are called field particles. Thus, we can say that the interactions between particles are described in terms of the exchange of field particle or quanta which are all bosons. For example, the electromagnetic force is mediated by photons called quanta of the electromagnetic field. Similarly, the strong nuclear force is mediated by field particles called gluons, the electro-weak force is mediated by the field particles called Bosons (W and Z) and the gravitational forces is mediated by field particles called gravitons.

2. String Theory

String Theory is an advanced concept in theoretical physics proposing that the fundamental particles of the universe, instead of being point-like, are actually tiny, vibrating strings. These strings can be open or closed loops, and their vibrations

For your Information



determine the properties of particles, including mass and force. String Theory framework, offering a potential theory of everything, still remains unproved experimentally.

12.12 THE STANDARD MODEL

The standard model is the collection of theories that describe the smallest experimentally observed particles of matter and interaction between energy and matter. Three categories of particles form the standard model are shown in Fig. 12.8. Matter, which makes up only 5% of the universe is composed of quarks and leptons. The fundamental bosons provide three forces: electromagnetism, the strong nuclear force and the weak nuclear force.

Fig. 12.8: Elementary particles in the standard model

The Higg's boson provides an explanation for how the other particles get mass.

This model is still considered incomplete. Currently, it is unable to explain many important features of the known universe such as: (i) gravity (ii) dark matter (27% of the universe (iii) dark energy (68% of the universe).

QUESTIONS

Multiple Choice Questions

Tick (✓) the correct answer.

12.1 Which one of the following is the fundamental particle?

(a) Proton (b) Neutron (c) Electron (d) Meson

12.2 The first discovered anti-particle is:

(a) anti-proton (b) anti-neutrino (c) anti-photon (d) anti-electron

12.3 Which one of the following pair of particles creates annihilation?

(a) proton-proton (b) proton-neutron
(c) neutron-photon (d) electron-positron

12.4 The strong nuclear force between the two particles is mediated by:

(a) gluons (b) photon (c) mesons (d) gravitons

12.5 Which one of the following forces interacts between two particles through photons?

(a) Strong nuclear force (b) Weak force
(c) Electromagnetic force (d) Gravitational force

12.6 When a neutron changes into a proton, then we will observe:
(a) β^--decay (b) β^+-decay (c) γ-decay (d) α-decay

12.7 Baryon is formed by combination of:
(a) 2 quarks (b) 3 quarks
(c) 4 quarks (d) A quark and an anti-quark

12.8 Which one of the following forces has negligible effect between the elementary particles?
(a) Strong nuclear force (b) Weak force
(c) Gravitational force (d) Electromagnetic force

12.9 Which particles are produced by strong interaction?
(a) Graviton (b) Leptons (c) Hadrons (d) Mesons

12.10 A strong nuclear force exists between the nucleons of:
(a) p – p (b) n – n (c) p – n (d) all of these

12.11 Which one of the following radiation/particles has the highest ionization power?
(a) α (b) β^+ (c) β^- (d) γ

12.12 Which one of the following radiation/particles has the highest penetrating power?
(a) α (b) β^+ (c) β^- (d) γ

12.13 A change occurs in atomic member of a nucleus but its mass number remains the same by decay of:
(a) α (b) β (c) γ (d) α and γ

12.14 In a nucleus, a neutron changes into a proton, the atomic number changes by one, the mass number will:
(a) decrease (b) increase (c) remain the same (d) none of these

12.15 The electroweak theory was introduced by:
(a) Dirac (b) Einstein (c) Anderson (d) Dr. Abdul Salam

12.16 The asymmetry of matter and anti-matter is due to imbalance number of:
(a) hadron (b) lepton (c) baryon (d) photons

12.17 Which one of the following particle is responsible for the mass of the fundamental particle?
(a) Quarks (b) Anti-quark (c) Lepton (d) Higgs boson

12.18 A proton is composed of up and down quarks, the order of quarks is:
(a) udd (b) udu (c) uud (d) dud

12.19 The number of quarks that composed of a neutron is:
(a) 2 (b) 3 (c) 4 (d) 5

Short Answer Questions

12.1 What do different isotopes of a given element have in common? How are they different?

12.2 Identify the element that has 87 nucleons and 50 neutrons.

12.3 What are the similarities and differences between the strong nuclear force and the electromagnetic force?

12.4 Fill in the missing particle or nucleus:

(a) $^{45}_{20}Ca \rightarrow ? + e^{-} + \overline{\nu}$

(b) $^{56}_{29}Cu^{*} \rightarrow ? + \gamma$

12.5 Why neutrino must be released in the positron emission?

12.6 Distinguish between fermions and bosons.

12.7 How does strong force hold the nucleus?

12.8 Can there be pair production for photons having energy 20 keV? Explain briefly.

12.9 What is the difference between beta particle and electron?

12.10 How do a proton and a neutron convert to each other?

12.11 Why does beta-decay have a continuous energy spectrum and alpha-decay have a discrete energy spectrum?

12.12 Differentiate between hadron and leptons with examples.

12.13 Why electron-positron pair cannot decay into a single photon?

12.14 State the role of Higgs Boson in the generation of mass in modern physics theories.

12.15 What are Mesons? Give examples.

Constructed Response Questions

12.1 Is meson, a boson or fermion? Give reason.

12.2 Why does an alpha emitter emit alpha particles instead of four separate nucleons?

12.3 Which is more energetic alpha decay or beta decay? Justify your answer.

12.4 A nucleus undergoes gamma decay, emitting gamma ray photon with energy 1.5 MeV. Calculate.

(i) frequency of gamma ray

(ii) wavelength of gamma ray

(iii) momentum of gamma ray

12.5 Why does the α-particles not make physical contact with the nucleus when headed directly towards it?

Comprehensive Questions

12.1 What is meant by radioactivity? Compare the properties and behaviour of three types of radiations.

12.2 Elaborate the phenomenon of beta-positive decay and beta-negative decay with examples.

12.3 What is the difference between matter and anti-matter? Discuss reasons why our universe is almost entirely composed of matter.

12.4 Explain the phenomenon of pair annihilation with an example. Explain the utility of its principle in the medical field.

12.5 Explain the law of conservation of energy and momentum in electron-positron pair annihilation.

12.6 Describe protons and neutrons in terms of their quark composition.

12.7 Describe four fundamental forces in nature.

12.8 Describe the classification of elementary particles.

Numerical Problems

12.1 Uranium-238 is an alpha emitter. In the process, it is transmutated into a daughter nucleus. What is the mass number A and charge number Z of the daughter nucleus? What is its chemical symbol? [Ans: A=234, Z=90, Thorium(Th)]

12.2 Polonium $^{218}_{84}Po$ is a beta minus emitter. What will be the mass number A and charge number Z of the daughter nucleus? (Ans: A=218, Z=85)

12.3 Nitrogen $^{14}_{7}N$ bombarded by alpha particle results into $^{17}_{8}O$. What is the product particle in this nuclear reaction? (Ans: $^{1}_{1}H$ proton)

12.4 Show that nucleon number N and charge number Z are conserved in the numerical question 12.3.

12.5 Determine the rest-mass energy of electron in eV. Its rest-mass is 0.000555 u? (Ans: 0.517 MeV)

12.6 Calculate the Q-value for the reaction taking place in Rutherford's experiment on artificial disintegration of nitrogen by bombardment with alpha particles. Relative masses are:

$^{14}_{7}N = 14.007515\,u$, $^{4}_{2}He = 4.003837\,u$, $^{17}_{8}O = 17.004533\,u$ and $^{1}_{1}H = 1.008142\,u$

(Ans: -1.23 MeV)

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